

Core – Data analysis

Q1ai The lowest average age = 25 years

Q1aii The median average age = 28.2 years

Q1b $Q_L = 29.9$, $Q_U = 31.0$,
 $IQR = Q_U - Q_L = 31.0 - 29.9 = 1.1$ years

Q1c $Q_L - 1.5 \times IQR = 29.9 - 1.5 \times 1.1 = 28.25$
 $26.0 < 28.25$, $\therefore 26.0$ is an outlier.

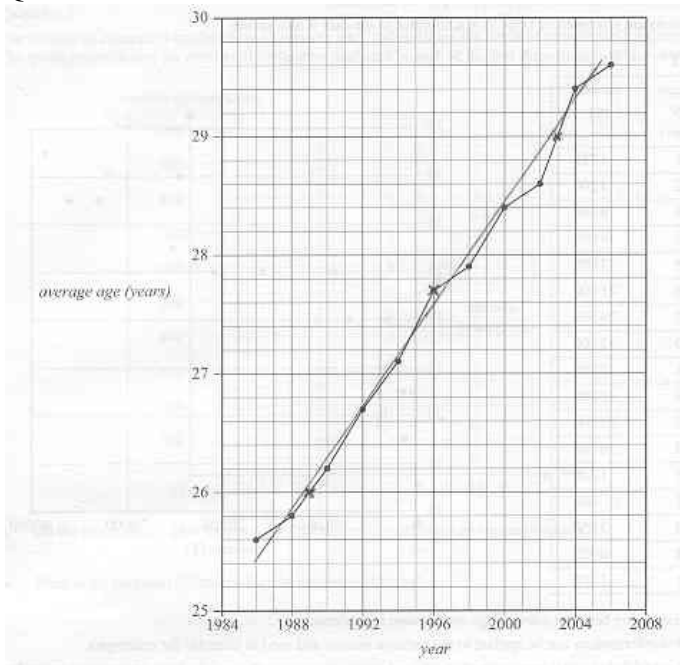
Q2a $42.1\% + 23.4\% = 65.5\%$

Q2b Yes. From 1986 to 2006, the age of women at first marriage has a negative correlation with year of marriage. Consider the 19 years and under group:

1986	1996	2006
8.5%	3.7%	2.0%

Q3a The average age of women at first marriage was fairly constant from 1915 to 1935. It had a decreasing trend from 1935 and tapered off towards 1970.

Q3bi and ii

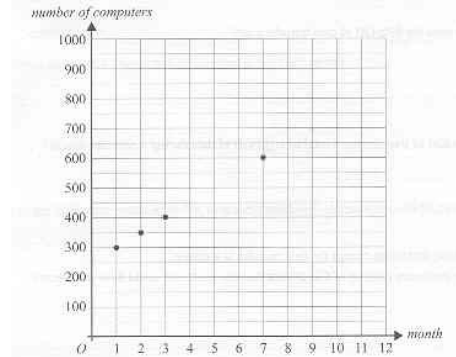


Q4a $average\ age = 2.39 + 5.89 \times \log(income)$

Q4b The average age of women at first marriage with an average yearly income of \$20000 is 27.7 years.

Module 1: Number patterns

Q1a



Q1b $d = 50$

Q1c $850 - 700 = 150$ or $50 \times 3 = 150$

Q2a Arithmetic sequence: $a = 4.00$, $d = -0.20$,
 $t_5 = 4.00 + 4(-0.20) = 3.20$, $\therefore \$3.20$

Q2b Arithmetic series: $S_8 = \frac{8}{2}(2(4.00) + 7(-0.20)) = 26.4$,
 $\therefore \$26.40$

Q2c It takes 20 GB to reduce the charge to \$0.20 per GB.
 $\therefore 20$ GB

Q2di $D_{n+1} = 1.5D_n - 1$
 $D_1 = 6$, $D_2 = 1.5D_1 - 1 = 8$, $D_3 = 1.5D_2 - 1 = 11$, $\therefore 11$ GB

Q2dii 11 GB in his the third month,
 $\therefore S_{11} = \frac{11}{2}(2(4.00) + 10(-0.20)) = 33.00$, $\therefore \$33.00$

Q2e $D_9 \approx 104.5$ GB, \therefore the 9th month

Q3a Charge for the second month:
 $95\% \times 50 = 0.95 \times 50 = 47.50$, $\therefore \$47.50$

Q3b Geometric sequence: $a = 50$, $r = 0.95$,
 $t_6 = 50 \times 0.95^5 = 38.689$, $difference = 50 - 38.69 = 11.31$,
 $\therefore \$11.31$

Q3c Geometric series: $S_{12} = \frac{50(1 - 0.95^{12})}{1 - 0.95} \approx 459.64$, $\therefore \$460$

Q4a Start of January: 200000
Start of February: $0.99 \times 200000 + 100 = 198100$

Q4b Given January is month 1,
 $\therefore C_{n+1} = 0.99C_n + 100$ and $C_1 = 200000$

Q4c $C_3 = 0.99C_2 + 100 = 0.99(198100) + 100 = 196219$
 $Difference = 200000 - 196219 = 3781$, $\therefore 3781$ more customers

Module 2: Geometry and trigonometry

Q1a Scale 1 cm : 30 m. From point A to the lighthouse the measured distance is 5 cm on the contour map, ∴ the actual distance is $30 \times 5 = 150$ m.

Q1b Average slope = $\frac{15}{75} = 0.2$

Q2ai $x^\circ = 180^\circ - 28^\circ = 152^\circ$

Q2aii $y^\circ = 360^\circ - 130^\circ - 152^\circ = 78^\circ$

Q2b Bearing of the lighthouse from Ship A is S28°W (208°T).

Q2c The triangle is isosceles, ∴ $\angle ALB = \frac{180^\circ - 78^\circ}{2} = 51^\circ$

∴ bearing of Ship B from the lighthouse: $28^\circ + 51^\circ = 79^\circ$,
∴ N79°E (079°T)

Q3a $\theta = \frac{360^\circ}{8} = 45^\circ$

Q3b Area of $\Delta POQ = \frac{1}{2} \times 2^2 \times \sin 45^\circ = \sqrt{2} \approx 1.4 \text{ m}^2$

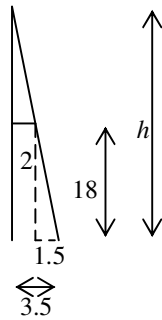
Q3c Minimum distance = $\frac{6.4 - 4}{2} = 1.2 \text{ m}$

Q3d Shaded area = $\pi \times 3.2^2 - 8 \times \sqrt{2} \approx 21 \text{ m}^2$

Q4a $\tan \alpha^\circ = \frac{18}{1.5}$, $\alpha^\circ = \tan^{-1}\left(\frac{18}{1.5}\right) \approx 85.2^\circ$

Q4bi

$\frac{h}{3.5} = \frac{18}{1.5}$, ∴ $h = 42 \text{ m}$



Q4bii Volume = $\frac{1}{3} \pi (3.5^2) 42 - \frac{1}{3} \pi (1.5^2) (42 - 18) \approx 438 \text{ m}^3$

Module 3: Graphs and relations

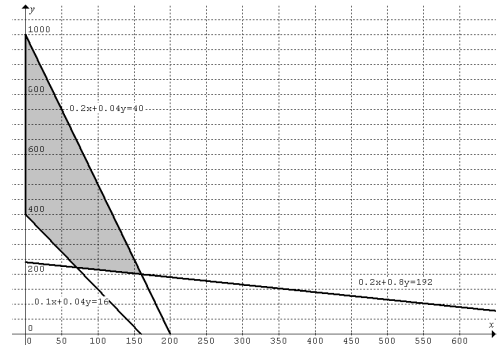
Q1a Weight of carbohydrate = $0.2 \times 180 + 0.8 \times 250 = 236 \text{ g}$

Q1b Let $n \text{ g}$ of raisins be added to the trail mix.

$0.2 \times 320 + 0.04n = 72$, ∴ $n = 200$, ∴ 200 g of raisins

Q1c $0.1x + 0.04y \geq 16$

Q1di and ii



Q1e (0,1000) satisfies Michael's dietary requirements and gives maximum weight ($0 + 1000 = 1000 \text{ g}$) of trail mix.

Q1f Solve $x + y = 500$ and $0.2x + 0.04y = 40$ for x .

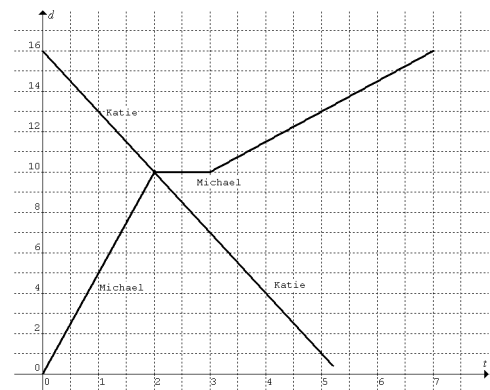
$x = 125$, ∴ 125 g of almonds

Q2a Average speed = $\frac{16}{7} \approx 2.3 \text{ km h}^{-1}$

Q2b Gradient $a = \frac{16 - 10}{7 - 3} = \frac{6}{4} = 1.5$ ∴ $d = 1.5t + b$

When $t = 3$, $d = 10$, ∴ $b = 5.5$

Q2c



When they pass each other they are the same distance from the national park office, i.e. $d = 10$ when $t = 2$, ∴ 2 hours

Q2d $d_{Katie} = -3t + 16$, $t \geq 0$

$d_{Michael} = 5t$, $0 \leq t \leq 2$

$d_{Katie} - d_{Michael} \leq 3$, ∴ $-8t + 16 \leq 3$, $-8t \leq -13$, ∴ $1.625 \leq t \leq 2$

From the graph, $d_{Michael} - d_{Katie} \leq 3$ for $2 \leq t \leq 3$

∴ they are not more than 3 km apart when $1.625 \leq t \leq 3$.

∴ duration = $3 - 1.625 = 1.375 \approx 1.38 \text{ hours}$

Module 4: Business-related mathematics

Q1a Let x be the cost without *GST*.

$$\left(1 + \frac{10}{100}\right)x = 3630, \therefore x = 3300,$$

$$\therefore \text{GST} = \$3630 - \$3300 = \$330$$

Q1b *Total cost* = $\$82 + \$220 + \$160 = \462

Q1c Let $r\%$ be the percentage discount.

$$220 \times \left(1 - \frac{r}{100}\right) = 202.40, \therefore r = 8, \therefore 8\% \text{ discount.}$$

Q2a End of 1st month,

$$\text{total amount} = \$5600 \times \left(1 + \frac{7.2}{100}\right) + \$200 = \$5833.60$$

Q2b *Value of investment* = $8000 \times \left(1 + \frac{7.2}{100}\right)$ dollars at the end of the 1st year.

Q2c End of 12 months, Pattie's account: \$8576.00

Tom's account: \$8497.58 by CAS.

\therefore Pattie has $\$8576.00 - \$8497.58 = \$78.42$ more than Tom.

Q2d $8000 \times \left(1 + \frac{r}{100}\right) = 9000, r = 12.5, \therefore$ annual compounding rate is 12.5%.

Q3a *Stamp duty*

$$= \$2870 + (\$300000 - \$130000) \times \frac{6}{100} = \$13070$$

Q3b *House value*

$$= \$300000 \times \left(1 + \frac{3.17}{100}\right)^5 = \$350661.76 \approx \$351000$$

Q3c $\$300000 \times \left(1 + \frac{3.17}{100}\right)^n = \$450000, \therefore n \approx 12.9924$

Close to the end of 2023 the house value will reach \$450000.

\therefore At the start of 2024 the house value will first exceed \$450000.

Q4ai By CAS $n \approx 299.57, \therefore 300$ monthly repayments.

Q4aii Amount owing after the 1st year: by CAS \$261305.75

Principal paid off: $\$265000 - \$261305.75 = \$3694.25$

Q4b Number of monthly repayments: $12 \times 19 = 228$

Amount owing: \$261305.75

Rate: 8.2%

By CAS new monthly repayment: \$2265.04

Module 5: Networks and decision mathematics

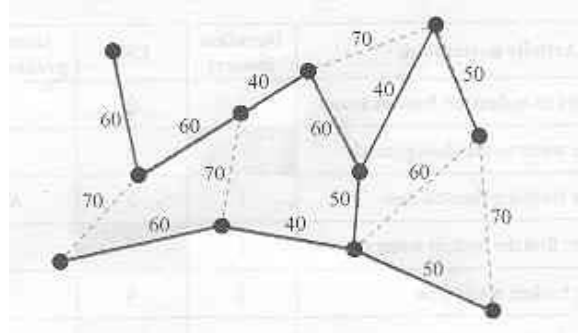
Q1a $60 + 140 = 200, \therefore 200$ km

Q1b From Farnham to Carrie: 6 different ways.

Q1c A possible path is DFEABCDEB and will finish at Bredon.

Q1d $130 + 110 = 240, \therefore 240$ km

Q2a



Q2b $60 + 60 + 40 + 60 + 40 + 50 + 50 + 50 + 40 + 60 = 510$
 $\therefore 510$ km

Q3a *Duration of activity B* = 2 hours

Q3b *EST for D* = 3 hours

Q3c Activities *F* and *H*

Q3d Shortest time: $2 + 1 + 1 + 4 + 4 + 1 = 13, \therefore 13$ hours

Q3e Shortest time: $2 + 1 + 1 + 4 + 5 + 1 = 14, \therefore 14$ hours

Q4a Storm water from Source 2 cannot reach Outlet 1

Q4b Outlet 1: 700 kL per min

Outlet 2: 700 kL per min

Q4c 300 kL per min

Module 6: Matrices

Q1ai birds eat lizards

Q1aii insects, birds or lizards do not eat birds

Q1b

$$Z = \begin{matrix} & \begin{matrix} I & B & L & F \end{matrix} \\ \begin{matrix} I \\ B \\ L \\ F \end{matrix} & \begin{bmatrix} & & & 1 \\ & & & 0 \\ & & & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Q2a

$$K = \begin{bmatrix} 100000 & 400 & 1000 & 800 \\ 0.995 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.025 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$$

$$= \begin{bmatrix} 99500 & 20 & 25 & 240 \end{bmatrix}$$

Q2b 20

$$Q2c \quad M = \begin{bmatrix} 99500 & 20 & 25 & 240 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 285 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Q2d If you are given $M = \begin{bmatrix} 285 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ only, 285 is the total number of creatures killed. If you know $M = KF$ as well, then 285 is the total number of birds, lizards and frogs killed.

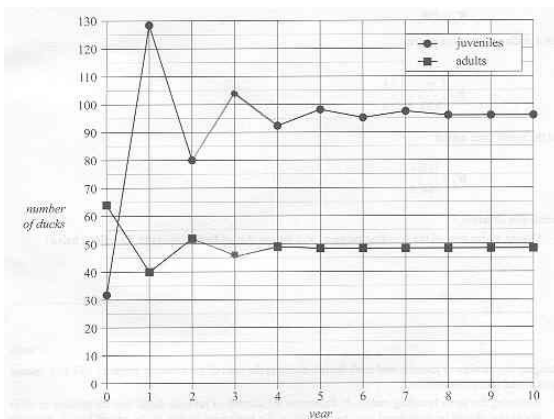
Q3a $32 + 64 = 96$, \therefore 96 female ducks

$$Q3b \quad W_1 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 32 \\ 64 \end{bmatrix} = \begin{bmatrix} 128 \\ 40 \end{bmatrix}$$

$$Q3ci \quad W_2 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 128 \\ 40 \end{bmatrix} = \begin{bmatrix} 80 \\ 52 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 80 \\ 52 \end{bmatrix} = \begin{bmatrix} 104 \\ 46 \end{bmatrix}$$

\therefore 104 juvenile female ducks and 46 adult female ducks



$$Q3cii \quad \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix}^n \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 96 \\ 48 \end{bmatrix} \text{ as } n \text{ is a large number.}$$

$96 + 48 = 144$, \therefore 144 female ducks

$$Q3d \quad W_0 = \begin{bmatrix} 32 \\ 64 \end{bmatrix}, \text{ total is } 96$$

$$\text{After 4 years, } W_4 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^4 \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 28 \\ 23 \end{bmatrix}, \text{ total is } 51$$

$$\text{After 5 years, } W_5 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^5 \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 23 \\ 18.5 \end{bmatrix}, \text{ total is } 41.5$$

\therefore during the 5th year

$$Q3e \quad \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^2 \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}, \therefore \begin{bmatrix} 0.25 & 0.5 \\ 0.125 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.25 & 0.5 \\ 0.125 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 400 \\ 0 \end{bmatrix}$$

\therefore 400 juvenile female ducks and no adult female duck

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors