2005 VCAA Specialist Mathematics Exam 1 Part I

Multiple-choice question 26

A particle initially at the origin starts from rest at $t = 0$. The particle moves in a **straight line** in such a way that its acceleration at time *t* is given by $e^{-0.1t}\,\boldsymbol{i} + (6t)\boldsymbol{j}$.

The velocity of the particle at time *t* is given by

A. $-\left(0.1e^{-0.1t}\right)\boldsymbol{i} + 6\,\boldsymbol{j}$

B. $-\left(10e^{-0.1t}\right)\boldsymbol{i} + \left(3t^2\right)\boldsymbol{j}$

C. $10\left(1 - e^{-0.1t}\right)\boldsymbol{i} + \left(3t^2\right)\boldsymbol{j}$

D. $0.1\left(1 - e^{-0.1t}\right)\boldsymbol{i} + \left(3t^2\right)\boldsymbol{j}$

E. $10\left(10 - t - 10e^{-0.1t}\right)\boldsymbol{i} + \left(t^3\right)\boldsymbol{j}$

None of the above choices for the velocity of the particle provides a straight line path. In fact a particle with the given acceleration will not move in a straight line.

VCAA suggested **C** is the correct answer, i.e. $\boldsymbol{v} = 10\left(1 - e^{-0.1t}\right)\boldsymbol{i} + \left(3t^2\right)\boldsymbol{j}$.
From there, $\boldsymbol{r} = 10\left(t + 10e^{-0.1t} - 10\right)\boldsymbol{i} + \left(t^3\right)\boldsymbol{j}$.
$\therefore x = 10\left(t + 10e^{-0.1t} - 10\right)$ and $y = t^3$.
$\therefore$ the cartesian equation for the path is $x = 10\left(\sqrt[3]{y} + 10e^{-0.1\times\sqrt[3]{y}} - 10\right)$.
The path is shown below. It is obviously not a straight line.