Physics notes – Materials and their use in structures

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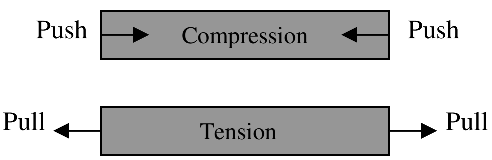
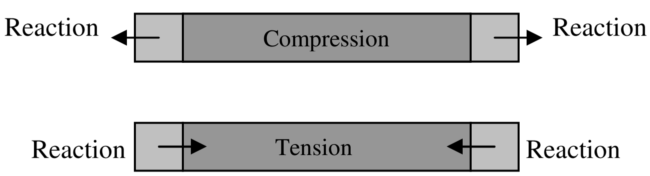
Compression and tension

A structure is in compression (tension) when it is pushed (pulled) at the two ends.

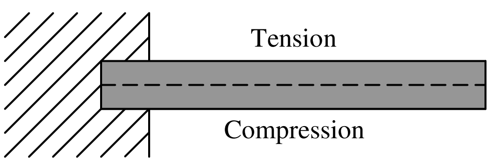


According to Newton's Third Law, the structure exerts a reaction force on the objects pushing or pulling it.

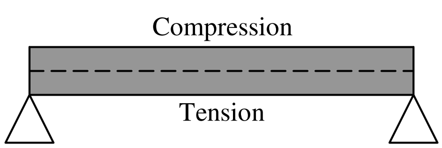


Compression and tension can co-exist within a structure:

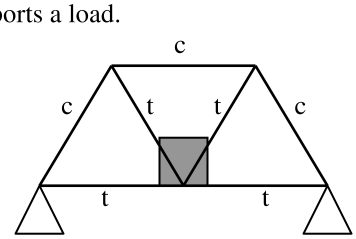
Example 1 A **cantilevered beam** bends under its own weight.



Example 2 A beam supported at its two ends, i.e. a **simply supported beam**, bends under its own weight.

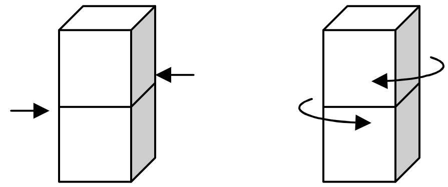


Example 3 A **truss**, a network of beams joined together in rigid triangles, supports a load.

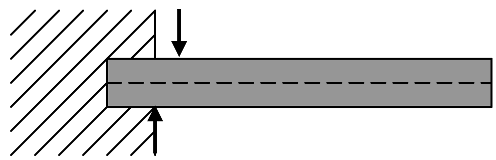


Shear

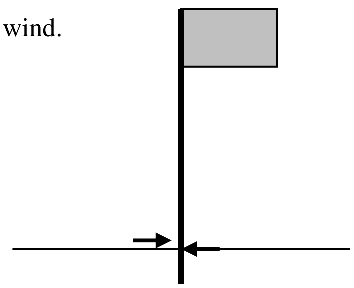
Shear refers to the sliding of a layer over another layer in a structure. It may occur when a structure experiences a sideways or twisting force.



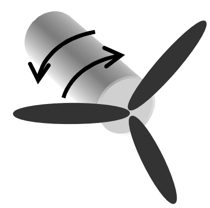
Example 1 Cantilevered beam under its own weight.



Example 2 Flag pole in the wind.

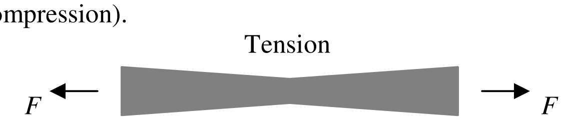


Example 3 A rotating propeller.



Stress

Consider a beam of non-uniform cross-section in tension (or compression).



Tension in the beam is the same along the entire length, which is equal to F, but the **stress** σ which is defined as $\sigma = \frac{F}{A}$, where A is the cross-sectional area, is different at different positions.

Since $\sigma \propto \frac{1}{A}$, therefore stress is highest at the narrowest section where fracture is most likely to occur.

The unit for stress σ is Nm^{-2}, or Pa (pascals).
Larger units are MPa (10^6Pa), GPa (10^9Pa).

Example 1 A cylindrical concrete column stands upright on solid ground. It is 3.5 m tall and has a radius of 0.2 m. Density of concrete is 2300 kg m^{-3}.
(a) What is the reaction force of the ground on the column?
(b) Calculate the compressive stress at the base of the column.
(c) Another upright concrete column has the same height but different radius. What is the stress at the base of the column?
(d) A third column has the same radius but is half as tall. What is the stress at the base?

(a) Weight of column = density × volume × g

$= 2300\times\pi\times 0.2^2\times 3.5\times 9.8 = 9.9\times 10^3$ N

Reaction force $= 9.9\times 10^3$ N.

(b) Stress $= \dfrac{F}{A} = \dfrac{9.9\times 10^3}{\pi\times 0.2^2} = 7.9\times 10^4$ Nm^{-2}.

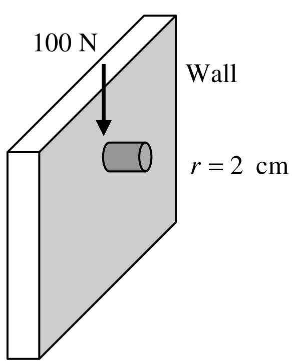
(c) Stress $= \dfrac{F}{A} = \dfrac{density\times A\times h\times g}{A} = density\times h\times g$.

$\therefore$ the stress is independent of the cross-sectional area A.

$\therefore$ stress $= 7.9\times 10^4$ Nm^{-2}.

(d) Since the stress is directly proportional to the height of the column (refer to part c), the stress is half of that in the first column, i.e. 3.9×10^4 Nm^{-2}.

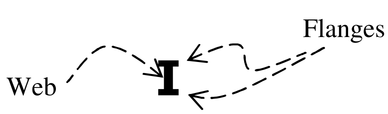
Example 2 Calculate the shear stress on the cylindrical rod due to a 100-N force exerted on it close to the wall.



Cross-sectional area of rod $= \pi\times 0.02^2 = 0.001257$ m^2.

Shear stress $= \dfrac{100}{0.001257} = 8.0\times 10^4$ Pa.

Example 3 In terms of tension, compression and shear, explain why in engineering steel beams usually have a cross-section of **I** or **[**.

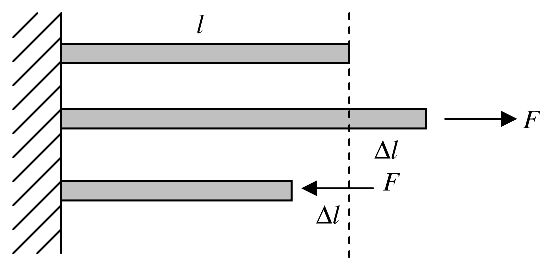


The flanges at the top and bottom of the beam are there to resist horizontal or longitudinal tension and compression, while the web in the middle is there to resist the vertical or shearing forces.

Strain

The length of a piece of material changes when it is loaded lengthwise (either in tension or compression). The fractional or percentage change in length is called **strain**. Strain ε is defined as $\varepsilon = \dfrac{\Delta l}{l}$, where l is the length of the material and Δl is the change in length.

ε is dimensionless, i.e. it has no unit. It is expressed as a decimal or per cent.



Example 1 Calculate the strain when a 8.00-m steel cable stretches by 1.00 cm under tensile stress. Express answer as a per cent.

$$\varepsilon = \frac{\Delta l}{l} = \frac{0.0100}{8.00} = 0.00125 = 0.125\%$$

Example 2 The compressive strain on a material is 0.25%.
(a) What is the decimal value of the ratio, *change in length* : *original length* before compression?
(b) What is the length of the material under the compressive strain if its original length is 20.00 cm?

(a) $\dfrac{\Delta l}{l} = \varepsilon = 0.25\% = 0.0025$

(b) $\Delta l = 0.0025l = 0.0025\times 20.00 = 0.05$ cm

Resulting length $= 20.00 - 0.05 = 19.95$ cm.

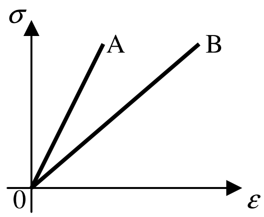
Relationship between stress and strain

There is a linear relationship between stress and strain for all materials under sufficiently small tensile or compressive stress, i.e.

$$\sigma = Y\varepsilon$$

where Y is a constant called **Young's modulus**. It is a measure of the **stiffness** of the material and the unit is Nm^{-2} (or Pa).

Different materials have different Y values. The gradient of σ-ε graph is the value of Y.



Material A is stiffer than B as shown by the gradients of the lines.

Example 1 Which of the following two materials is stiffer?

Material P:

σ (MPa)	30	60
ε	0.001	0.002

Material Q:

σ (MPa)	2	4
ε	0.00001	0.00002

For both P and Q, there is a linear relationship between σ and ε,

i.e. $\sigma = Y\varepsilon$, where $Y_P = \dfrac{30}{0.001} = 30000$ MPa,

and $Y_Q = \dfrac{2}{0.00001} = 200000$ MPa.

$Y_Q > Y_P$, material Q is stiffer than material P.

$\sigma = Y\varepsilon$ and $F = kx$

The first equation is essentially Hooke's Law:

In $\sigma = Y\varepsilon$, $\sigma = \dfrac{F}{A}$ and $\varepsilon = \dfrac{\Delta l}{l} = \dfrac{x}{l}$.

$\therefore \dfrac{F}{A} = Y\dfrac{x}{l}$, $\therefore F = \dfrac{YA}{l}x$, i.e. $F = kx$, where $k = \dfrac{YA}{l}$.

The value of the force constant k depends on the type and shape of the material under consideration. It is directly proportional to the cross-sectional area A and inversely proportional to the length l of the material.

Example 1 Young's modulus for rubber is about 7 MPa. Calculate the force constant for a rectangular piece of rubber measuring 2cm × 2cm × 3cm when it is compressed
(a) parallel to its length and
(b) perpendicular to its length.

(a) $A = 0.02 \times 0.02 = 0.0004\,\text{m}^2$, $l = 0.03$ m,

$k = \dfrac{YA}{l} = \dfrac{7\times10^6 \times 0.0004}{0.03} \approx 9\times10^4\,\text{Nm}^{-1}$.

(b) $A = 0.02 \times 0.03 = 0.0006\,\text{m}^2$, $l = 0.02$ m,

$k = \dfrac{YA}{l} = \dfrac{7\times10^6 \times 0.0006}{0.02} \approx 2\times10^5\,\text{Nm}^{-1}$.

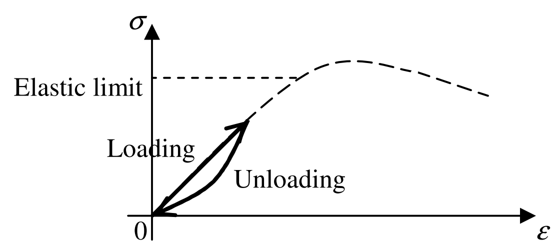
Example 2 A piece of cylindrical material has a force constant of 150 Nm^{-1}. What is the force constant of three pieces identical to the one just described
(a) when they are joined end to end to form a longer piece?
(b) when they are placed side by side to form a thicker piece?

(a) $k = \dfrac{YA}{l}$, $k \propto \dfrac{1}{l}$ for constant A. $\therefore$ when l is 3 times as long as before, k is $\dfrac{1}{3}$ of the value before, i.e. $k = \dfrac{1}{3}\times150 = 50\,\text{Nm}^{-1}$.

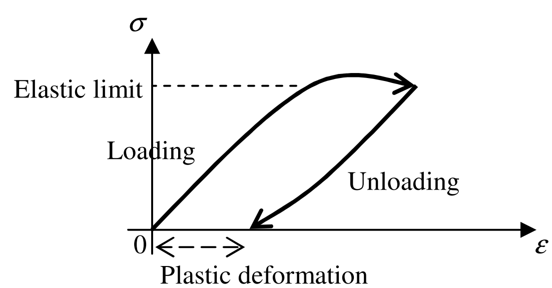
(b) $k \propto A$ for constant l. $\therefore$ when A is 3 times as large as before, k is 3 times the value before, i.e. $k = 3\times150 = 450\,\text{Nm}^{-1}$.

Elastic materials and elastic limit

If applied stress (loading) is below some limiting stress called the **elastic limit** of the material, the material deforms and returns to its original shape when the stress is removed (unloading). The material is said to show **elastic behaviour**.

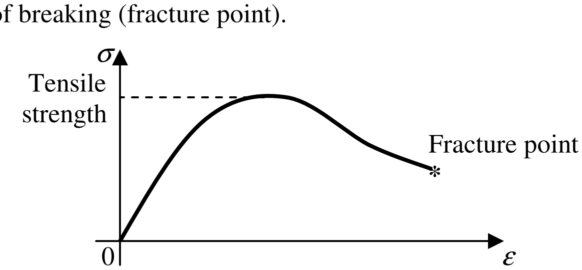


If the elastic limit is exceeded during loading, the material becomes permanently deformed and does not return to its original shape. The permanent change in shape is called **plastic deformation**, and it is said to show **plastic behaviour**.



Tensile and compressive strength

When loading of a material continues, eventually it reaches the point of breaking (fracture point).

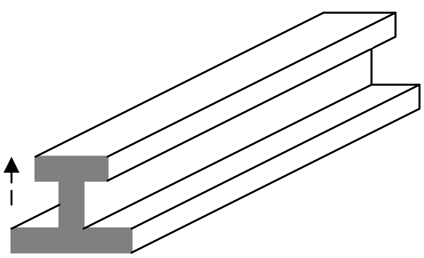


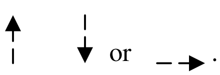
The maximum stress a material can withstand before breaking is called the **tensile** (or **compressive**) **strength** of the material.

The same material can behave differently under tension and compression. Its tensile strength can be different from the compressive strength, e.g. concrete is very strong under compression (high compressive strength) but weak under tension (low tensile strength).

Cast iron is weaker in tension than in compression while wood is weaker in compression than in tension.

Example 1 A cast iron beam has the following cross-section. If it is to be used as a simply supported beam, which way is the best to place the beam? Explain.

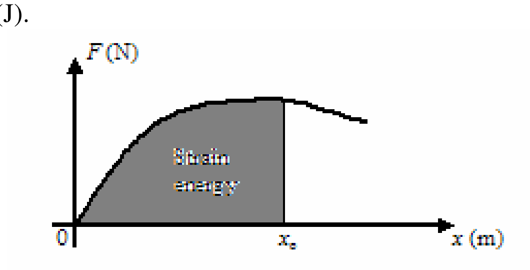




When used as a simply supported beam (upper section in compression, lower section in tension), it should be positioned as shown, because cast iron is weaker in tension than in compression, hence a larger flange is needed at the lower section.

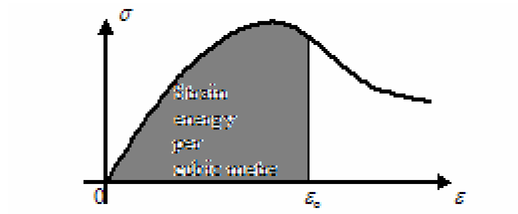
Strain energy

Strain energy is the amount of potential energy stored in the material when it is loaded. It can be estimated from the area under the force-extension (compression) graph. It is measured in joules (J).



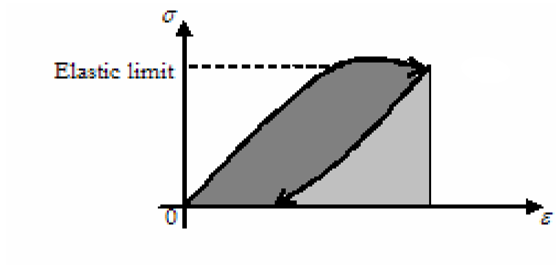
Alternatively, it can be obtained indirectly from the area under the stress-strain graph. This area gives the strain energy for each cubic metre of the material and has the unit Jm^{-3}.

$\therefore$ strain energy (J)
= volume of the material (m^3) $\times$ area under σ-ε graph.



If the stress is below the elastic limit, the potential energy stored in the material is called **elastic strain energy**. This stored energy is returned in other forms when the stress is removed.

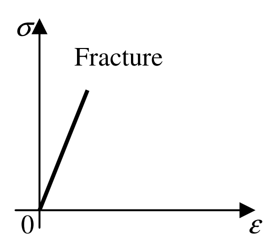
If the stress exceeds the elastic limit, some of the strain energy is put into plastic deformation and changes to heat and other form of potential energy. Only a portion is returned when the material is unloaded.

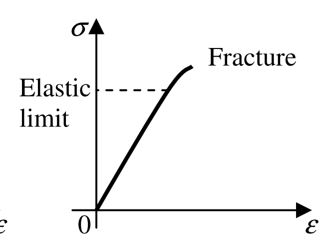


Area = energy absorbed in deforming a m^3 of the material
Area = energy returned for each m^3 of the material

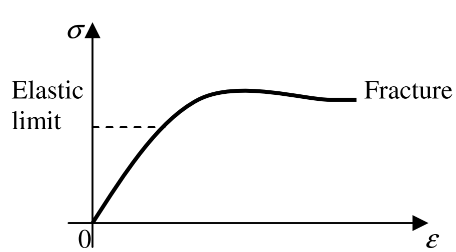
Brittleness, ductility and toughness

Materials behave differently under stress. Some materials fracture at the linear section or near the elastic limit, e.g. glass, ceramics. They are classified as **brittle** materials.





If a material fractures after it is well past the elastic limit and has undergone plastic deformation, it is called a **ductile** material, e.g. aluminium, steel.



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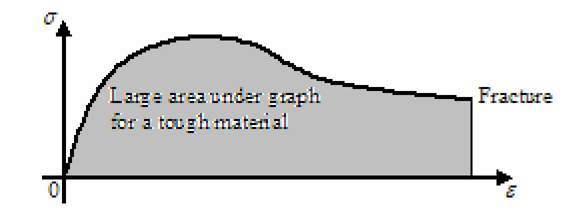
A **tough** material is ductile and it absorbs a large amount of strain energy before it fractures, e.g. polyethylene. The total area under the stress-strain graph gives a good indication of toughness when comparing materials.



Steel is the preferred material in building cranes and bridges etc, because it is relatively stiff (refer to Young's modulus) and tough.

Composite materials

Composite materials are made from two or more component materials that can be separated mechanically.

Clay with added straw is a composite material in making bricks. The purpose of putting chopped straw in clay is to prevent cracking when the wet clay is dried rapidly in the sun. In this case, clay is reinforced with straw.

Concrete is weak under tension because of the existence of small cracks. These cracks propagate easily when the material is stretched.
Concrete can be strengthened by placing steel rods or mesh in it when it is poured. It is called **reinforced concrete** and it is considered as a composite material.

Example 1 Explain why steel mesh (rods) is/are placed at the underside of a concrete driveway but at the upperside of a concrete cantilevered verandah.

Underside of a concrete driveway is in tension, and it requires steel mesh reinforcement to prevent cracks from propagating upwards causing the appearance of surface cracking.

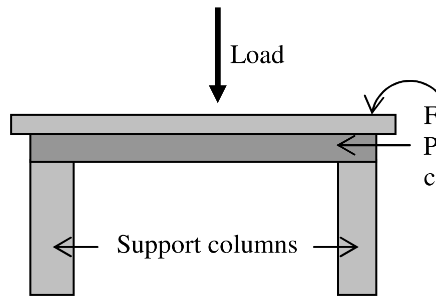
Upperside of a concrete cantilevered verandah is in tension, and it requires steel rods reinforcement to prevent cracks from propagating downwards causing the verandah to collapse.

Another way to strengthen concrete is to keep it in compression all the time so that cracks cannot propagate. One way to achieve this is to keep steel rods in tension while concrete is poured and allowed to dry before the tension is released. Contraction of the rods after the release of the tension compresses the concrete. The rods have a very rugged surface texture to prevent slipping after the tension is released. This strengthened concrete is called **pre-stressed concrete**.

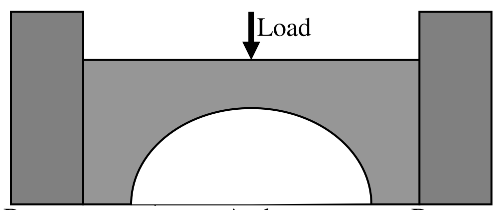
Concrete can also be strengthened by compression after it is set. This requires the steel rods to be smooth so that they can slip through the dry concrete when they are stretched and anchored at the ends of the concrete. This strengthened concrete is called **post-stressed concrete**.

They are always in compression provided the loads on them are below the recommended limit.

Example 1 Used as a horizontal beam in supporting a floor.



Pre-stressed concrete is frequently used in modern buildings. In older structures **arches** made with stones and bricks were employed. Such a structure is always under compression, due to its own weight and the load it supports, which keeps the stones or bricks in position together, e.g. a bridge.



The buttresses prevent the arch from widening leading to the collapse of the bridge.

Factors of safety

To ensure safety a structure is built to withstand loads that are many times what it actually carries. The number of times is called the **factor of safety**. In general industrial practice it is between 3 and 10.

It is defined as:

Factor of safety = $\dfrac{tensile(compressive)strength}{average.stress}$

for brittle materials.

Factor of safety = $\dfrac{elastic.\lim it}{average.stress}$ for ductile materials.

Example 1 A concrete column is designed to support an average stress of 2.5 MPa. What is the factor of safety allowed for the design?

Factor of safety = $\frac{compressive strength}{average.stress} = \frac{20}{2.5} = 8$.

Example 2 A steel (elastic limit 450MPa) cable is used to lift an average load of 800kg. What is the radius of the cable that will ensure a factor of safety of 5?

Average stress $= \frac{800 \times 9.8}{\pi r^2} = \frac{2495.5}{r^2}$.

Factor of safety = $\frac{elastic.\lim it}{average.stress}$, $5 = \frac{450 \times 10^6}{\frac{2495.5}{r^2}}$,

$r = 5.3 \times 10^{-3}$ m $= 5.3$ mm.

Choosing a suitable material

The following table shows some typical values in round figures.

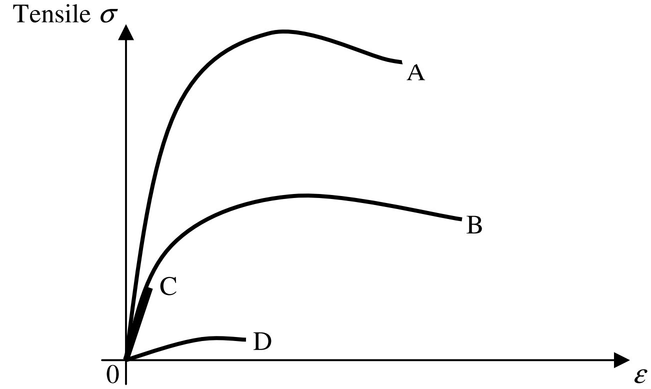
Material	Density (gcm^{-3})	Y's mod (GPa)	Elastic Limit (MPa)	Tensile Streng. (MPa)	Compres Strength (MPa)
Cast iron	8	-	200	200	650
Steel	8	200	450	600	600
Al. alloy	3	80	240	300	-
Concrete	4	18	4	4	20
Glass	4	70	100	100	-
Wood (pine)	0.5	15	35	40	35
Polyethylene	1	2	25	35	-

Cast iron contains about 2-4% of carbon while steel contains less than 1%.

Cast iron: For building iron arch bridges or similar structures.
Steel: Good for structures such as buildings where you do not want the structure to change shape under stress (such as wind stress and weight stress).
Aluminium alloy: For making window and door frames.
Concrete: For making slabs and panels in buildings.
Glass: For windows, doors and enclosures.
Wood: For building house frames.
Polyethylene: Not suitable as a structural material due to its relatively low modulus.

Example 1 Consider the following stress-strain graphs up to the fracture points for four building materials from the table.

Identify the material for each graph.
Which one is the (a) toughest? (b) most ductile? (c) most brittle? (d) stiffest? (e) strongest?



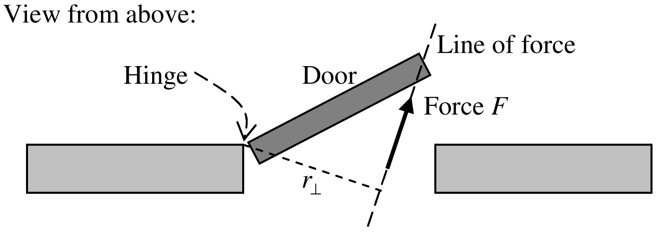
A: Steel B: Aluminium alloy C: Glass D: Pine wood

(a) A (b) B (c) C (d) A (e) A

Torques

A torque has the tendency to change the state of rotation of a structure. It is defined as the product of the force on a structure and the perpendicular distance from a chosen convenient point to the line of force.

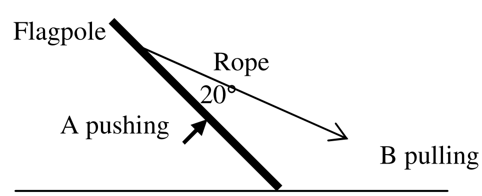
View from above:



The torque of F on the door about the hinge is $\tau = r_\perp \times F$.

Torque is a vector quantity and has the unit Nm. In two dimensions its direction is either clockwise or anticlockwise.

Example 1 A small flagpole is raised by two people, A and B.



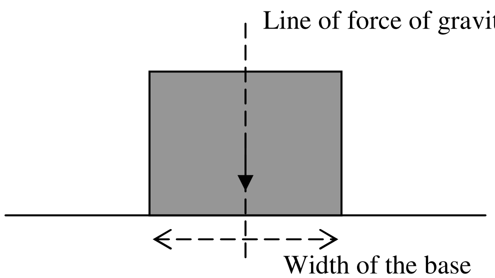
A exerts a force of 100 N perpendicularly to the pole at 1.5 m from the base. The tension in the rope is 80 N. The rope is fastened to the pole at an angle of 20° and 3.0 m from the base. Calculate the net torque exerted by A and B on the pole about the base.

Both A and B exert clockwise torque on the flagpole. Perpendicular distance from the base to the line of force of B $= 3.0 \sin 20° = 1.0$ m.
$\tau_{net} = 100 \times 1.5 + 80 \times 1.0 = 230$ Nm.

Structures in equilibrium

A structure in a state of balance is said to be in equilibrium. In this section our concern is with structures which are at rest. Such structures are in **static equilibrium**.

When a structure is in static equilibrium, the line of force of gravity passes through its base.

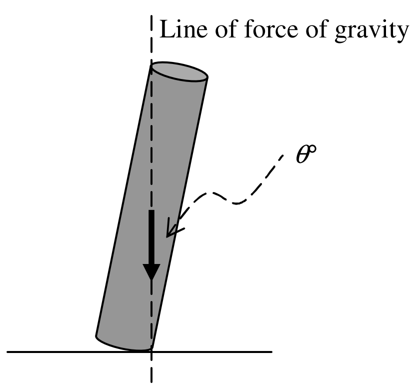


Static equilibrium can be **stable** or **unstable**, e.g.

Both structures above are in static equilibrium, but the one on the right is unstable because a small displacement at the top will cause it to topple over.

One way to determine its stability is to check the width of the base of the structure relative to the height. Increasing the base width (or decreasing the height) increases its stability.

Example 1 The leaning Tower of Pisa is 54 m high and 7.0 m in diameter. Assume that the tower is a circular cylinder of uniform density, what leaning angle will bring the tower to the verge of toppling over?



$\tan\theta° = \dfrac{7.0}{54}$, $\theta \approx 7.4$.

Equilibrium conditions for structures

There are two possible types of motion that can occur to a structure. They are translation and rotation. The former occurs if the net force on the structure is non-zero, and the latter if the net torque is non-zero. The two conditions for static equilibrium are:

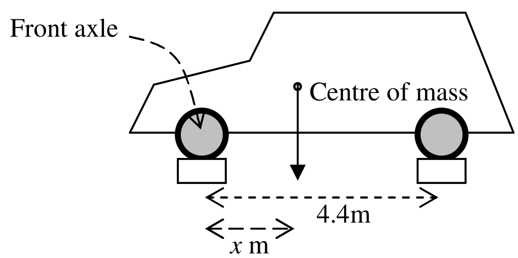
$$F_{net} = 0,\ \tau_{net} = 0$$

In two dimensional situations, equation (1) is split into *x* and *y* components:

$$F_{x,net} = 0,\ F_{y,net} = 0$$

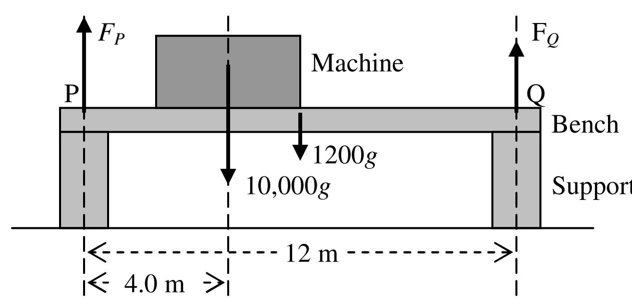
Therefore three equations can be set up to analyse a structure in static equilibrium.

Example 1 A 1200-kg car is supported by two scales. The one supporting the two front wheels has a reading of 700 kg. Determine the horizontal position of the centre of mass of the car from the front axle.



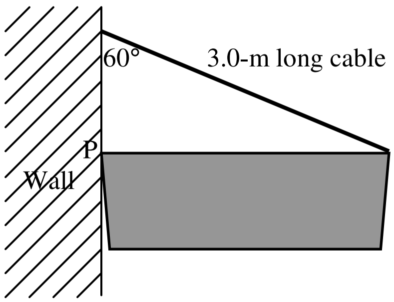
Reading on the scale supporting the two rear wheels $= 1200 - 700 = 500$ kg.
$\tau_{net} = 0$ about the front axle,
$\therefore 500g \times 4.4 - 1200gx = 0$, $x \approx 1.8$ m.
The centre of mass of the car is 1.8 m from the front axle.

Example 2 A horizontal uniform 1200-kg bench supports a 10,000-kg machine as shown in the diagram below.
Which part is considered as the structure on which the forces act?
Calculate the force and hence the stress on each of the two supports (cross-sectional area of each support is 2500 cm^2).



The bench is the structure on which the forces act.
$\tau_{net} = 0$ about point P, $F_Q \times 12 - 10000g \times 4.0 - 1200 \times 6.0 = 0$.
$F_{y,net} = 0$, $F_P + F_Q - 10000g - 1200g = 0$.
$F_Q \approx 3.9\times10^4$ N, $\sigma_Q = 3.9\times10^4 / 0.25 \approx 1.5\times10^5$ Nm^{-2}.
$F_P \approx 7.1\times10^4$ N, $\sigma_P = 7.1\times10^4 / 0.25 \approx 2.8\times10^5$ Nm^{-2}.

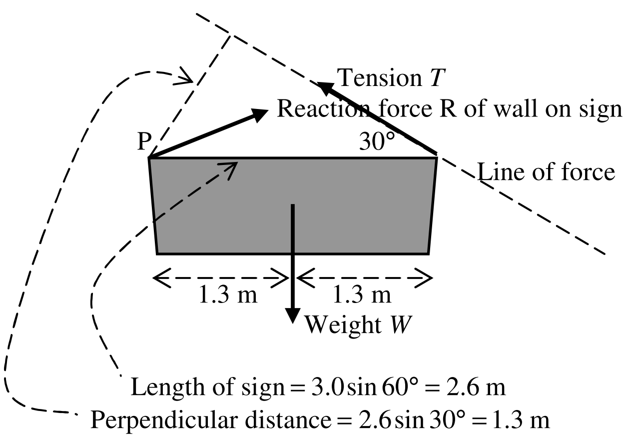
Example 3 One end of a 20-kg shop sign is attached to a wall at point P, and the sign is supported by a cable (of negligible mass) at the other end. The 3.0-m long cable makes a 60^0 angle with the wall.



(a) Name the structure. Is it the wall, the sign, the cable or a combination of two or more?
(b) Show and label all the forces on the structure.
(c) Calculate the tension in the cable.

(a) The sign.

(b)



(c) $\tau_{net} = 0$ about point P, $T \times 1.3 - 20g \times 1.3 = 0$, $T \approx 200\,\text{N}$.

