

**2018 Year 11 math topic test: Exponential and logarithmic functions** © itute 2018

Q1 Rewrite each one of the following exponent/logarithm in exact form using base 2.

a. $\log_{10} 1000$ 1 mark

b. $\log_e 1000$ 1 mark

c. 10^3 1 mark

d. e^3 1 mark

Q2 Consider logarithmic function $f(x) = \log_4(2x + 1)$.

a. Rewrite $y = \log_4(2x + 1)$ in exponential form with x as the subject of the equation. 2 marks

b. Write down f^{-1} . 1 mark

c. State the domain and range of f^{-1} . 2 marks

d. Without using CAS/calculators, find the solution/s to $f^{-1}(x) = f(x)$. 3 marks



Q3 Solve the following equations for the exact value/s of x .

a. $(10^{2x+1} - 1)(10^{2x+1} - 1) = 10^{2x+1} - 1$ 3 marks

b. $\log_{10} x^4 - \log_{10} x^2 + \log_{10}(2x^2) = 0$ 3 marks

Q4 Show

a. $(\log_a b)(\log_b c)(\log_c a) = 1$ 2 marks

b. $(\log_b a)^{-1} + (\log_c b)^{-1} + (\log_a c)^{-1} = \log_a b + \log_b c + \log_c a$ 2 marks

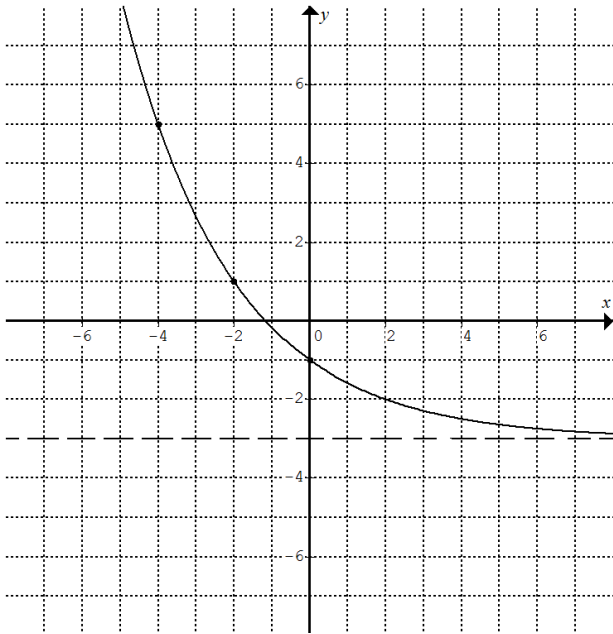
c. $\log_a b + \log_b c + \log_c a = (\log_c b)(\log_a c) + (\log_a c)(\log_b a) + (\log_b a)(\log_c b)$ 2 marks



Q5 The graph of function f shown below has equation $y = a \times 2^{bx} + c$, where $a, b, c \in R$.

a. Determine the value of each of a, b , and c .

3 marks



b. Determine the exact value of the x -intercept in the form $x = \log_p(q) + r$, where $p, q, r \in R$.

3 marks

c. Sketch the graph of f^{-1} on the diagram above.

Show asymptote with equation and points with coordinates.

3 marks

d. Determine the equation of f^{-1} .

3 marks

e. The given graph above can be made to pass through the origin by changing the value of a **only**.

Find that value of a .

2 marks



Q6 The magnitude M of an earthquake on the Richter scale is given by

$$M = \begin{cases} \log_{10} A + 1.6\log_{10} D - 0.15 & \text{for } D < 200 \\ \log_{10} A + 3.0\log_{10} D - 3.38 & \text{for } 200 < D < 600 \end{cases}$$

where A is the seismograph signal amplitude in mm and D is the seismograph distance in km from the epicentre of an earthquake.

a. Determine the magnitude (correct to 1 decimal place) of an earthquake; the epicentre is 500 km from the seismograph showing signal amplitude of 50 mm.

2 marks

b. If the magnitude of an earthquake is 5.0, and the signal amplitude is 27 mm, determine the distance (correct to the nearest km) of the seismograph from the epicentre.

2 marks

Q7 The value $\$V$ of an investment $\$A$ grows continuously and it will be doubled every 8 years.

a. Write an equation that allows the value $\$V$ to be calculated after t years.

3 marks

b. Calculate the value of $\frac{\$V}{\$A}$ after 24 years.

1 mark

c. If the value of the investment grows to $\$200\,000$ after 12 years, how much (correct to the nearest $\$10$) was invested initially?

1 mark

d. Find the number of years (correct to the nearest whole number) required for the investment to increase by 570%.

3 marks