

$$Q1 \frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i} = \frac{4cis\left(\frac{\pi}{6}\right)}{2cis\left(-\frac{\pi}{3}\right)} = 2cis\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = 2cis\left(\frac{\pi}{2}\right)$$

$$Q2a \ f(z) = z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i$$

$$= z^2(z - (\sqrt{5} - i)) + 4(z - (\sqrt{5} - i))$$

$$= (z - (\sqrt{5} - i))(z^2 + 4)$$

$\therefore f(\sqrt{5} - i) = 0$ . Hence  $\sqrt{5} - i$  is a solution of

$$z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$$

Q2b The other two solutions are:

$$z^2 + 4 = 0, \ z = -i \text{ or } i$$

Q3 By implicit differentiation:

$$3x^2 - 4xy - 2x^2 \frac{dy}{dx} + 4y \frac{dy}{dx} = 0,$$

$$\frac{dy}{dx} = \frac{3x^2 - 4xy}{2x^2 - 4y}$$

At  $P(2,3)$ ,  $\frac{dy}{dx} = 3$ .

Equation of tangent:  $y - 3 = 3(x - 2)$ ,  $\therefore y = 3x - 3$

$$Q4 \ V = \int_{-\frac{1}{2}}^0 \pi y^2 dx = \pi \int_{-\frac{1}{2}}^0 \frac{1}{1-x^2} dx$$

$$= \frac{\pi}{2} \int_{-\frac{1}{2}}^0 \left( \frac{1}{1-x} + \frac{1}{1+x} \right) dx \quad (\text{partial fractions})$$

$$= \frac{\pi}{2} [-\log_e(1-x) + \log_e(1+x)]_{-\frac{1}{2}}^0$$

$$= \frac{\pi}{2} \left[ \log_e \frac{1+x}{1-x} \right]_{-\frac{1}{2}}^0 = \frac{\pi}{2} \left( -\log_e \frac{1}{3} \right) = \frac{\pi}{2} \log_e 3.$$

Q5a  $u = +4$ ,  $s = +3$ ,  $v = 0$ , use  $v^2 = u^2 + 2as$  to find  $a$ .

$$\therefore a = \frac{v^2 - u^2}{2s} = \frac{0 - 16}{6} = -\frac{8}{3} \text{ ms}^{-2}$$

Q5b Newton's second law:  $R = ma$ ,

$$-\mu N = 6 \left( -\frac{8}{3} \right), \quad -\mu(6g) = -6 \left( \frac{8}{3} \right), \quad \mu = \frac{8}{3g}.$$

$$Q6a \ r(t) = \int (-4 \sin(2t)\mathbf{i} + 6 \cos(2t)\mathbf{j}) dt,$$

$$r(t) = 2 \cos(2t)\mathbf{i} + 3 \sin(2t)\mathbf{j} + \mathbf{c}.$$

Given  $r(0) = 2\mathbf{i}$ ,  $\mathbf{c} = \mathbf{0}$ .  $\therefore r(t) = 2 \cos(2t)\mathbf{i} + 3 \sin(2t)\mathbf{j}$

Q6b  $x = 2 \cos(2t)$ ,  $y = 3 \sin(2t)$ .

$$\therefore \frac{x^2}{4} = \cos^2(2t) \text{ and } \frac{y^2}{9} = \sin^2(2t)$$

$$\therefore \frac{x^2}{4} + \frac{y^2}{9} = 1.$$

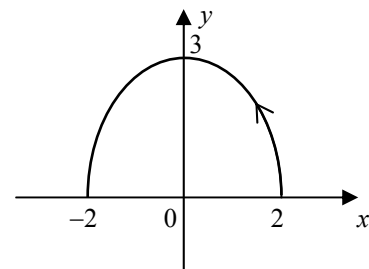
Given  $0 \leq t \leq \frac{\pi}{2}$ , when  $t = 0$ ,  $x = 2$ ,  $y = 0$ ;

when  $t = \frac{\pi}{4}$ ,  $x = 0$ ,  $y = 3$ ;

when  $t = \frac{\pi}{2}$ ,  $x = -2$ ,  $y = 0$ .

$$\therefore -2 \leq x \leq 2 \text{ and } 0 \leq y \leq 3.$$

Q6c



Q7a

$$x_0 = 1, \quad y_0 = 1$$

$$x_1 = 1.1, \quad y_1 \approx 1 + 0.1 \times \frac{1}{1} = 1 + 0.1 = 1.1$$

$$x_2 = 1.2, \quad y_2 \approx 1.1 + 0.1 \times \frac{1}{1.1} = 1.1 + \frac{0.1}{1.1} = \frac{1.31}{1.1} = \frac{131}{110}.$$

Q7b  $\frac{dy}{dx} = \frac{1}{x}$ ,  $y = \int \frac{1}{x} dx = \log_e x + c$ , where  $x > 0$ .

Since  $y(1) = 1$ ,  $\therefore c = 1$  and  $y = \log_e x + 1$ .

When  $x = 1.2$ ,  $y = \log_e(1.2) + 1$ .

Q8a

$y$	-2	-1	0	1	2
$\frac{dy}{dx}$	2.5	1	0.5	1	2.5

Note:  $\frac{dy}{dx} = \frac{1+y^2}{2}$  is independent of  $x$ .

Graph in part c.

$$\text{Q8b } \frac{dx}{dy} = \frac{2}{1+y^2}, \quad x = 2 \int \frac{1}{1+y^2} dy, \quad x = 2 \tan^{-1} y + c.$$

Given  $y = -1$  when  $x = 0$ ,

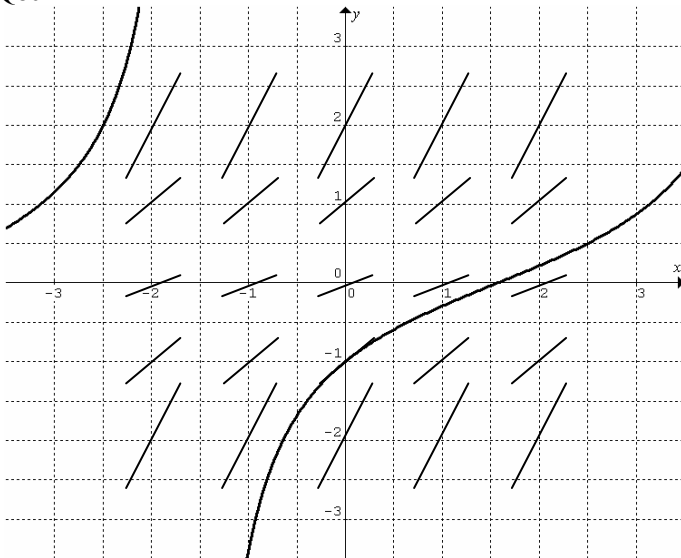
$$0 = 2 \tan^{-1}(-1) + c, \quad 0 = 2 \left( -\frac{\pi}{4} \right) + c.$$

$$\therefore c = \frac{\pi}{2} \text{ and } x = 2 \tan^{-1} y + \frac{\pi}{2}.$$

$$\text{Hence } y = \tan \frac{1}{2} \left( x - \frac{\pi}{2} \right).$$

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Q8c



$$\text{Q9 } \mathbf{r} = x\mathbf{i} + y\mathbf{j}, \quad \mathbf{v} = \frac{d}{dt} \mathbf{r} = \frac{dx}{dt} \mathbf{i} + \frac{dy}{dt} \mathbf{j}.$$

$$\text{Given } \mathbf{v} = -y\mathbf{i} + x\mathbf{j}, \quad \therefore \frac{dx}{dt} = -y \text{ and } \frac{dy}{dt} = x.$$

$$\mathbf{a} = \frac{d}{dt} \mathbf{v} = -\frac{dy}{dt} \mathbf{i} + \frac{dx}{dt} \mathbf{j}.$$

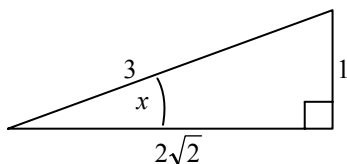
$$\therefore \mathbf{a} = -x\mathbf{i} - y\mathbf{j} = -(\mathbf{r}).$$

Hence  $\mathbf{a} = -\mathbf{r}$ .

$$\text{Q10 } \tan(2x) = \frac{4\sqrt{2}}{7}, \quad \frac{2 \tan x}{1 - \tan^2 x} = \frac{4\sqrt{2}}{7}, \text{ where } 0 \leq x < \frac{\pi}{4}.$$

$$\therefore \tan^2 x + \frac{7}{2\sqrt{2}} \tan x - 1 = 0, \text{ where } 0 \leq x < \frac{\pi}{4}.$$

$$\tan x = \frac{-\frac{7}{2\sqrt{2}} + \sqrt{\frac{49}{8} + 4}}{2} = \frac{1}{2\sqrt{2}}. \quad (\text{Quadratic formula})$$



From diagram,  $\sin x = \frac{1}{3}$ .