2006 VCAA Mathematical Methods Exam 1 Solutions

Q1 $f(x) = x^2 + 1$, $g(x) = 2x + 1$,
$f(g(x)) = (g(x))^2 + 1 = (2x+1)^2 + 1 = 4x^2 + 4x + 2$

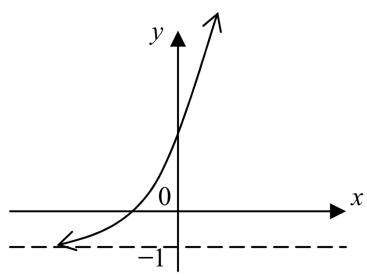
Q2a Equation of the original function: $y = 3e^{2x} - 1$

Equation of the inverse: $x = 3e^{2y} - 1$, $\therefore e^{2y} = \frac{x+1}{3}$,

$\therefore y = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$

Rule of the inverse function is $f^{-1}(x) = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$.

Q2b Graph of f



Domain of f^{-1} is the range of *f*, i.e. $(-1, \infty)$.

Q3a $f(x) = e^{\cos x}$, a composite function.
Apply the chain rule to find $f'(x) = (-\sin x)e^{\cos x}$

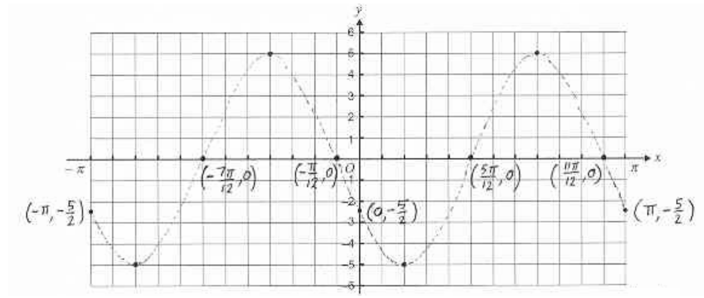
Q3b $y = x\tan x$, product of functions.
Apply the product rule to find $\frac{dy}{dx} = \tan x + x\sec^2 x$.

When $x = \frac{\pi}{6}$,

$$\frac{dy}{dx} = \tan\frac{\pi}{6} + \frac{\pi}{6} \times \frac{1}{\left(\cos\frac{\pi}{6}\right)^2} = \frac{1}{\sqrt{3}} + \frac{\pi}{6} \times \frac{4}{3} = \frac{3\sqrt{3} + 2\pi}{9}.$$

Q4a Amplitude is 5, period $= \frac{2\pi}{2} = \pi$.

Q4b y-coordinate of end point at $x = \pi$:
$y = 5\cos 2\left(\pi + \frac{\pi}{3}\right) = 5\cos\frac{8\pi}{3} = 5\cos\frac{2\pi}{3} = -\frac{5}{2}$.
Same value for end point at $x = -\pi$.



Q5a $\Pr(X > 80) = \Pr\left(Z > \frac{80-72}{8}\right) = \Pr(Z > 1) = 1 - \Pr(Z < 1)$
$= 1 - 0.84 = 0.16$

Q5b $\Pr(64 < X < 72) = \Pr(-1 < Z < 0)$
$= \Pr(0 < Z < 1) = \Pr(Z < 1) - \Pr(Z < 0) = 0.84 - 0.50 = 0.34$

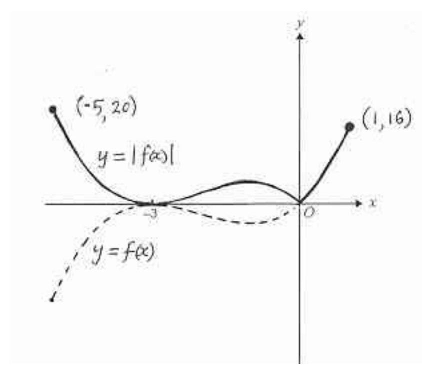
Q5c $\Pr((X < 64) | (X < 72)) = \frac{\Pr((X<64) \cap (X<72))}{\Pr(X<72)}$
$= \frac{\Pr(X<64)}{\Pr(X<72)} = \frac{\Pr(X>80)}{\Pr(X<72)} = \frac{0.16}{0.5} = 0.32$

Q6a $\Pr(X < 3) = \int_1^3 \frac{x}{12}dx = \left[\frac{x^2}{24}\right]_1^3 = \frac{9}{24} - \frac{1}{24} = \frac{1}{3}$

Q6b $\Pr(X \ge a) = \frac{5}{8}$, $\therefore \int_a^5 \frac{x}{12}dx = \frac{5}{8}$, $\therefore \left[\frac{x^2}{24}\right]_a^5 = \frac{5}{8}$,

$\frac{25}{24} - \frac{a^2}{24} = \frac{5}{8}$, $a^2 = 10$, $\therefore a = \sqrt{10}$.

Q7a



Q7b When $x = -5$,
$y = f(-5) = (-5)^3 + 6[-5)^2 + 9(-5) = -20$,
$\therefore y = |f(-5)| = 20$.
When $x = 1$, $y = f(1) = (1)^3 + 6[1)^2 + 9(1) = 16$,
$\therefore y = |f(1)| = 16$.
From graph, the range of $|f(x)|$ and domain $[-5, 1]$ is $[0, 20]$.

Q8 $y=\sqrt{x}$, $\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{x}}$.

Gradient of normal $=-4$, $\therefore$ gradient of tangent $=-\dfrac{1}{-4}=\dfrac{1}{4}$,

i.e. $\dfrac{1}{2\sqrt{x}}=\dfrac{1}{4}$, $\therefore x=4$ and $y=2$.

$y=-4x+a$ is the normal to the curve at $(4,2)$.

Hence $2=-4(4)+a$, $\therefore a=18$.

Q9a $A=(2a)b=2ab$ where $a>0$ and $b=9-3a^2$,

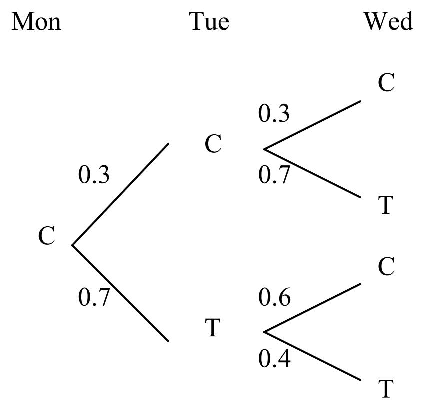
$\therefore A=2a(9-3a^2)=18a-6a^3$

Q9b $\dfrac{dA}{da}=18-18a^2$. At stationary points, $\dfrac{dA}{da}=0$,

i.e. $18-18a^2=0$, $\therefore a=1$.

At $a=1$, $A_{\max}=18(1)-6(1)^3=12$ square units

Q10



Pr(tea on Wed) $=0.3\times0.7+0.7\times0.4=0.49$

Q11 Shaded area $=\displaystyle\int_0^3\left(-x^2+ax+12\right)dx=45$,

$\therefore\left[-\dfrac{x^3}{3}+\dfrac{ax^2}{2}+12x\right]_0^3=45$,

$\therefore -9+\dfrac{9a}{2}+36=45$, $\therefore a=4$.

$\therefore f(x)=-x^2+4x+12$.

To find the x-intercepts: Let $-x^2+4x+12=0$

Factorise: $(-x-2)(x-6)=0$, $\therefore x=-2$ or 6.

Hence $a=4$, $m=6$ and $n=-2$.

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