= Year 12 fm = Matrices (Transition) = Worksheet 4

Questions 1, 2, 3 and 4 are related

1. The following transition matrix can be used to predict the population in each of 4 towns *A*, *B C* and *D* in subsequent years. In 2010 the populations of *A*, *B C* and *D* are 10000, 20000, 30000 and 40000 respectively.

$$\begin{matrix} A & B & C & D \end{matrix}$$
$$\begin{bmatrix} 0.95 & 0 & 0.02 & 0.04 \\ 0.01 & 0.98 & 0.01 & 0 \\ 0.02 & 0.01 & 0.97 & 0 \\ 0.02 & 0.01 & 0 & 0.96 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

Find the steady state population of *D*.

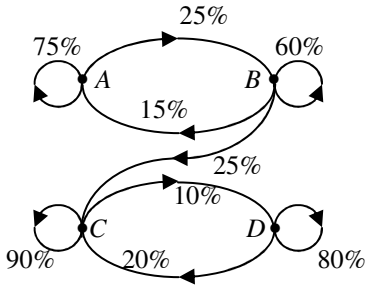
2. Eventually what percentage of the population in each town will remain in each town?

3. Predict the populations of the 4 towns in 2020. What were the populations in 2009?

4. In which year did *A* have its first resident?

5. Complete the transition matrix corresponding to the transition diagram.

$$\begin{matrix} A & B & C & D \end{matrix}$$
$$\begin{bmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & 0.25 & \_ & \_ \\ \_ & \_ & \_ & \_ \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$



6. Refer to Q5.

The state $\begin{bmatrix} 222 \\ 333 \\ 444 \\ 111 \end{bmatrix}$ becomes $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ after 2 transitions.

Write down the matrix equation to represent the transitions.
Find *a*, *b*, *c* and *d*.

7. Refer to Q5 and Q6.
Find the steady state.

8. (i) Draw a transition diagram for the transition matrix.

(ii) Find the steady state, given the initial state is $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

$$\begin{matrix} A & B & C & D \end{matrix}$$
$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.1 & 0.2 \\ 0.4 & 0.1 & 0.2 & 0.3 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

Numerical, algebraic and worded answers.

1. 20000

2. 26.67%, 26.67%, 26.67%, 26.67%

3. (i) 21763, 20417, 26765, 31055
(ii) 8145, 20013, 30554, 41289

4. 2006

5.
$$\begin{bmatrix} 0.75 & 0.15 & 0 & 0 \\ 0.25 & 0.60 & 0 & 0 \\ 0 & 0.25 & 0.90 & 0.20 \\ 0 & 0 & 0.10 & 0.80 \end{bmatrix}$$

6.
$$\begin{bmatrix} 0.75 & 0.15 & 0 & 0 \\ 0.25 & 0.60 & 0 & 0 \\ 0 & 0.25 & 0.90 & 0.20 \\ 0 & 0 & 0.10 & 0.80 \end{bmatrix}^2 \begin{bmatrix} 222 \\ 333 \\ 444 \\ 111 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$
$a = 201, b = 207, c = 545, d = 157$

7.
$$\begin{bmatrix} 0 \\ 0 \\ 740 \\ 370 \end{bmatrix}$$

8(ii)
$$\begin{bmatrix} 0.25(a+b+c+d) \\ 0.25(a+b+c+d) \\ 0.25(a+b+c+d) \\ 0.25(a+b+c+d) \end{bmatrix}$$