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Specialist Mathematics

2017

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

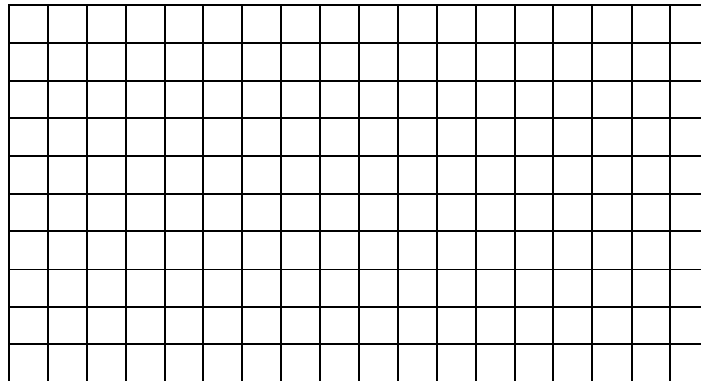
Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1

Consider relation $S = \{(x, y) : |y| = \sqrt{|x|}, -9 < x < 9\}$.

a. Sketch the graph of S on the Cartesian plane.

2 marks



b. Find $\frac{dy}{dx}$ in terms of x . Express your answer in simplest form (least number of pro-numerals).

3 marks

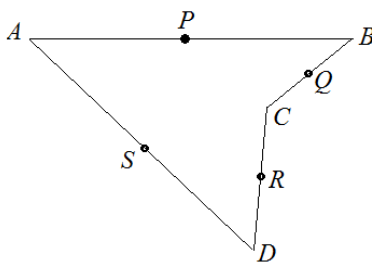
c. State the domain of $\frac{dy}{dx}$.

1 mark

Question 2

$ABCD$ is a quadrilateral. P, Q, R and S are the midpoints of sides AB, BC, CD and DA respectively.

Let vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ and \overrightarrow{DA} be $\tilde{a}, \tilde{b}, \tilde{c}$ and \tilde{d} respectively.



- a. Express \tilde{d} in terms of \tilde{a}, \tilde{b} and \tilde{c} .

1 mark

- b. Show that $PQRS$ is a parallelogram.

2 marks

Question 3

Given $z \in \mathbb{C}$ and $|z| = 1$. Let $u = (1+i)z$ and $v = \frac{\bar{z}}{1-i}$.

- a. Show that $z^2 = u\bar{v}$.

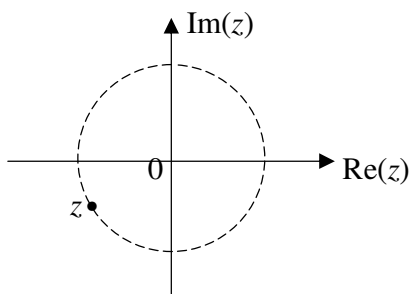
2 marks

- b. Express uv in simplest form.

2 marks

- c. $z \in \mathbb{C}$ is shown in the Argand diagram below. Plot complex number uvz on the same diagram.

1 mark



Question 4

The position of a particle moving in a straight line from a fixed origin is given by $x = \sqrt{3} \sin 2t + \cos 2t$, where x is in metres and $t > 0$ is in seconds.

- a. Find the earliest time when it reaches maximum speed.

2 marks

- b. Find the maximum speed of the particle.

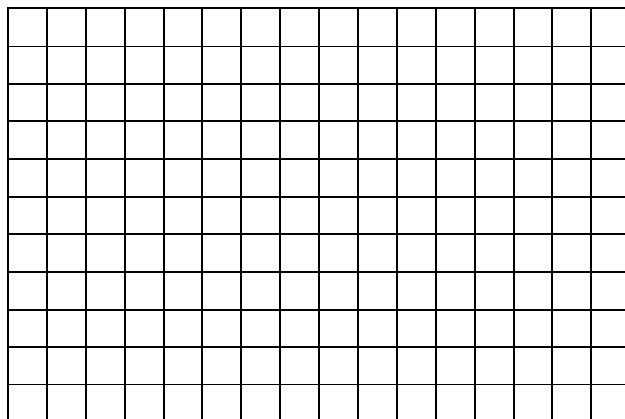
2 marks

Question 5

The position of a particle from a fixed origin at time t seconds, $t \geq 0$, is given by $\tilde{r} = (2 \sin 2t)\tilde{i} - (2 \cos 2t)\tilde{j}$, where components are in metres.

- a. Sketch the path of the particle for $0 \leq t \leq \frac{\pi}{4}$.

3 marks



- b. Find the length of the path of the particle for $0 \leq t \leq \frac{\pi}{4}$.

1 mark

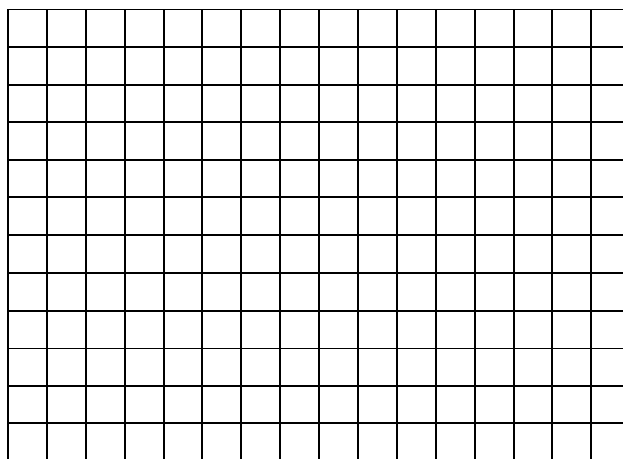
Question 6

$(2, 1)$ and $(-2, -1)$ are points on the solution curve(s) to the differential equation $\frac{dy}{dx} - \frac{x}{2y} = 0$.

- a. Find the Cartesian equation of the solution curve(s).

2 marks

- b. Sketch the solution curve(s). Show and label the important features with coordinates or equations. 2 marks



Question 7

In 2016 a particular subject had a mean score of 32 with a standard deviation of 8 for female students, and a mean score of 29 with a standard deviation of 10 for male students. The ratio of female to male students sat for the examination was 3 : 2. A random sample of 10 students (irrespective to genders) was taken from all the students who sat for the examination. Let random variable \bar{X} be the mean score of a sample. It happened by chance that there were 6 female and 4 male students in the sample.

- a. Use the relevant information to find $E(\bar{X})$.

1 mark

- b. Use the relevant information to find $sd(\bar{X})$.

2 marks

Question 8

One of the sixth roots of -1 is -1 .

- a. Write down the other sixth roots of -1 in $x + yi$ form.

2 marks

- b. Hence solve $(z - 2i)^6 + 1 = 0$ for z in $x + yi$ form.

2 marks

Question 9

Let $\tilde{p} = 2\tilde{i} + \tilde{j} + 2\tilde{k}$, $\tilde{q} = -\tilde{i} - 2\tilde{j} + 2\tilde{k}$ and $\tilde{r} = 2\tilde{i} - 2\tilde{j} - \tilde{k}$ be vectors in 3-dimensional space defined by perpendicular unit vectors \tilde{i} , \tilde{j} and \tilde{k} .

- a. Find $\tilde{s} = -2\tilde{p} + 3\tilde{q} - \tilde{r}$ in terms of \tilde{i} , \tilde{j} and \tilde{k} .

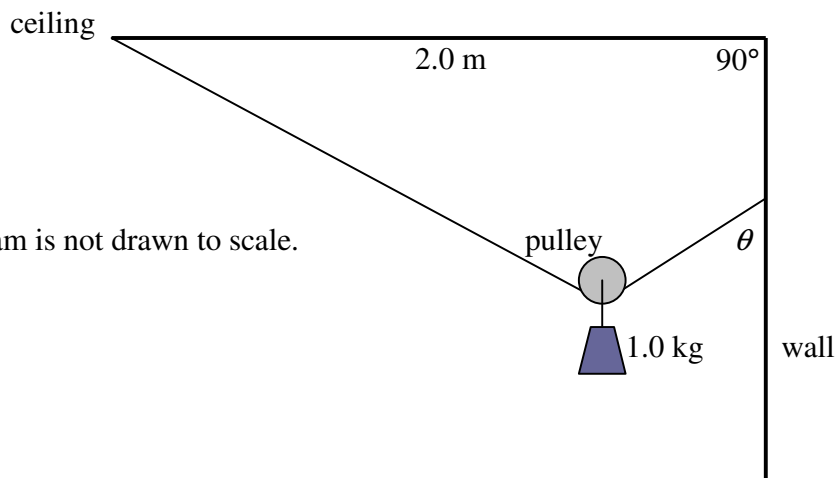
1 mark

- b. Determine whether \tilde{p} , \tilde{q} and \tilde{r} are linearly dependent or independent vectors. Explain.

2 marks

Question 10

One end of a 3.0 m long string is fastened to the ceiling and the other end to a wall as shown below. A pulley has a 1.0 kg mass attached to it and rolls along the string. Assume that the pulley is a point and has no mass.



Note: The diagram is not drawn to scale.

The system is in equilibrium.

Let θ be the acute angle between the string and the wall.

a. Show that $\sin \theta = \frac{2}{3}$.

2 marks

b. Hence find the tension in the string when the system is in equilibrium, in terms of g N.

2 marks

End of Exam 1