2018 Year 10 math topic test: Relations and transformations

Q1 Consider the relation $y = x^2$.

a. Explain why $y = x^2$ is different from $\frac{y}{x^2} = 1$. 1 mark

b. The graph of $y = x^2$ is dilated by the same factor k in both x and y directions, where $k \in R$. Write down the equation of the graph after the two dilations in simplest form.

2 marks

c. The graph of $y = x^2$ is dilated by a factor of 2 in the x direction and then translated by 2 units in the positive x direction. Write down the equation of the graph after the transformations in simplest form.

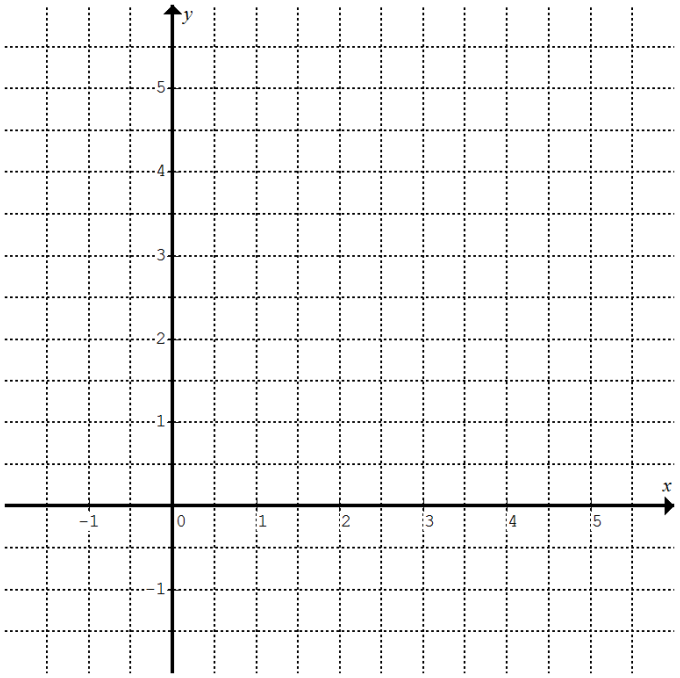
2 marks

d. The graph of $y = x^2$ is translated by 2 units in the positive x direction and then dilated by a factor of 2 in the x direction. Write down the equation of the graph after the transformations in simplest form.

2 marks

e. Sketch the graph of $y = x^2$ and the **transformed graphs** of $y = x^2$ in parts c and d on the same set of axes shown below, showing correctly the turning points and axis-intercepts.

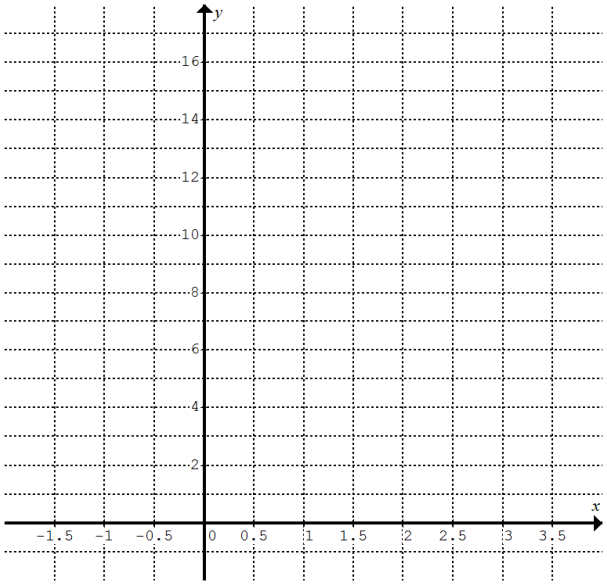
5 marks



Q2 Consider $y = 2^x$ and $y = 4^x$.

a. Sketch the graphs of $y = 2^x$ and $y = 4^x$ on the same set of axes shown below. Show correctly the points at $x = -1, 0, 1, 2$ for each graph.

4 marks



b. Circle the correct one of the following statements related to $y = 2^x$ and $y = 4^x$. 1 mark

A. $y = 4^x$ is the dilation of $y = 2^x$ by a factor of 2 in the x direction.

B. $y = 4^x$ is the dilation of $y = 2^x$ by a factor of $\frac{1}{2}$ in the x direction.

C. $y = 4^x$ is the dilation of $y = 2^x$ by a factor of 2 in the y direction.

D. $y = 4^x$ is the dilation of $y = 2^x$ by a factor of $\frac{1}{2}$ in the y direction.

c. Justify your answer to part b. 2 marks

d. Circle the correct one of the following statements related to $y = 2^x$ and $y = \left(\frac{1}{2}\right)^{-x}$. 1 mark

A. $y = \left(\frac{1}{2}\right)^{-x}$ is the reflection of $y = 2^x$ in the x-axis.

B. $y = \left(\frac{1}{2}\right)^{-x}$ is the reflection of $y = 2^x$ in the y-axis.

C. $y = \left(\frac{1}{2}\right)^{-x}$ and $y = 2^x$ are the same relation.

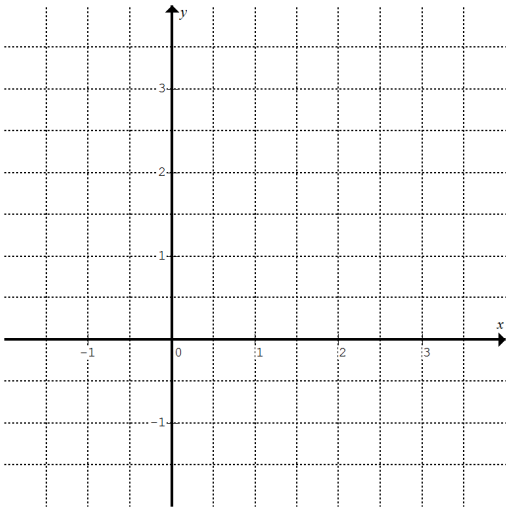
D. $y = 2^x$ is the reflection of $y = \left(\frac{1}{2}\right)^{-x}$ in the x-axis.

e. Justify your answer to part d. 2 marks

Q3 Consider $y = \dfrac{1}{x-\sqrt{2}}$ and $y = \dfrac{1}{x} + \sqrt{2}$.

a. Sketch the graphs of $y = \dfrac{1}{x-\sqrt{2}}$ and $y = \dfrac{1}{x} + \sqrt{2}$ on the same set of axes shown below.

Clearly show and label with equations the asymptotes of each graph. 4 marks



b. Write down two transformations of $y = \dfrac{1}{x-\sqrt{2}}$ to change it to $y = \dfrac{1}{x} + \sqrt{2}$. 2 marks

c. Calculate the coordinates of the intersection/s of the graphs of $y = \dfrac{1}{x-\sqrt{2}}$ and $y = \dfrac{1}{x} + \sqrt{2}$.

4 marks

d. Show by calculations the translation of $y = \dfrac{1}{x-\sqrt{2}}$ by $2\sqrt{2}$ units in the negative x direction is the same as the reflections of $y = \dfrac{1}{x-\sqrt{2}}$ in both axes. 3 marks

Q4 Consider the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2(x + y) + 1 = 0$.

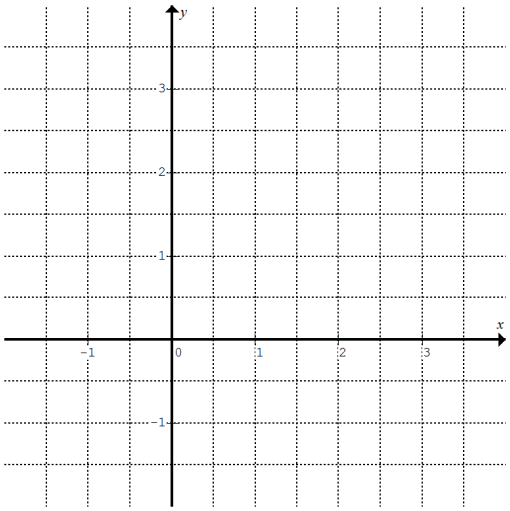
a. Determine the radius of $x^2 + y^2 - 2(x + y) + 1 = 0$. 3 marks

b. Explain the fact that $x^2 + y^2 - 2(x + y) + 1 = 0$ is the translation of $x^2 + y^2 = 1$ by $\sqrt{2}$ units along the line $y = x$.

2 marks

c. Sketch the graphs of $x^2 + y^2 = 1$ and $x^2 + y^2 - 2(x + y) + 1 = 0$ on the same set of axes shown below.

2 marks



d. Write down the coordinates of the intersection/s of $x^2 + y^2 = 1$ and $x^2 + y^2 - 2(x + y) + 1 = 0$ .1 mark

e. **Calculate** the coordinates of the intersection/s of $x^2 + y^2 = 1$ and $y = \frac{1}{x}$. 3 marks

f. $x^2 + y^2 - 2(x + y) + 1 = 0$ and $y = \frac{1}{x + p} - p$ intersect on the axes. Find the exact value/s of p.

4 marks

