2015 NSW BOS Mathematics Extension 2 Solutions

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Section I

1	2	3	4	5	6	7	8	9	10
D	A	A	C	B	C	A	B	D	C

Q1 $e=\frac{\sqrt{13}}{3}>1$, a hyperbola, $a^2=3^2$ and $b^2=2^2$ **D**

Q2 $(4-3i)^2=7-24i$ **A**

Q3 The graph of $y=(x-1)^2(x+3)^5$ has turning point at $x=1$ and a stationary point of inflection at $x=-3$. **A**

Q4 $P(x)=x^3+x^2-5x+3$, $P(1)=0$

$\therefore\ P(x)=(x-1)(x^2+2x-3)=(x-1)(x-1)(x+3)\ \therefore\ \alpha=1$ **C**

Q5 $z=1-i=\sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)$

$z^3=\left(\sqrt{2}\right)^3\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)^3$

$=2\sqrt{2}\left(\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right)$ **B**

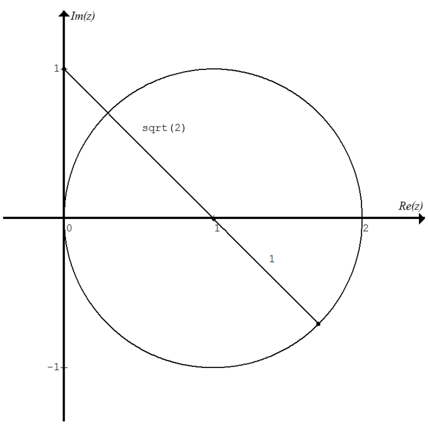
Q6 $x^2\sin x=\frac{d}{dx}\left(-x^2\cos x+\int 2x\cos x\,dx\right)$ **C**

Q7 In each permutation 1 is either on the left of 2 or 2 is on the left of 1. For each permutation with 1 on the left of 2, there is a corresponding permutation with 2 on the left of 1.

$\Pr(1\text{ left of }2)=\Pr(2\text{ left of }1)=\frac{1}{2}$ **A**

Q8 A is incorrect because it is defined for all $x\in R$.
C is incorrect because $y\geq 0$.
D is incorrect because $f\left(\sqrt{a}\right)\neq 0$. **B**

Q9 $\sqrt{2}+1$ **D**



Q10 $\left(1+x+x^2+\cdots+x^{100}\right)\left(1+2x+3x^2+\cdots+101x^{100}\right)$

Coefficient of x^{100}

$=101+100+99+\cdots+2+1=\frac{101}{2}(101+1)=5151$ **C**

Section II

Q11a $\frac{(4+3i)(2+i)}{(2-i)(2+i)}=\frac{5+10i}{5}=1+2i$

Q11bi $z=-\sqrt{3}+i$, $|z|=\sqrt{\left(-\sqrt{3}\right)^2+1^2}=2$

Q11bii $\arg(z)=\tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)=\frac{5\pi}{6}$ in the second quadrant

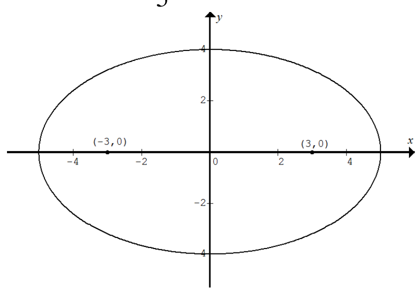
Q11biii $\arg\left(\frac{z}{w}\right)=\arg(z)-\arg(w)=\frac{5\pi}{6}-\frac{\pi}{7}=\frac{29\pi}{42}$

Q11c $A(x^2+2)+(Bx+C)x=1$

Compare coefficients: $A+B=0$, $C=0$, $2A=1$

$\therefore\ A=\frac{1}{2}$, $B=-\frac{1}{2}$ and $C=0$

Q11d $16=25(1-e^2)$, $e=\frac{3}{5}$, foci are $(\pm ae,0)=(\pm 3,0)$



Q11e $x+x^2y^3=-2$

By implicit differentiation: $1+2xy^3+x^2 3y^2\frac{dy}{dx}=0$

$\frac{dy}{dx}=-\frac{1+2xy^3}{3x^2y^2}=\frac{1}{4}$ at $(2,-1)$

Q11fi $\cot\theta+\operatorname{cosec}\theta=\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}=\frac{\cos\theta+1}{\sin\theta}$

$=\frac{2\cos^2\left(\frac{\theta}{2}\right)}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}=\frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}=\cot\left(\frac{\theta}{2}\right)$

Q11fii $\int(\cot\theta+\operatorname{cosec}\theta)\,d\theta=\int\frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}d\theta$

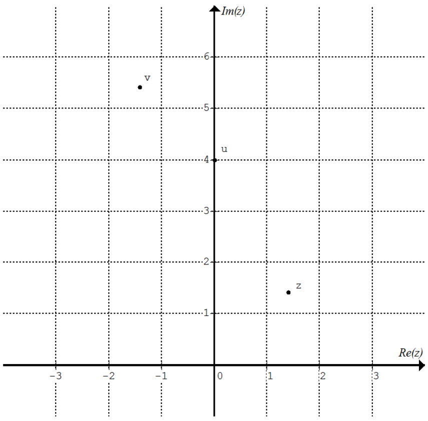
$=\int\frac{2\,du}{u}=2\ln|u|+c=2\ln\left|\sin\left(\frac{\theta}{2}\right)\right|+c$

Let $u=\sin\left(\frac{\theta}{2}\right)$

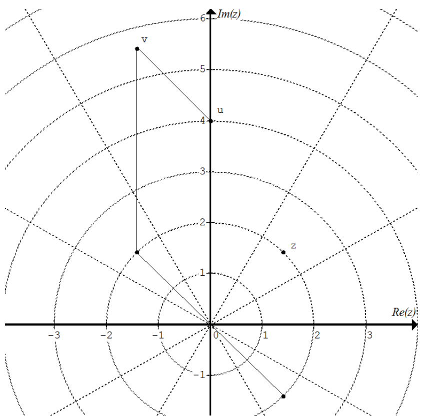
$2\frac{du}{d\theta}=\cos\left(\frac{\theta}{2}\right)$

Q12ai

$z = \sqrt{2} + \sqrt{2}\,i$, $u = z^2 = 4i$, $v = z^2 - \bar{z} = -\sqrt{2} + \left(4 + \sqrt{2}\right) i$



Alternatively:



Q12bi Conjugate roots exist for real coefficients.

.: $(a+ib)(a-ib)(a+2ib)(a-2ib) = 10$ and

$(a+ib)+(a-ib)+(a+2ib)+(a-2ib) = 4$

.: $\left(a^2+b^2\right)\left(a^2+4b^2\right) = 10$ and $4a = 4$

Solve simultaneously: $a = 1$ and $b = 1$ or -1

.: the roots are $1 \pm i$, $1 \pm 2i$

Q12bii $(x-(1+i))(x-(1-i)) = x^2 - 2x + 2$

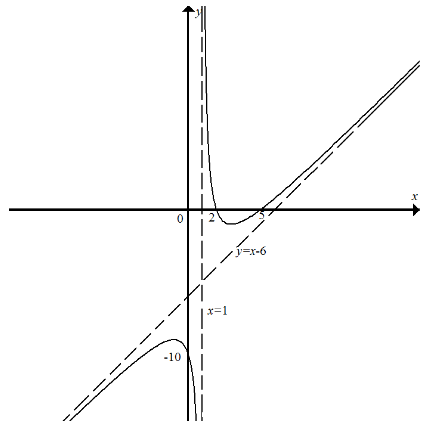
Q12ci $\dfrac{(x-2)(x-5)}{x-1} = \dfrac{x^2-7x+10}{x-1}$

Let $P(x) = x^2 - 7x + 10$, remainder $P(1) = 4$

.: $\dfrac{x^2-7x+10}{x-1} = \dfrac{(x-6)(x-1)+4}{x-1} = x - 6 + \dfrac{4}{x-1}$

The oblique asymptote is $y = x - 6$.

Q12cii



Q12d Translate the graph to the left by 3 units so that the rotation is about the y-axis: $y = \sqrt{x+4}$

Reflect the graph in the y-axis: $y = \sqrt{4-x}$

$V = \int_0^3 2\pi x\sqrt{4-x}\,dx$

Let $u^2 = 4 - x$,
$x = 4 - u^2$
$dx = -2u\,du$

$= \int_1^2 4\pi u^2\left(4-u^2\right)du = 4\pi\left[\dfrac{4u^3}{3} - \dfrac{u^5}{5}\right]_1^2$

$= \dfrac{188\pi}{15}$ cubic units

Q13ai $\dfrac{(a\tan\theta)^2}{a^2} - \dfrac{(b\sec\theta)^2}{b^2} = \tan^2\theta - \sec^2\theta = -1$

.: point $Q(a\tan\theta, b\sec\theta)$ satisfies the equation for H_2.

Q13aii $m_{PQ} = \dfrac{b\tan\theta - b\sec\theta}{a\sec\theta - a\tan\theta} = -\dfrac{b}{a}$

Equation of line PQ: $y - b\sec\theta = -\dfrac{b}{a}(x - a\tan\theta)$

.: $bx + ay = ab(\tan\theta + \sec\theta)$

Q13aiii Gradient of perpendicular to $PQ = -\dfrac{1}{m_{PQ}} = \dfrac{a}{b}$

Equation of the perpendicular (to PQ) through the origin O is $y = \dfrac{a}{b}x$.

The perpendicular lines intersect at $T(x, y)$ where

$\dfrac{a^2}{b}x + bx = ab(\tan\theta + \sec\theta)$, .: $x = \dfrac{ab^2(\tan\theta+\sec\theta)}{a^2+b^2}$ and

$y = \dfrac{a^2 b(\tan\theta+\sec\theta)}{a^2+b^2}$.: $OT = \sqrt{x^2+y^2} = \dfrac{ab(\tan\theta+\sec\theta)}{\sqrt{a^2+b^2}}$

$PQ = \sqrt{(a\sec\theta - a\tan\theta)^2 + (b\tan\theta - b\sec\theta)^2}$

$= \sqrt{a^2+b^2}\sqrt{(\tan\theta - \sec\theta)^2}$

.: area of $\Delta OPQ = \dfrac{1}{2}OT \times PQ = \dfrac{ab}{2}\sqrt{\left(\tan^2\theta - \sec^2\theta\right)^2} = \dfrac{ab}{2}$

independent of θ.

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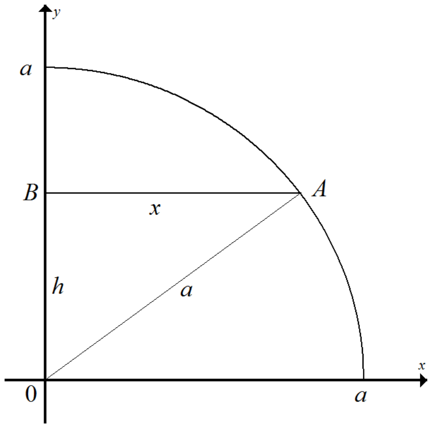
Q13bi $AB = x = \sqrt{a^2 - h^2}$



Q13bii

$$V = \int_0^a x^2 dh = \int_0^a \left(a^2 - h^2\right) dh = \left[a^2 h - \frac{h^3}{3}\right]_0^a = \frac{2a^3}{3} \text{ cubic units}$$

Q13ci $S = 4\pi r^2$, $\frac{dS}{dr} = 8\pi r$, $\frac{dr}{dt} = \frac{dr}{dS} \times \frac{dS}{dt} = \frac{1}{8\pi r}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}$

Q13cii $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dr} = 4\pi r^2$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi r^2 \times \frac{1}{8\pi r}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)^{\frac{1}{3}} = \frac{1}{2} V^{\frac{1}{3}}$$

Q13ciii The balloon will burst at $t = T$ seconds when the volume increases from 8000 (at $t = 0$) to 64000 cm^3.

$$\frac{dt}{dV} = 2V^{-\frac{1}{3}}$$

$$T = \int_{8000}^{64000} 2V^{-\frac{1}{3}} dV = \left[3V^{\frac{2}{3}}\right]_{8000}^{64000} = 3600 \text{ seconds} = 1 \text{ hour}$$

Q14ai $\frac{d}{d\theta}\sin^{n-1}\theta\cos\theta = (n-1)\sin^{n-2}\theta\cos^2\theta - \sin^n\theta$

$= (n-1)\sin^{n-2}\theta\left(1 - \sin^2\theta\right) - \sin^n\theta$

$= (n-1)\left(\sin^{n-2}\theta - \sin^n\theta\right) - \sin^n\theta$

$= (n-1)\sin^{n-2}\theta - n\sin^n\theta$

Q14aii $\int_0^{\frac{\pi}{2}}\left((n-1)\sin^{n-2}\theta - n\sin^n\theta\right)d\theta = \left[\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} = 0$

$\therefore \int_0^{\frac{\pi}{2}}(n-1)\sin^{n-2}\theta\, d\theta - \int_0^{\frac{\pi}{2}} n\sin^n\theta\, d\theta = 0$

$\therefore n\int_0^{\frac{\pi}{2}}\sin^n\theta\, d\theta = (n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\, d\theta$

$\therefore \int_0^{\frac{\pi}{2}}\sin^n\theta\, d\theta = \frac{(n-1)}{n}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\, d\theta$

Q14aiii By $\int_0^{\frac{\pi}{2}}\sin^n\theta\, d\theta = \frac{(n-1)}{n}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\, d\theta$

$$\int_0^{\frac{\pi}{2}}\sin^4\theta\, d\theta = \frac{3}{4}\int_0^{\frac{\pi}{2}}\sin^2\theta\, d\theta = \frac{3}{4}\times\frac{1}{2}\int_0^{\frac{\pi}{2}} d\theta = \frac{3}{8}[\theta]_0^{\frac{\pi}{2}} = \frac{3\pi}{16}$$

Q14bi $\alpha\beta + \beta\gamma + \gamma\alpha = -p$ and $\alpha + \beta + \gamma = 0$

$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 0$

$\therefore 16 + 2(-p) = 0$, $p = 8$

Q14bii The cubic equation can be expressed as $x^3 = 8x - q$.

$\therefore \alpha^3 = 8\alpha - q$, $\beta^3 = 8\beta - q$, $\gamma^3 = 8\gamma - q$

$\therefore \alpha^3 + \beta^3 + \gamma^3 = 8(\alpha + \beta + \gamma) - 3q$, $\therefore -9 = -3q$, $q = 3$

Q14biii Now $x^3 = 8x - 3$, $\therefore x^4 = 8x^2 - 3x$

$\therefore \alpha^4 = 8\alpha^2 - 3\alpha$, $\beta^4 = 8\beta^2 - 3\beta$, $\gamma^4 = 8\gamma^2 - 3\gamma$

$\therefore \alpha^4 + \beta^4 + \gamma^4 = 8\left(\alpha^2 + \beta^2 + \gamma^2\right) - 3(\alpha + \beta + \gamma) = 8 \times 16 = 128$

Q14ci Vertical components: $N\cos\theta - \mu N\sin\theta - mg = 0$

$\therefore N = \frac{mg}{\cos\theta - \mu\sin\theta}$

Horizontal components: $N\sin\theta + \mu N\cos\theta = \frac{mv^2}{r}$

$\therefore \frac{mv^2}{r} = \frac{mg(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$, $\therefore v^2 = \frac{rg(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$

$v^2 = \frac{rg(\sin\theta + \mu\cos\theta)/\cos\theta}{(\cos\theta - \mu\sin\theta)/\cos\theta} = rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)$

Q14cii At speed V, where $V^2 = rg$, $\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = 1$

$\therefore \mu = \frac{1 - \tan\theta}{1 + \tan\theta}$

Since $\tan\theta > 0$ for $0 < \theta < \frac{\pi}{2}$, $\therefore 1 - \tan\theta < 1 + \tan\theta$, $\therefore \mu < 1$

Q15ai $1a = -kv^2$, $\frac{dv}{dt} = -kv^2$, $\therefore kt = \int \frac{-1}{v^2} dv$, $kt = \frac{1}{v} + c_1$

When $t = 0$, $v = u$, $\therefore c_1 = -\frac{1}{u}$, $\therefore \frac{1}{v} = kt + \frac{1}{u}$

Q15aii $1a = -\left(g + kw^2\right)$, $\frac{dw}{dt} = -k\left(\frac{g}{k} + w^2\right)$,

$\therefore -kt = \int \frac{1}{\frac{g}{k} + w^2} dw = \sqrt{\frac{k}{g}}\tan^{-1}\left(w\sqrt{\frac{k}{g}}\right) + c_2$

When $t = 0$, $w = u$, $\therefore c_2 = -\sqrt{\frac{k}{g}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$

$\therefore t == \sqrt{\frac{1}{gk}}\left(\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right) - \tan^{-1}\left(w\sqrt{\frac{k}{g}}\right)\right)$

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Q15aiii When $w=0$, $t=\sqrt{\frac{1}{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$ and $v=V$.

$\frac{1}{v}=kt+\frac{1}{u}$, .: $\frac{1}{V}=\frac{1}{u}+\sqrt{\frac{k}{g}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$

Q15aiv As $u\to\infty$, $\frac{1}{u}\to 0$ and $\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)\to\frac{\pi}{2}$

.: $\frac{1}{V}\approx\frac{\pi}{2}\sqrt{\frac{k}{g}}$, $V\approx\frac{2}{\pi}\sqrt{\frac{g}{k}}$ for very large u.

Q15bi Given $x\geq 0$, .: $1-x^2\leq 1\leq 1+x$

.: $\frac{1-x^2}{1+x}\leq\frac{1}{1+x}\leq\frac{1+x}{1+x}$, .: $1-x\leq\frac{1}{1+x}\leq 1$

Q15bii $\int_0^{\frac{1}{n}}(1-x)dx\leq\int_0^{\frac{1}{n}}\frac{1}{1+x}dx\leq\int_0^{\frac{1}{n}}1\,dx$

.: $\frac{1}{n}-\frac{1}{2n^2}\leq\ln\left(1+\frac{1}{n}\right)\leq\frac{1}{n}$.: $1-\frac{1}{2n}\leq n\ln\left(1+\frac{1}{n}\right)\leq 1$

Q15biii $1-\frac{1}{2n}\leq n\ln\left(1+\frac{1}{n}\right)\leq 1$, .: $1-\frac{1}{2n}\leq\ln\left(1+\frac{1}{n}\right)^n\leq 1$

.: $e^{1-\frac{1}{2n}}\leq\left(1+\frac{1}{n}\right)^n\leq e$

As $n\to\infty$, $e^{1-\frac{1}{2n}}\to e$, .: $e\leq\left(1+\frac{1}{n}\right)^n\leq e$ for very large n

.: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$

Q15ci Given $\sqrt{xy}\leq\frac{x+y}{2}$, where $x>0$ and $y>0$.

$\left(\sqrt{xy}\right)^2\leq\left(\frac{x+y}{2}\right)^2$, $xy\leq\frac{x^2+2xy+y^2}{4}$, $4xy\leq x^2+2xy+y^2$

$xy\leq\frac{x^2+y^2}{2}$, $\sqrt{xy}\leq\sqrt{\frac{x^2+y^2}{2}}$

Q15cii $\sqrt{xy}\leq\sqrt{\frac{x^2+y^2}{2}}$, let $x=\sqrt{ab}$ and $y=\sqrt{cd}$

$\sqrt{\sqrt{ab}\sqrt{cd}}\leq\sqrt{\frac{\left(\sqrt{ab}\right)^2+\left(\sqrt{cd}\right)^2}{2}}\leq\sqrt{\frac{\frac{a^2+b^2}{2}+\frac{c^2+d^2}{2}}{2}}$

.: $\sqrt[4]{abcd}\leq\sqrt{\frac{a^2+b^2+c^2+d^2}{4}}$ for positive real numbers a, b, c and d.

Q16ai 3 ways to put a black counter in the first column, 3 ways to put another black counter in the second column etc.

.: 3^5 ways to place the 5 black counters, one in each column.

There are $\frac{15!}{5!10!}=3003$ ways in total in placing the counters in the cells without restrictions.

.: $\Pr(\text{exactly one black counter in each column})=\frac{3^5}{3003}=\frac{81}{1001}$

Q16aii n ways to put a black counter in the first column, n ways to put another black counter in the second column etc.

.: n^q ways to place the q black counters, one in each column.

There are $\frac{(nq)!}{q!(nq-q)!}=\binom{nq}{q}$ ways in total in placing the counters in the cells without restrictions.

.: $\Pr(\text{exactly one black counter in each column})=P_n=\frac{n^q}{\binom{nq}{q}}$

Q16aiii $P_n=\frac{n^q}{\binom{nq}{q}}=\frac{n^q}{\frac{(nq)!}{q!(nq-q)!}}=\frac{n^q q!(nq-q)!}{(nq)!}$

$=\frac{n^q q!(nq-q)!}{(nq)(nq-1)(nq-2)\cdots(nq-q+1)(nq-q)!}$

$=\frac{n^q q!}{(nq)(nq-1)(nq-2)\cdots(nq-q+1)}$

$=\frac{q!}{(q)\left(q-\frac{1}{n}\right)\left(q-\frac{2}{n}\right)\cdots\left(q-\frac{q-1}{n}\right)}$

.: $\lim_{n\to\infty}P_n=\frac{q!}{q^q}$

Q16bi By de Moivre's theorem :

$(\cos\alpha+i\sin\alpha)^{2n}=\cos(2n\alpha)+i\sin(2n\alpha)$

By binomial expansion and group real and imaginary parts:

$$(\cos\alpha+i\sin\alpha)^{2n}=\cos^{2n}\alpha-\binom{2n}{2}\cos^{2n-2}\alpha\sin^2\alpha+\binom{2n}{4}\cos^{2n-4}\alpha\sin^4\alpha+\cdots+(-1)^{n-1}\binom{2n}{2n-2}\cos^2\alpha\sin^{2n-2}+(-1)^n\sin^{2n}\alpha+i(\text{imaginary part})$$

Hence

$$\cos(2n\alpha)=\cos^{2n}\alpha-\binom{2n}{2}\cos^{2n-2}\alpha\sin^2\alpha+\binom{2n}{4}\cos^{2n-4}\alpha\sin^4\alpha+\cdots+(-1)^{n-1}\binom{2n}{2n-2}\cos^2\alpha\sin^{2n-2}+(-1)^n\sin^{2n}\alpha$$

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Q16bii Let $\alpha = \cos^{-1} x$, .: $\cos\alpha = x$, $\sin^2 x = 1 - x^2$

$T_{2n}(x) = \cos(2n\alpha)$

$= x^{2n} - \binom{2n}{2} x^{2n-2}(1-x^2) + \binom{2n}{4} x^{2n-4}(1-x^2)^2 + \cdots + (-1)^n (1-x^2)^n$

Q16biii The $2n$ roots of $T_{2n}(x) = \cos(2n\cos^{-1} x)$ are:

$2n\cos^{-1} x = \frac{\pi}{2} + k\pi$, .: $\cos^{-1} x = \frac{(2k+1)\pi}{4n}$

.: $x = \cos\frac{(2k+1)\pi}{4n}$ where $k = 0, 1, 2, \cdots, 2n-1$

The product of the roots $= \cos\frac{\pi}{4n} \cos\frac{3\pi}{4n} \cdots \cos\frac{(4n-1)\pi}{4n}$

$T_{2n} = x^{2n} - \binom{2n}{2} x^{2n-2}(1-x^2) + \binom{2n}{4} x^{2n-4}(1-x^2)^2 - \cdots + (-1)^n (1-x^2)^n$

$= x^{2n} - \binom{2n}{2} x^{2n-2}(1-x^2) + \binom{2n}{4} x^{2n-4}(1-2x^2+x^4) - \cdots + (-1)^n (1 \cdots + (-1)^n x^{2n})$

$= x^{2n} + \binom{2n}{2}(\cdots + x^{2n}) + \binom{2n}{4}(\cdots + x^{2n}) + \cdots + (\cdots + x^{2n})$

$= x^{2n}\left(1 + \binom{2n}{2} + \binom{2n}{4} + \cdots + \binom{2n}{2n}\right) + \cdots + (-1)^n$

.: $\cos\frac{\pi}{4n} \cos\frac{3\pi}{4n} \cdots \cos\frac{(4n-1)\pi}{4n} = \frac{(-1)^n}{1 + \binom{2n}{2} + \binom{2n}{4} + \cdots + \binom{2n}{2n}}$

Since, by binomial expansion,

$2^{2n} = (1+1)^{2n} = 1 + \binom{2n}{1} + \binom{2n}{2} + \cdots + \binom{2n}{2n}$ and

$0^{2n} = (1-1)^{2n} = 1 - \binom{2n}{1} + \binom{2n}{2} - \cdots + \binom{2n}{2n}$

.: $2^{2n} + 0^{2n} = 2\left(1 + \binom{2n}{2} + \binom{2n}{4} + \cdots + \binom{2n}{2n}\right)$

.: $\frac{2^{2n}}{2} = 1 + \binom{2n}{2} + \binom{2n}{4} + \cdots + \binom{2n}{2n}$

.: $1 + \binom{2n}{2} + \binom{2n}{4} + \cdots + \binom{2n}{2n} = 2^{2n-1}$

.: $\cos\frac{\pi}{4n} \cos\frac{3\pi}{4n} \cdots \cos\frac{(4n-1)\pi}{4n} = \frac{(-1)^n}{2^{2n-1}}$

Q16biv

$T_{2n} = x^{2n} - \binom{2n}{2} x^{2n-2}(1-x^2) + \binom{2n}{4} x^{2n-4}(1-x^2)^2 - \cdots + (-1)^n (1-x^2)^n$

$T_{2n}(x) = \cos(2n\cos^{-1} x)$

Let $\cos^{-1} x = \frac{\pi}{4}$, .: $x = \frac{1}{\sqrt{2}}$

$T_{2n}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2^n} - \binom{2n}{2}\frac{1}{2^n} + \binom{2n}{4}\frac{1}{2^n} - \cdots + (-1)^n \frac{1}{2^n} = \cos\left(\frac{n\pi}{2}\right)$

.: $1 - \binom{2n}{2} + \binom{2n}{4} - \cdots + (-1)^n \binom{2n}{2n} = 2^{2n}\cos\left(\frac{n\pi}{2}\right)$

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