

Q1a  $\sqrt[3]{\frac{651}{4\pi}} \approx 3.728$  by calculator.

Q1b  $\frac{n^2 - 25}{n - 5} = \frac{(n - 5)(n + 5)}{n - 5} = n + 5$

Q1c  $2^{2x+1} = 32, 2^{2x+1} = 2^5, 2x+1 = 5, x = 2$

Q1d  $\frac{d}{dx}(\ln(5x+2)) = \frac{5}{5x+2}$

Q1e  $2 - 3x \leq 8, -3x \leq 6, x \geq \frac{6}{-3}, x \geq -2$

Q1f  $\frac{4}{\sqrt{5} - \sqrt{3}} = \frac{4}{(\sqrt{5} - \sqrt{3})} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})} = \frac{4(\sqrt{5} + \sqrt{3})}{2} = 2(\sqrt{5} + \sqrt{3})$

Q1g  $800 \times 0.02 = 16$  defective items

Q2ai  $\alpha + \beta = 6$

Q2aii  $\alpha\beta = 2$

Q2aiii  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{6}{2} = 3$

Q2b  $2\sin x = -\sqrt{3}$  and  $0 \leq x \leq 2\pi, \sin x = -\frac{\sqrt{3}}{2}$

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

Q2c  $y = (2x+1)^4, \frac{dy}{dx} = 8(2x+1)^3$

At  $x = -1, y = 1, \frac{dy}{dx} = -8$

Tangent:  $y - 1 = -8(x + 1), y = -8x - 7$

Q2d  $y = x^2e^x, \frac{dy}{dx} = 2xe^x + x^2e^x$  by the product rule

Q2e  $\int \frac{1}{3x^2} dx = -\frac{1}{3x} + c$

Q3ai  $t_1 = 3, d = 0.5, t_{25} = 3 + 24 \times 0.5 = 15, \$15$  million

Q3aii  $S_{110} = \frac{110}{2}(2 \times 3 + 109 \times 0.5) = 3327.5, \$3327.5$  million

Q3b The vertex of the parabola is equidistant from its focus (3,2) and directrix  $y = -4, \therefore$  the vertex is (3,-1).

Q3ci  $\ell_1: 3x + 4y - 12 = 0$

Let  $x = 0, y = 3, \therefore B(0,3)$

Q3cii  $m_1 = -\frac{3}{4}, m_2 = \frac{4}{3}, m_1m_2 = -1, \therefore \ell_1$  and  $\ell_2$  are  $\perp$

Q3ciii  $\ell_1: 3x + 4y - 12 = 0$ . Let  $y = 0, x = 4, \therefore A(4,0)$

$\therefore AB = \sqrt{3^2 + 4^2} = 5$

$\Delta OAB$  and  $\Delta OEB, \therefore \frac{OE}{OA} = \frac{OB}{AB}, \frac{OE}{4} = \frac{3}{5}, OE = \frac{12}{5}$

Q3civ  $\frac{BE}{BO} = \frac{OE}{OA}, \frac{BE}{3} = \frac{\frac{12}{5}}{4}, BE = \frac{9}{5}$

Q3cv Area of  $\Delta BOE = \frac{1}{2} \times \frac{12}{5} \times \frac{9}{5} = \frac{54}{25}$  unit squares

Q4a  $\frac{d}{dx}\left(\frac{x}{\sin x}\right) = \frac{\sin x - x \cos x}{\sin^2 x}$

Q4b  $\int_e^{e^3} \frac{5}{x} dx = [5 \ln x]_e^{e^3} = 5 \ln e^3 - 5 \ln e = 15 - 5 = 10$

Q4c  $\frac{dy}{dx} = 6x - 2, y = \int (6x - 2) dx = 3x^2 - 2x + c$

$(-1, 4), 4 = 3(-1)^2 - 2(-1) + c, c = -1$

$\therefore y = 3x^2 - 2x - 1$

Q4di  $y = \sqrt{9 - x^2}, \frac{dy}{dx} = \frac{1}{2\sqrt{9 - x^2}} \times (-2x) = -\frac{x}{\sqrt{9 - x^2}}$

Q4dii  $-\frac{x}{\sqrt{9 - x^2}} = \frac{dy}{dx}, \frac{6x}{\sqrt{9 - x^2}} = -6 \frac{dy}{dx}$

$\int \frac{6x}{\sqrt{9 - x^2}} dx = -6 \int \frac{dy}{dx} dx = -6 \int dy = -6y + c = -6\sqrt{9 - x^2} + c$

Q4e  $y \leq 4 - x^2$  and  $y \geq |x| - 2$

Q5ai  $t_1 = a = 27$ ,  $r = 2$ ,  $t_{12} = ar^{12-1} = 27 \times 2^{11} = 55296$

Q5aii  $t_n = 27 \times 2^{n-1} > 10000000$ ,  $2^{n-1} > \frac{10000000}{27}$ ,  
 $n > 19.5$ ,  $\therefore n = 20$ , i.e. Day 20.

Q5aiii  $S_{12} = \frac{27(2^{12} - 1)}{2 - 1} = 110565$ ,  
 $earnings = \$0.005 \times 110565 \approx \$553$

Q5bi  $\Pr(\text{red\_shirt\_Monday}) = \frac{3}{5}$

Q5bii  $\Pr(\text{red\_red\_red}) = \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}$

Q5biii  $\Pr(\text{red\_yellow\_red}) + \Pr(\text{yellow\_red\_yellow})$   
 $= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{3}{10}$

Q5c  $\int_0^{20} v dt \approx \frac{5}{3} (173 + 4 \times 81 + 2 \times 127 + 4 \times 195 + 168) = \frac{8495}{3} \text{ m}$

Q6ai  $\angle CDE = \frac{3 \times 180^\circ}{5} = 108^\circ$

Q6aii  $\angle CED = \frac{180^\circ - 108^\circ}{2} = 36^\circ$   
 $\angle EPC = 360^\circ - 3 \times 108^\circ = 36^\circ$   
 $\therefore \triangle EPC$  is isosceles

Q6b  $\overline{PA}^2 + \overline{PB}^2 = 40$ ,  $(x+1)^2 + y^2 + (x-3)^2 + y^2 = 40$ ,  
 $2x^2 - 4x + 10 + 2y^2 = 40$ ,  $x^2 - 2x + 5 + y^2 = 20$ ,  
 $x^2 - 2x + 1 + y^2 = 16$ ,  $(x-1)^2 + y^2 = 4^2$   
 $\therefore$  the locus is a circle of radius 4 centred at (1,0).

Q6ci  $P(0,2)$

Q6cii  $\int_0^{\frac{\pi}{2}} 2 \cos x dx = [2 \sin x]_0^{\frac{\pi}{2}} = 2 \sin \frac{\pi}{2} - 2 \sin 0 = 2$

Q6ciii  $C$

Q6civ  $Area = 2 \times 4 = 8$  unit squares

Q6cv  $\int_{\frac{\pi}{2}}^{2\pi} 2 \cos x dx = -2 - 2 + 2 = -2$

Q7ai  $f(x) = x^3 - 3x + 2$ ,  $f'(x) = 3x^2 - 3 = 3(x+1)(x-1)$   
 $f'(x) = 0$  at  $x = -1$ ,  $y = 4$  or  $x = 1$ ,  $y = 0$

Stationary points are  $(-1,4)$  and  $(1,0)$

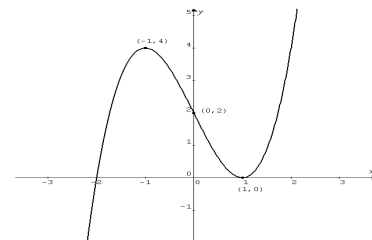
Since the coefficient of the  $x^3$  term is a positive value, the shape of  $f(x)$  is

OR

$x$	-2	-1	0	1	2
$f'(x)$	+	0	-	0	+

$\therefore (-1,4)$  is a local maximum and  $(1,0)$  is a local minimum.

Q7aii



Q7bi  $\dot{x} = 8 - 8e^{-2t}$

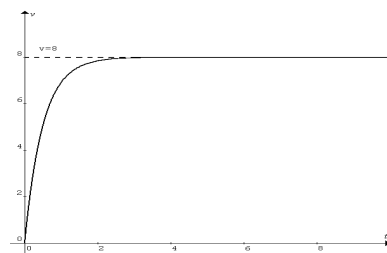
When  $t = 0$ ,  $v = \dot{x} = 8 - 8e^0 = 0$ ,  $\therefore$  the particle is initially at rest.

Q7bii  $a = \ddot{x} = 16e^{-2t} > 0$ ,  $\therefore$  the acceleration is always positive.

Q7biii The particle starts from rest and its acceleration is always positive,  $\therefore$  its velocity is positive for all  $t > 0$ , i.e. the particle moves in the positive direction for all  $t > 0$ .

Q7biv  $v = \dot{x} = 8 - 8e^{-2t} \rightarrow 8$  as  $t \rightarrow \infty$ . The value of the constant is 8.

Q7bv



Q8ai  $x^2 + 20^2 - 2(x)(20)\cos 60^\circ = 22^2$   
 $x^2 + 400 - 20x = 484$ ,  $\therefore x^2 - 20x - 84 = 0$

Q8aii  $x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-84)}}{2} = \frac{20 \pm \sqrt{736}}{2}$

$\therefore PL = \frac{20 + \sqrt{736}}{2} \approx 24 \text{ km}$

Q8bi  $V = \int_0^h \pi x^2 dy = \int_0^h \pi y dy = \left[ \frac{\pi y^2}{2} \right]_0^h = \frac{1}{2} \pi h^2$

Q8bii  $V_{cylinder} = \pi(\sqrt{h})^2 h = \pi h^2$

$\therefore \frac{V_{paraboloid}}{V_{cylinder}} = \frac{\frac{1}{2}\pi h^2}{\pi h^2} = \frac{1}{2}$

Q8ci  $R = 1 + \frac{6}{12 \times 100} = 1.005$

End of $n$ th month	$\$P_n$
1	$100 \times 1.005^1$
2	$100 \times 1.005^1 + 100 \times 1.005^2$
3	$100 \times 1.005^1 + 100 \times 1.005^2 + 100 \times 1.005^3$
...	...
420	$\frac{100 \times 1.005(1.005^{420} - 1)}{0.005} \approx 143183$

Q8cii1

End of $n$ th month	$\$A_n$
1	$29227 \times 1.005 + M \times 1.005$
2	$(29227 \times 1.005 + M + M \times 1.005) \times 1.005$ $= 29227 \times 1.005^2 + M(1.005 + 1.005^2)$

Q8cii2 At the end of 240<sup>th</sup> month after starting the new contributions,  $A_{240}$   
 $= 29227 \times 1.005^{240} + M(1.005 + 1.005^2 + \dots + 1.005^{240}) = 800000$   
 $\therefore M = \frac{800000 - 29227 \times 1.005^{240}}{1.005 + 1.005^2 + \dots + 1.005^{240}} = \frac{703252.6538}{464.3511} = 1514.48$

Q9ai  $\triangle ABC$  and  $\triangle ADE$  are similar because

$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE} = \frac{1}{2}$

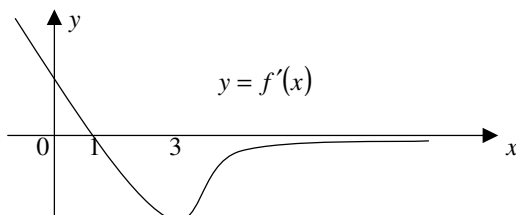
Q9aii  $BC$  and  $DE$  are parallel,  $\therefore \angle BCD = \angle CDE$  and  $\angle CBE = \angle BED$ ,  $\therefore \triangle FBC$  and  $\triangle FED$  are similar.

Since  $BC : DE = 1 : 2$ ,  $\therefore BF : FE = 1 : 2$

Q9bi  $\left(2 + \frac{t^2}{t+1}\right) - \left(1 + \frac{1}{t+1}\right) = 1 + \frac{t^2 - 1}{t+1} = \frac{t^2 + t}{t+1} = \frac{t(t+1)}{t+1} = t$

Q9bii  $\Delta V = \int_0^4 t dt = \left[\frac{t^2}{2}\right]_0^4 = 8$  litres

Q9c



Q9di  $\frac{1}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}} \times \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+1} - \sqrt{n}}$   
 $= \frac{\sqrt{n+1} - \sqrt{n}}{n+1-n} = \sqrt{n+1} - \sqrt{n}$

Q9dii  $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}}$   
 $= \sqrt{2} - \sqrt{1} + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \dots + \sqrt{100} - \sqrt{99}$   
 $= \sqrt{100} - \sqrt{1} = 9$

Q10ai  $I = 10^{-12} \times e^{0.1L} = 10^{-12} \times e^{0.1 \times 110} \approx 6.0 \times 10^{-8}$  watt/m<sup>2</sup>  
 (Comment: The correct formula is  $I = 10^{-12} \times 10^{0.1L} = 10^{0.1L-12}$ )

Q10aai  $I = 10^{-12} \times e^{0.1L} = 8.1 \times 10^{-9}$ ,  $e^{0.1L} = 8.1 \times 10^3$   
 $L = \frac{\ln(8.1 \times 10^3)}{0.1} \approx 90$  decibels

Q10aiii  $I = 10^{-12} \times e^{0.1L}$ ,  $L = 10 \ln(10^{12} I)$   
 Let  $L_i = 10 \ln(10^{12} I_i)$  and  $L_f = 10 \ln(10^{12} I_f)$   
 $\Delta L = L_f - L_i = 10 \ln(10^{12} I_f) - 10 \ln(10^{12} I_i) = 10 \ln \frac{I_f}{I_i}$

If  $I_f = 2I_i$ ,  $\Delta L \approx 6.9$  decibels

Q10bi Length of fencing  $P = \text{arc length} + 2 \text{ times radius}$   
 $\therefore P = r\theta + 2r = r(\theta + 2)$

Q10bii  $r\theta = P - 2r$

Sector area  $A = \frac{1}{2} r^2 \theta = \frac{1}{2} r(P - 2r) = \frac{1}{2} P.r - r^2$

Q10biii Let  $\frac{dA}{dr} = \frac{1}{2} P - 2r = 0$ ,  $r = \frac{1}{4} P$  to maximise  $A$ .

Q10biv  $P = r(\theta + 2)$ ,  $P = \frac{1}{4} P(\theta + 2)$ ,  $1 = \frac{1}{4}(\theta + 2)$ ,  $\theta = 2$

Q10bv Since  $0 < \theta < 2\pi$ ,  $0 < r\theta < 2\pi r$ ,  
 $0 + 2r < r\theta + 2r < 2\pi r + 2r$ ,  $\therefore 2r < P < 2r(\pi + 1)$

$\therefore r < \frac{P}{2}$  and  $\frac{P}{2(\pi + 1)} < r$

$\therefore \frac{P}{2(\pi + 1)} < r < \frac{P}{2}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.