



Frames of reference

A frame of reference is a set of coordinate axes fixed to some body (or group of bodies) such as the earth, a moving train, the moon, spacecraft etc. Every measurement must be made with respect to a frame of reference. Many measurements are made with respect to the earth and it should be stated as the frame of reference. However, it is not always specified for the sake of simplicity.

Example 1 Consider a train travelling at 80 km h^{-1} in the same direction as a car travelling at 100 km h^{-1} . Both speeds have the same frame of reference, i.e. the earth (the ground). They are recorded by an observer standing on the ground. An observer in the moving car sees the train moving backwards at 20 km h^{-1} . In this case the frame of reference used to measure the speed of the train is the moving car. The earth is considered as a stationary frame of reference while the car is a moving frame of reference.

Can you describe the motion of the car seen by an observer in the train?

Everyday motion can be explained in terms of the **Newtonian model**.

The three laws in the Newtonian model

Newton's first law (also known as the **law of inertia**): If an object experiences no net force due to other bodies, it either remains at rest or remains in motion at the same speed in a straight line.

The tendency of an object to maintain its state of motion (i.e. at rest or in uniform straight-line motion) is called its **inertia**.

A frame of reference, in which Newton's first law is valid, is called an **inertial frame** of reference. Stationary and moving (at constant speed in a straight line, i.e. at constant velocity) frames of reference are inertial frames of reference.

A **non-inertial frame** of reference is one in which Newton's first law does not hold. A frame of reference which is moving with increasing or decreasing speed and/or in a curve is a non-inertial frame of reference.

Example 2 Explain why an accelerating car is a non-inertial frame of reference.

Inside the accelerating car you observe a passenger accelerates backwards with no force acting on the person. Thus Newton's first law is violated, hence the accelerating car is a non-inertial frame of reference.

Newton's second law: The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to its mass. The direction of the acceleration is the same as the direction of the net force.

$$a \propto F_{\text{net}}, a \propto \frac{1}{m}, \vec{a} \text{ and } \vec{F}_{\text{net}} \text{ are in the same direction}$$

These 3 ideas can be summarised as a single equation $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

or $\vec{F}_{\text{net}} = m\vec{a}$ which is known as Newton's second law.

Example 3 An empty truck has a maximum acceleration of 3.0 m s^{-2} .

- (a) What is its maximum acceleration when it is loaded with goods two times its mass?
 (b) What is the value of the ratio, the mass of the loaded truck M to the mass of the empty truck m , if the maximum acceleration is 2 m s^{-2} when loaded?

(a) The total mass is 3 times the mass of the truck when it is empty. \therefore the acceleration is $\frac{1}{3}$ of the acceleration when the

truck is empty, i.e. $\frac{1}{3} \times 3 = 1 \text{ m s}^{-2}$.

(b) $M \times 2 = m \times 3, \therefore \frac{M}{m} = \frac{3}{2}$.

Force of friction

Without friction between the car tyres and the road surface, a car cannot change its velocity, i.e. no acceleration is possible. It cannot speed up, slow down or change its direction.

During braking, the friction force on the tyres is opposite to the direction of motion. When the car is speeding up, the friction force is in the direction of motion. This is the force which propels the car forward and it is called the **driving force**. When the car is turning at constant speed, the friction force is perpendicular to the direction of motion (given by the velocity vector) and it is towards the centre of the turn.

Example 4 The wheels of a car of mass 1200 kg were locked in a sudden braking. It came to a stop from a speed of 30 m s^{-1} in 15 m . Determine the average friction force between each tyre and the road surface.

$$v^2 = u^2 + 2a_{\text{av}}s, 0 = 30^2 + 2a_{\text{av}}(15), a_{\text{av}} = -30 \text{ m s}^{-2}$$

$\vec{F}_{\text{net,av}} = m\vec{a}_{\text{av}} = 1200 \times -30 = -36000 \text{ N}$ (The negative sign means the friction force is opposite to the direction of motion).

For each tyre, average friction force = -9000 N .

Example 5 A 1.5 tonnes car towing a 1.5 tonnes caravan has an acceleration of 1.2 m s^{-2} . The total resistance to the motion of the car is 100 N , and to the motion of the caravan is 120 N .

- (a) Calculate the driving force of the car.
 (b) Calculate the tension in the towbar.

(a) Consider the car and the caravan together. The driving force of the car overcomes the resistance and causes both the car and the caravan to accelerate.

$$\vec{F}_{\text{net}} = \vec{F}_{\text{drive}} - 220, \text{ total mass } m = 1500 + 1500 = 3000 \text{ kg.}$$

$$\vec{F}_{\text{net}} = m\vec{a}, F_{\text{drive}} - 220 = 3000 \times 1.2, F_{\text{drive}} \approx 3.8 \times 10^3 \text{ N.}$$

(b) Consider the caravan alone. The force of the towbar overcomes the resistance on the caravan and causes it to accelerate.

$$\vec{F}_{\text{net}} = \vec{F}_{\text{towbar}} - 120, \vec{F}_{\text{net}} = m\vec{a}, \vec{F}_{\text{towbar}} - 120 = 1500 \times 1.2,$$

$$F_{\text{towbar}} \approx 1.9 \times 10^3 \text{ N.}$$

Example 6 A 2.0 tonnes truck slows down uniformly from 30 m s^{-1} to 20 m s^{-1} in 3.0 s .

- (a) Calculate the restraining force of a fastened seatbelt on the 80 kg driver.
 (b) What is the restraining force of a seatbelt on a 40 kg passenger?
 (c) The passenger holds a helium balloon inside the cabin with the windows closed. In which direction will the balloon move during the slow down?

(a) $v = u + at$, $20 = 30 + a \times 3.0$, $a = \frac{-10}{3.0}$.

Consider the 80 kg driver: $\vec{F}_{net} = m\vec{a}$,

$\vec{F}_{belt} = 80 \times \frac{-10}{3.0} \approx -2.7 \times 10^3 \text{ N}$ (-267 N)

(b) Consider the 40 kg passenger: Since $F_{net} \propto m$ and the mass of the passenger is $\frac{1}{2}$ of the mass of the driver, $\therefore \vec{F}_{belt}$ on the

passenger is $\frac{1}{2}$ of the force on the driver, i.e.

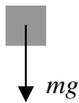
$\vec{F}_{belt} \approx \frac{1}{2} \times -2.67 \times 10^3 = -1.3 \times 10^3 \text{ N}$.

(c) Air has a higher density than helium. The air rushes to the front when the truck slows down and forces the helium balloon to the back of the cabin.

Force of gravity

When an m kg object falls (assuming that there is no air resistance) near the surface of the earth, the only force on it is the force of gravity $m\vec{g}$, where \vec{g} is the gravitational field strength 9.8 N kg^{-1} towards the centre of the Earth.

The force of gravity is also known as the **weight** of the object. It is the only force on the object, $\therefore \vec{F}_{net} = m\vec{g}$ newtons downward.



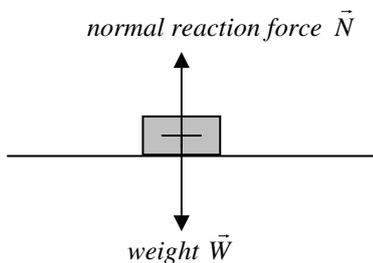
Example 7 Calculate the weight of an apple which has a mass of 120 grams. What is the force of gravity on the apple?

Weight = $mg = 0.120 \times 9.80 \approx 1.18 \text{ N}$

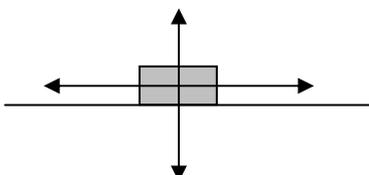
Force of gravity $\approx 1.18 \text{ N}$

Normal reaction force, net force

When the object rests on a horizontal surface, the surface supports the object by exerting a vertically upward force on it. This upward force has the same magnitude as the downward force of gravity, therefore the net force on the object $F_{net} = 0$. This explains why it remains at rest according to Newton's first law. The upward force is perpendicular to the surface and it is called the **normal reaction** of the surface on the object.



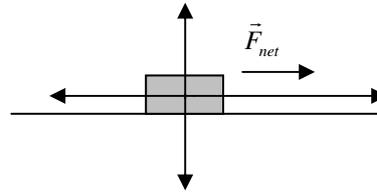
If there is a small pulling force and the surface is rough, friction exists and it prevents the object from sliding. In this case, the pulling force and the force of friction are equal but opposite. The vector sum of the four forces, i.e. the net force is still zero.



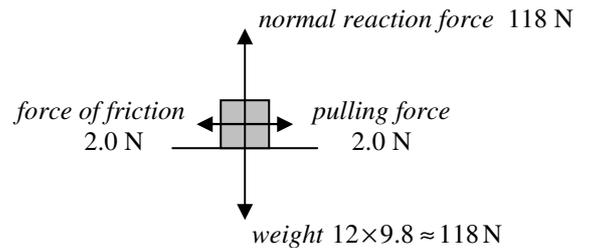
As long as the object remains at rest, force of friction is always equal in magnitude to the pulling (pushing) force.

However, there is an upper limit to the amount of friction.

If the pulling force exceeds the maximum force of friction, the net force is no longer zero and causes a change in motion, i.e. acceleration, according to Newton's second law.



Example 8 A 12 kg box is at rest on a rough horizontal surface while a 2.0 N pulling force acts on it. Draw a labelled force diagram to show the forces on the box. Determine the magnitude of each force.



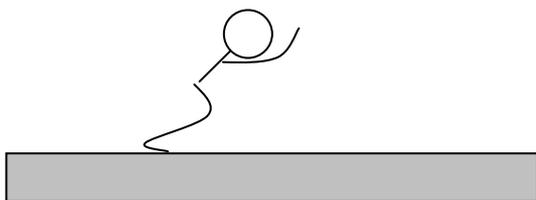
Example 9 If the pulling force in Example 8 is increased to 15 N, the box accelerates at 0.8 m s^{-2} . Determine the force of friction on the box.

$\vec{F}_{net} = m\vec{a} = 12 \times 0.8 = +9.6 \text{ N}$

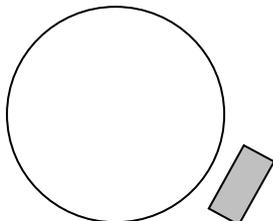
$\therefore +15 + \vec{F}_f = +9.6$, $\therefore \vec{F}_f = -5.4 \text{ N}$

The negative sign shows the friction force is opposite to the direction of motion.

Q1 Draw an arrow to show the force exerted by your feet on the ground due to friction when you take a forward leap.



Q3 Draw an arrow to show the force exerted by the tyres on the road due to friction when a car travels at a roundabout at *constant speed*, assuming that there is no air resistance or rolling resistance.



Q5 A 5-kg box slides on a horizontal plane. Its speed decreases 1.5 ms^{-1} in a second. Calculate the force of friction on the box.

Q7 A 900 kg car slows down uniformly from 20 m s^{-1} to 10 m s^{-1} in 3.0 s.

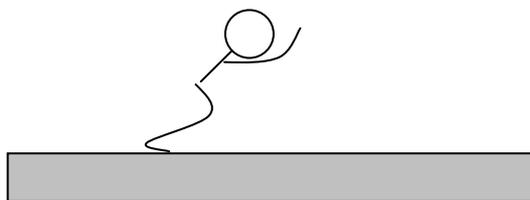
(a) Calculate the restraining force of a fastened seatbelt on the 70 kg driver.

(b) What is the braking force on the car with the driver inside?

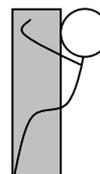
Q9 A kid tries to shift a 100-kg crate at rest on a rough floor with a horizontal force of 25 N. Determine the weight of the crate, the normal reaction force of the floor on the crate, and the force of friction on the crate.

Q11 Name the difference between an inertial frame of reference and a non-inertial frame.

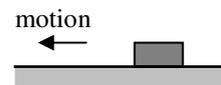
Q2 Draw an arrow to show the force exerted by the ground on your feet due to friction when you take a forward leap.



Q4 Draw an arrow to show the force exerted by your hands on a vertical pole due to friction when you climb up the pole.



Q6 A 3-kg box slides along a smooth (i.e. frictionless) horizontal surface. Draw an arrow to show the direction of the reaction force of the surface on the box. What is the magnitude of the reaction force?



Q8 What is the weight of a 2-kg bag of oranges resting on a table? What is the normal reaction force of the table on it?

Q10 The driving force causes a 1-tonne car to accelerate at 2.0 m s^{-2} against an average resistive force of 100 N. Calculate the magnitude of the average driving force.

Numerical, algebraic and worded answers:

1. Arrow points backwards. 2. Arrow points forwards.
3. Arrow points away from the centre of the circle.
4. Arrow points downwards. 5. 7.5 N 6. Arrow points upwards, 29.4 N
- 7a. 233 N 7b. 3230 N approx. 8. 19.6 N, 19.6 N 9. 980 N, 980 N, 25 N
10. 2100 N 11. Newton's first and second laws are true in an inertial frame of reference, but not in a non-inertial frame.