

Q1a 3.7

Q1b $x^3 - 27 = x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

Q1c $4x + \tan x$

Q1d $\frac{(2x-3)}{2} - \frac{(x-1)}{5} = \frac{5(2x-3) - 2(x-1)}{10} = \frac{8x-13}{10}$

Q1e $|x-3| \leq 1, 2 \leq x \leq 4$

Q1f $x^2 = 8(y-1) = 4 \times 2(y-1)$, vertex (0,1), focus (0,3)

Q2a $\cos \theta = \frac{1}{\sqrt{2}}, \theta = \frac{\pi}{4}, \frac{7\pi}{4}$

Q2bi $\frac{d}{dx}(x \sin x) = x \cos x + \sin x$

Q2bii $\frac{d}{dx}\left(\frac{x^2}{x-1}\right) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$

Q2ci Let $u = x^3 + 1, \frac{du}{dx} = 3x^2, \int \frac{6x^2}{x^3+1} dx$
 $= \int \frac{2}{u} \frac{du}{dx} dx = \int \frac{2}{u} du = 2 \log_e u + C = 2 \log_e (x^3 + 1) + C$

Q2cii $\int_0^{\pi} \cos 3x dx = \left[\frac{\sin 3x}{3} \right]_0^{\pi} = \frac{1}{3} \sin \frac{\pi}{2} - 0 = \frac{1}{3}$

Q2d $y = \log_e x, m_T = \frac{dy}{dx} = \frac{1}{x} = \frac{1}{e}$

Equation of tangent: $y - 1 = \frac{1}{e}(x - e), \therefore y = \frac{1}{e}x$

Q3a $\sum_{n=3}^5 (2n+1) = (2 \times 3 + 1) + (2 \times 4 + 1) + (2 \times 5 + 1) = 27$

Q3bi $\cos \theta = \frac{7^2 + 8^2 - 13^2}{2 \times 7 \times 8}, \theta = 120^\circ$

Q3bii $Area = \frac{1}{2} \times 7 \times 8 \sin 120^\circ = 24.2 \text{ cm}^2$

Q3ci $m_{AD} = m_{BC} = \frac{6-0}{12-9} = 2,$

Equation of line AD: $y - 0 = 2(x - 6), y = 2x - 12$

Q3cii Solve $x - 2y = 0$ and $y = 2x - 12$ simultaneously to find the intersection D: $x - 2(2x - 12) = 0, \therefore 3x = 24, x = 8, y = 4,$
 D is (8,4).

Q3ciii DE is 3 units long, E is (11,4).

Q3civ $\angle CDE = \angle DOA$ (Corresponding angles)

$\angle DCE = \angle ODA$ (Corresponding angles)

$\therefore \angle DEC = \angle OAD, \therefore \triangle OAD$ and $\triangle DEC$ are similar.

Q3cv $AD : EC = OA : DE = 6 : 3 = 2 : 1$

or $\frac{AD}{EC} = 2$

Q4ai Length of arc $AB = r\theta = 90 \times 0.6 = 54 \text{ cm}$

Q4aii Length of $AB = 2 \times 90 \sin 0.3 = 53.2 \text{ cm}$

Q4aiii Sector area $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 90^2 \times 0.6 = 2430 \text{ cm}^2$

Q4bi $f(x) = (x+3)(x^2 - 9) = (x+3)(x+3)(x-3) = 0$
 $\therefore x = -3$ or 3

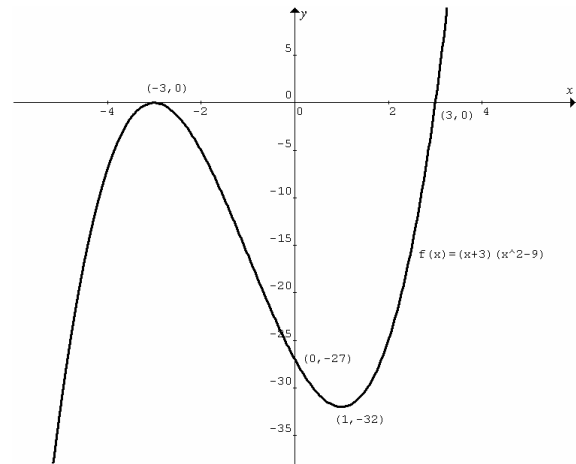
Q4bii $f'(x) = (x^2 - 9) + 2x(x+3) = (x+3)(3x-3)$

Stationary points: $(x+3)(3x-3) = 0, \therefore x = -3$ and $y = 0$ or $x = 1$ and $y = -32$

Nature of stationary points:

At $x =$	-4	-3	0	1	2
$f'(x)$	> 0	= 0	< 0	= 0	> 0
Nature		Local max.		Local min.	

Q4biii



Q4biv $f'(x) = 3x^2 + 6x - 9, f''(x) = 6x + 6 = 0, x = -1.$
 $f''(x) < 0$ for $x < -1, \therefore f(x)$ is concave down for $x < -1.$

Q5a $\log_3 7 = \frac{\log_{10} 7}{\log_{10} 3} = 1.77$

Q5b $\angle ADC = 180 - 120 = 60^\circ$ ($\angle ADC, \angle DAB$ are co-interior angles). $\therefore \angle BCE = \angle ADC = 60^\circ$ (Corresponding angles).
 $BC = AD = BE, \therefore \triangle BCE$ is isosceles and
 $\angle BEC = \angle BCE = 60^\circ. \therefore \angle CBE = 60^\circ.$ Hence $\triangle BCE$ is equilateral.

Q5c At point P, $\frac{dy}{dx} = 2e^x + 3 = 5, \therefore e^x = 1, x = 0$ and $y = 2.$
 P is (0,2).

Q5di $\Pr(\text{all red}) = \frac{{}^{100}C_3}{{}^{300}C_3} = 0.0363$

Q5dii $\Pr(\text{at least one not red}) = 1 - \Pr(\text{all red}) = 1 - 0.0363 = 0.9637$

Q5diii $\Pr(\text{one of each colour}) = \frac{({}^{100}C_1)^3}{{}^{300}C_3} = 0.2245$

Q6a

$$\int_0^{20} f(x) dx \approx \frac{20-0}{3 \times 4} [f(0) + 4f(5) + 2f(10) + 4f(15) + f(20)]$$

$$= \frac{5}{3} [15 + 100 + 44 + 72 + 10] = \frac{1205}{3} \approx 402$$

Q6bi At $t = 10$ min, $V = 3600 \left(1 - \frac{1}{6}\right)^2 = 2500$ litres

Q6bii At $t = 20$ min, $\frac{dV}{dt} = 2 \times \frac{-1}{60} \times 3600 \left(1 - \frac{t}{60}\right)$

$$= -120 \left(1 - \frac{t}{60}\right) = -120 \left(1 - \frac{20}{60}\right) = -80$$

Drainage rate = 80 litres per min.

Q6biii $\frac{dV}{dt} = 2t - 120$, the fastest rate is -120 that occurs at $t = 0$.

Q6ci Solve $y = x^2$ and $y = 12 - 2x^2$ simultaneously:
 $x^2 = 12 - 2x^2$, $3x^2 = 12$, $x = -2$ and $y = 4$, or $x = 2$ and $y = 4$. The intersecting points are $(-2, 4)$ and $(2, 4)$.

Q6cii $Volume = \int_0^4 \pi(x_1)^2 dy + \int_4^{12} \pi(x_2)^2 dy$

$$= \int_0^4 \pi y dy + \int \pi \frac{1}{2}(12 - y) dy = \left[\frac{\pi y^2}{2}\right]_0^4 + \left[\frac{\pi(12 - y)^2}{-4}\right]_4^{12}$$

$$= 8\pi + 16\pi = 24\pi \text{ cubic units}$$

Q7ai $a = 50000$, $d = 2500$, $t_{13} = 50000 + 12(2500) = 80000$

Q7aii $a = 50000$, $r = 1.04$, $t_{13} = 50000(1.04)^{12} = 80051.61$

Q7aiii Kay: $S_{20} = \frac{50000(1.04^{20} - 1)}{0.04} = 1488903.93$

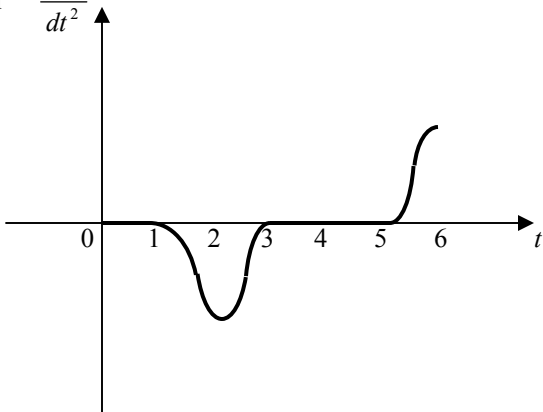
Anne: $S_{20} = \frac{20}{2}(2(50000) + 19(2500)) = 1475000$

Kay's total exceeds Anne's total by
 $1488903.93 - 1475000 = \$13903.93$

Q7bi $t = 2$

Q7bii $t = 4$

Q7biii $\frac{d^2x}{dt^2}$



Q8ai $x^2 = R^2 - h^2$, $V = \pi x^2(2h) = 2\pi h(R^2 - h^2)$

Q8aii Max. volume when

$$\frac{dV}{dh} = (R^2 - h^2)(2\pi) + (-2h)(2\pi h) = 0,$$

$$2\pi(R^2 - 3h^2) = 0, \therefore h = \frac{R}{\sqrt{3}}.$$

Q8b Area of shaded region = area of quarter circle -

$$\int_0^1 (x^2 - 3x + 2) dx = \pi - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1 = \pi - \frac{5}{6}$$

Q8ci $A_1 = (3 \times 10^6)(1.12) - 4.8 \times 10^5$,

$$A_2 = A_1(1.12) - 4.8 \times 10^5 = ((3 \times 10^6)(1.12) - 4.8 \times 10^5)(1.12) - 4.8 \times 10^5$$

$$= (3 \times 10^6)(1.12)^2 - (4.8 \times 10^5)(1 + 1.12)$$

Q8cii $A_n = (3 \times 10^6)(1.12)^n - (0.48 \times 10^6)(1 + 1.12 + \dots + 1.12^{n-1})$

$$= 10^6 \left(3(1.12)^n - 0.48 \left(\frac{1(1.12^n - 1)}{0.12} \right) \right)$$

$$= 10^6 (3(1.12)^n - 4(1.12)^n + 4) = 10^6 [4 - (1.12)^n]$$

Q8ciii After the final payment, $A_n = 0$, $10^6 [4 - (1.12)^n] = 0$,

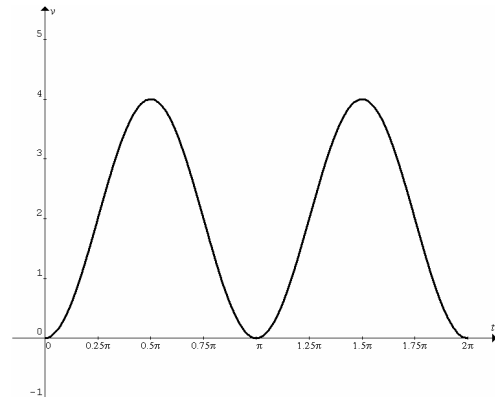
$$\therefore (1.12)^n = 4, n = \frac{\log_e 4}{\log_e 1.12} = 12.23, \therefore \text{year 2017.}$$

Q9ai At $t = 0$, $x = 0$, $\dot{x} = 0$.

At time t , $\dot{x} = \int 4 \sin 2t dt = -2 \cos 2t + C$, $\therefore 0 = -2 + C$,

i.e. $C = 2$. $\therefore \dot{x} = 2 - 2 \cos 2t$

Q9aii



First comes to rest at $t = \pi$.

Q9aiii Distance = magnitude of displacement = area under v-t graph from $t = 0$ to $t = \pi$.

$$\int_0^\pi (2 - 2 \cos 2t) dt = [2t - \sin 2t]_0^\pi = 2\pi$$

Q9bi $BD = 6 \sin \theta$, $\angle DBE = \angle EDF = \angle FEG$ etc.

$$DE = BD \sin \theta = 6 \sin^2 \theta, EF = DE \sin \theta = 6 \sin^3 \theta.$$

Q9bii $BD + EF + GH + \dots = 6 \sin \theta + 6 \sin^3 \theta + 6 \sin^5 \theta + \dots$

$$= 6(\sin \theta + \sin^3 \theta + \sin^5 \theta + \dots) = 6 \left(\frac{\sin \theta}{1 - \sin^2 \theta} \right) = 6 \left(\frac{\sin \theta}{\cos^2 \theta} \right)$$

$$= 6 \left(\frac{1}{\cos \theta} \times \frac{\sin \theta}{\cos \theta} \right) = 6 \sec \theta \tan \theta$$

Q10ai $m = \frac{\alpha^2 - \beta^2}{\alpha - \beta} = \frac{(\alpha - \beta)(\alpha + \beta)}{(\alpha - \beta)} = \alpha + \beta$

$A(\alpha, \alpha^2)$ lies on the line $y = mx + b$, $\therefore \alpha^2 = (\alpha + \beta)\alpha + b$,
 $\therefore \alpha\beta = -b$

Q10aii

$$AB = \sqrt{(\alpha - \beta)^2 + (\alpha^2 - \beta^2)^2} = \sqrt{(\alpha - \beta)^2 [1 + (\alpha + \beta)^2]}$$

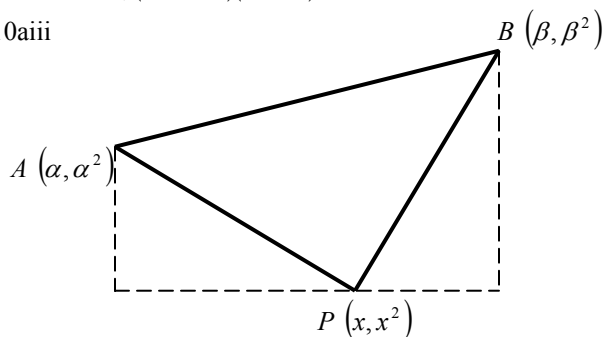
Since $\alpha + \beta = m$, $\therefore m^2 = (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$$\therefore m^2 = \beta^2 - 2\alpha\beta + \alpha^2 + 4\alpha\beta = (\beta - \alpha)^2 - 4b$$

$$\therefore (\alpha - \beta)^2 = (\beta - \alpha)^2 = m^2 + 4b$$

Hence $AB = \sqrt{(m^2 + 4b)(1 + m^2)}$

Q10aiii



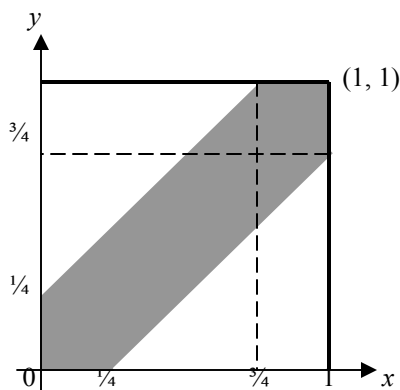
Area of $\triangle ABP$ = Area of trapezium – areas of two smaller \triangle s

$$\begin{aligned} &= \frac{1}{2}[(\alpha^2 - x^2) + (\beta^2 - x^2)](\beta - \alpha) - \frac{1}{2}(\alpha^2 - x^2)(x - \alpha) - \frac{1}{2}(\beta^2 - x^2)(\beta - x) \\ &= \frac{1}{2}((\alpha^2 - x^2)(\beta - x) + (\beta^2 - x^2)(x - \alpha)) \quad \text{expand and simplify} \\ &= \frac{1}{2}((\beta^2 - \alpha^2)x - (\beta - \alpha)x^2 - (\beta - \alpha)\alpha\beta) \\ &= \frac{1}{2}((\beta + \alpha)x - x^2 - \alpha\beta)(\beta - \alpha) \\ &= \frac{1}{2}(mx - x^2 + b)\sqrt{m^2 + 4b} \end{aligned}$$

Q10aiv $\frac{dA}{dx} = \frac{1}{2}\sqrt{m^2 + 4b}(m - 2x) = 0$, $\therefore x = \frac{m}{2}$ and

$y = \frac{m^2}{4}$. Hence P is $(\frac{m}{2}, \frac{m^2}{4})$.

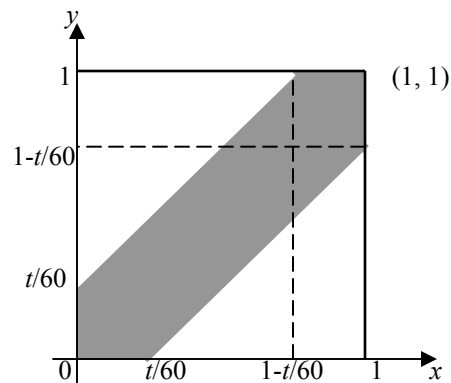
Q10bi



The shaded region satisfies all the inequalities and is inside the square. \therefore they will meet.

Q10bii $\Pr(\text{meet}) = \text{area of shaded region} = 1 - \left(\frac{3}{4}\right)^2 = \frac{7}{16}$

Q10biii



$\Pr(\text{meet}) = 1 - \left(1 - \frac{t}{60}\right)^2 = 0.5$, $\left(1 - \frac{t}{60}\right)^2 = 0.5$,

$1 - \frac{t}{60} = 0.7071$, $t = 17.6$ minutes.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.