VCAA Mathematical Methods 34
Sample exam 1 solutions 2006

Q1a Rule of the original function $f(x) = 2\log_e(x+1)$, equation of the original function $y = 2\log_e(x+1)$. Interchange *x* and *y* to obtain equation of inverse function $x = 2\log_e(y+1)$. Make *y* the subject of the equation, $\log_e(y+1) = \frac{x}{2},\ y+1 = e^{\frac{x}{2}},\ y = e^{\frac{x}{2}} - 1$.

Hence rule of inverse function $f^{-1}(x) = e^{\frac{x}{2}} - 1$.

Q1b The domain of f^{-1} is the range of f, which is *R*.

Q2a $y = 3x^4 \tan x$, use the product rule,

$\frac{dy}{dx} = 12x^3 \tan x + 3x^4 \sec^2 x$.

Q2b $f'(x) = \frac{1}{x-2},\ f(x) = \int \frac{1}{x-2}dx = \log_e|x-2| + C$.

Given $f(1) = 6,\ \therefore \log_e|-1| + C = 6,\ \therefore C = 6$.

Hence $f(x) = \log_e|x-2| + 6$.

Since $x < 2,\ \therefore f(x) = \log_e(2-x) + 6$.

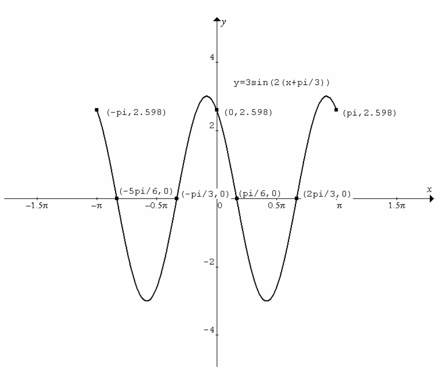
Q3 $\sin x = \sqrt{3}\cos x,\ -\pi \le x \le \pi.\ \frac{\sin x}{\cos x} = \sqrt{3},\ \therefore \tan x = \sqrt{3}$.

$\therefore x = -\frac{2\pi}{3}$ or $\frac{\pi}{3}$.

Q4a $f(x) = 3\sin\left(2\left(x+\frac{\pi}{3}\right)\right)$, amplitude 3, period $= \frac{2\pi}{2} = \pi$.

Q4b *y*-intercept: Let $x = 0,\ y = 3\sin\left(\frac{2\pi}{3}\right) = \frac{3\sqrt{3}}{2} = 2.598$.

x-intercepts: Before translation, $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$. After translation, $x = -\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$.



Q5a Given $\mu = 4,\ \sigma = 2,\ \Pr(X > 4) = 0.5$.

Q5b Since a normal distribution is symmetrical about its mean, $\therefore b = 3$.

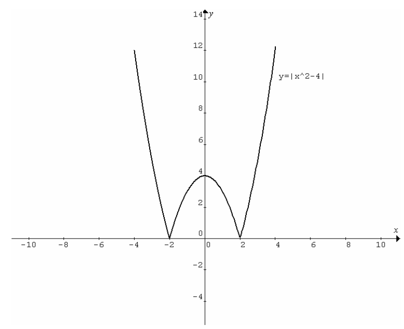
Q6a Since $f(x)$ is a probability density function,

$\therefore \int_{-\infty}^{\infty} f(x)dx = \int_0^2 ax(2-x)dx = 1,\ \left[a\left(x^2 - \frac{x^3}{3}\right)\right]_0^2 = 1$,

$\therefore a\left(\frac{4}{3}\right) = 1,\ a = \frac{3}{4}$.

Q6b $\Pr\left(X < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} \left(\frac{3}{4}x(2-x)\right)dx = \left[\frac{3}{4}\left(x^2 - \frac{x^3}{3}\right)\right]_0^{\frac{1}{2}} = \frac{15}{96}$.

Q7a



Q7b Area $= -4\left(\int_0^2 (x^2-4)dx\right) = -4\left[\frac{x^3}{3} - 4x\right]_0^2$

$= -4\left(\frac{2^3}{3} - 4(2)\right) = \frac{64}{3}$.

Q8a $f(x) = x^2 + 1,\ g(x) = \log_e(x)$,

$g(f(x)) = \log_e(f(x)) = \log_e(x^2+1)$.

Q8b $\frac{d}{dx}g(f(x)) = \frac{dg}{df} \times \frac{df}{dx} = \frac{1}{f(x)} \times 2x = \frac{2x}{x^2+1}$.

Q8c $\therefore \int \frac{2x}{x^2+1}dx = g(f(x)) = \log_e(x^2+1)$,

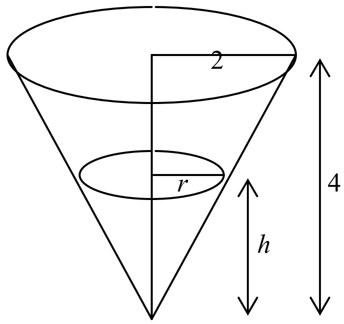
$\therefore 2\int \frac{x}{x^2+1}dx = \log_e(x^2+1)$,

$\therefore \int \frac{x}{x^2+1}dx = \frac{1}{2}\log_e(x^2+1)$.

Q9 Gradient of tangent $= \frac{dy}{dx} = 4,\ \therefore 4x^3 = 4,\ \therefore x = 1$.

At $x = 1,\ y = 4(1) - 1 = 3,\ \therefore 3 = 1^4 + c,\ \therefore c = 2$.

Q10



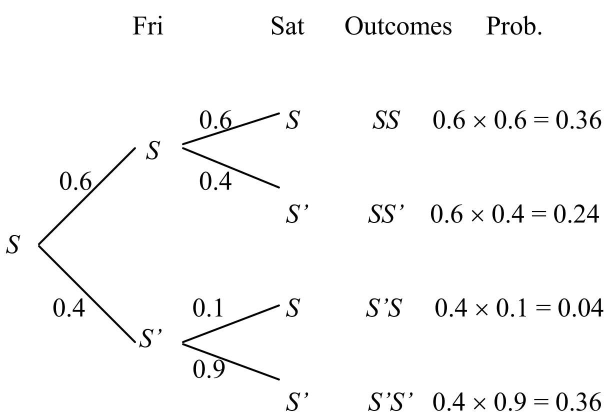
$\frac{r}{h}=\frac{2}{4}, \therefore r=\frac{h}{2}.$

Volume of water $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h=\frac{\pi}{12}h^3$.

$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}, \therefore 3=\frac{\pi}{4}h^2\times\frac{dh}{dt}, \therefore \frac{dh}{dt}=\frac{12}{\pi h^2}.$

When $h=3$, $\frac{dh}{dt}=\frac{12}{\pi 3^2}=\frac{4}{3\pi}$ metre per min.

Q11 Set up a tree diagram.



Probability that it will not snow on the following Saturday
$=\Pr(SS')+\Pr(S'S')=0.24+0.36=0.60.$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors