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Newton's second law in terms of impulse and momentum (in one dimension)

Only constant net force is considered to simplify the situation, otherwise calculus is required for the analysis.

Consider an object of mass m kg experiencing a net force, F_{net} newtons over a time interval Δt seconds. The net force changes its velocity from u m s^{-1} to v m s^{-1} during this time interval.

Since the net force is constant, the acceleration is also constant and given by $\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}-\vec{u}}{\Delta t}$.

Substitute $\vec{a} = \frac{\vec{v}-\vec{u}}{\Delta t}$ *in* $\vec{F}_{net} = m\vec{a}$ *to obtain* $\vec{F}_{net}\Delta t = m\vec{v} - m\vec{u}$.

The quantity $\vec{F}_{net}\Delta t$ *is called impulse* $\vec{I}$ *given to the object by the net force.* $\vec{I}$ *is measured in Ns and is in the same direction as the net force,* $\vec{I} = \vec{F}_{net}\Delta t$.

The two quantities $m\vec{v}$ *and* $m\vec{u}$ *are called final momentum* $\vec{p}_f$ *and initial momentum* $\vec{p}_i$ *respectively.*
The difference $m\vec{v} - m\vec{u}$ *is the change in momentum* $\Delta\vec{p}$.
Momentum is measured in kg m s^{-1} and it is a vector in the same direction as velocity.

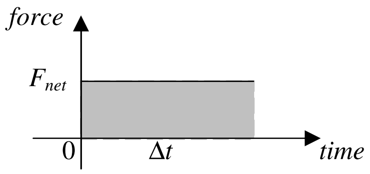
$\therefore \vec{p}_i = m\vec{u}$, $\vec{p}_f = m\vec{v}$ and $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v} - m\vec{u}$

Note: The unit for impulse Ns is equivalent to that for momentum kg m s^{-1}.

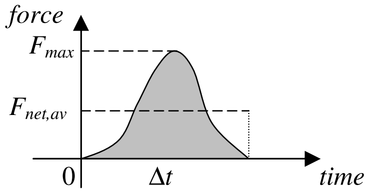
Now we have Newton's second law in terms of impulse and momentum, i.e. $\vec{I} = \Delta\vec{p}$ *instead of* $\vec{F}_{net} = m\vec{a}$.

Note: $\vec{I}$ *and* $\Delta\vec{p}$ *are in the same direction.*

The area under a force-time graph represents impulse (which is also equal to the change in momentum).



The same idea applies to non-constant force, e.g. force during a collision.



The area under the curve equals the area under the line $F_{net,av}$

$\therefore \vec{F}_{net,av}\Delta t = \Delta\vec{p}$ or $\vec{F}_{net,av} = \frac{\Delta\vec{p}}{\Delta t}$

$\therefore F_{net,av} \propto \frac{1}{\Delta t}$ *for constant Δp.*

If Δt is smaller, $F_{net,av}$ becomes larger. This explains, for example, why a person hitting an airbag sustains lesser injuries than a person hitting a strong rigid windscreen in a car accident. Both persons (of the same mass) have the same change in momentum, but the person hitting the windscreen has a shorter impact time and therefore experiences a stronger impact force than the other person hitting the airbag that helps to extend the stopping time.

Example 1 In a laboratory test a car without a crumple zone hits a strong barrier and comes to a halt. Two crash test dummies of the same mass 60 kg are placed at the front without seatbelts on. One of the dummies hits the windscreen and the other hits an airbag at 50 km h^{-1}. The measured stopping times are 0.003 s and 0.036 s respectively. Estimate the average net force on each dummy.

The following calculation is an oversimplification of the real situation because different parts of the dummies hit the car interior at different places and at different times. Assume that the effective mass hitting the windscreen or airbag is 7 kg.

50 km h^{-1} = 13.9 m s^{-1}

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - 7 \times {}^{+}13.9 = {}^{-}97$ kg m s^{-1}

Windscreen: $\vec{F}_{net,av} = \frac{\Delta\vec{p}}{\Delta t} = \frac{{}^{-}97}{0.003} \approx {}^{-}3\times10^{4}$ N

Airbag: $\vec{F}_{net,av} = \frac{{}^{-}97}{0.036} \approx {}^{-}3\times10^{3}$ N

Modern cars have a crumple zone at the front end. This effectively lengthens the stopping time of a car. If seatbelt is fastened, the occupant will have the same motion as the car and therefore about the same stopping time (0.1-0.2 s) as the car. Hence the average force on the occupant is smaller.

Example 2 In another crash test, a 60 kg dummy is fastened to the front seat with a standard seatbelt. The car has a crumple zone at the front end and hits a strong barrier at 50 km h^{-1}. It comes to a halt. The recorded stopping time of the driver compartment is 0.15 s. Estimate the average net force on the dummy exerted by the seatbelt.

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - 60 \times {}^{+}13.9 \approx {}^{-}830$ kg m s^{-1}

$\vec{F}_{net,av} = \frac{\Delta\vec{p}}{\Delta t} = \frac{{}^{-}830}{0.15} \approx {}^{-}6\times10^{3}$ N

Note: The forces calculated above must be considered together with the contact areas. The quantity *pressure* $\left(=\frac{F}{A}\right)$ is used to compare the effects of these forces.

Windscreen: Estimated contact area ≈ 0.01 m^2,

pressure ≈ $\frac{3\times10^{4}}{0.01} = 3\times10^{6}$ N m^{-2}

Airbag: Estimated contact area ≈ 0.03 m^2,

pressure ≈ $\frac{3\times10^{3}}{0.03} = 1\times10^{5}$ N m^{-2}

Seatbelt: Estimated contact area ≈ 0.05 m^2,

pressure ≈ $\frac{6\times10^{3}}{0.05} \approx 1\times10^{5}$ N m^{-2}

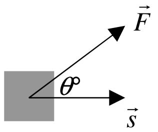
The pressure exerted by the windscreen is about 30 times that by the airbag or seatbelt.

Work done by a constant force

Work done W by a constant force $\vec{F}$ on an object over a displacement $\vec{s}$ is defined as $W = Fs$ if $\vec{F}$ and $\vec{s}$ are in the same direction. W in this situation is a positive amount of work and the object gains energy.

If $\vec{F}$ and $\vec{s}$ are in opposite directions. W is a negative amount of work, $W = -Fs$ and the object loses energy.

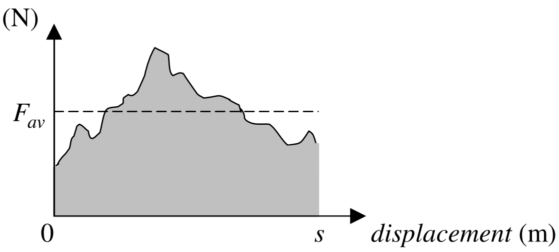
If $\vec{F}$ and $\vec{s}$ are at an angle $\theta°$, the work done is given by $W = Fs\cos\theta°$.



Work and energy are scalar quantities and they are measured in joules (J).

Work done by a variable force

When force $\vec{F}$ on an object changes with its displacement $\vec{s}$, area under the force-displacement graph represents work done by $\vec{F}$ if $\vec{F}$ and $\vec{s}$ are in the same direction. Estimate the area if it cannot be determined by simple calculation.



The estimated area $\approx F_{av}s$.

Example 3 A crate is pulled along a rough level surface with a rope at an angle of 25° above the horizontal. The tension in the rope is 100 N and friction force against motion is 80 N.
(a) Find the work done by the pulling force when the crate moves a distance of 1.2 m.
(b) Find the work done by the friction force over the 1.2 m.
(c) Find the work done by the normal reaction on the crate.
(d) Find the work done by the net force over the 1.2 m.

(a) $W = Fs\cos\theta° = 100\times1.2\times\cos25° \approx 110$ J (108.8 J)

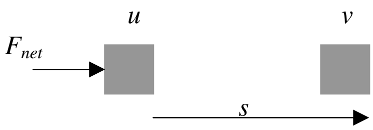
(b) $W = -Fs = -80\times1.2 = -96$ J. The negative sign indicates that energy is taken out of the system.

(c) $W = Fs\cos\theta° = N\times1.2\times\cos90° = 0$, where N is the normal reaction force on the crate.

(d) $\vec{F}_{net} = {}^{+}100\cos25° + {}^{-}80 = {}^{+}10.63$ N
$W = F_{net}s = 10.63\times1.2 \approx 13$ J
Alternatively, $W = 108.8 - 96 \approx 13$ J

Newton's second law in terms of work and energy

To simplify the situation consider a constant net force F_{net} acting on an object of mass m over a displacement s.



Since F_{net} is constant, acceleration a is also constant.

$$\therefore v^2 = u^2 + 2as \quad \therefore a = \frac{v^2 - u^2}{2s}$$

Newton's second law: $F_{net} = ma$, $\therefore F_{net}s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

The quantity $F_{net}s$ is the work W done by F_{net} on the object.

The quantities $\frac{1}{2}mv^2$ and $\frac{1}{2}mu^2$ are defined as the final kinetic energy and the initial kinetic energy respectively. The difference $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ is the change in kinetic energy ΔE_k.

Therefore Newton's second law can be stated as ***work done*** *by net force equals* ***change in kinetic energy*** *of the object.*

$$W = \Delta E_k$$

This idea of work done by the net force on an object equals its change in kinetic energy can also explain why crumple zone is an important safety feature of a car. The crumple zone extends the stopping distance s of an occupant wearing a seatbelt and thus lowers the force on the person.

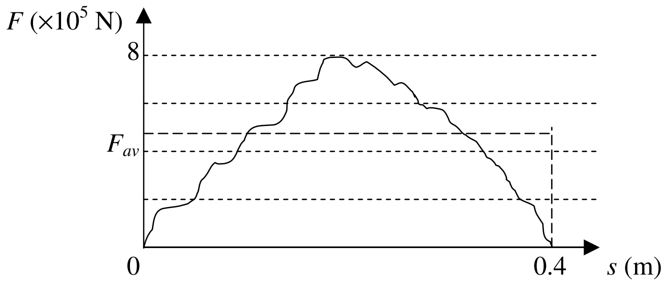
$F_{net,av} = \frac{|\Delta E_k|}{s}$, and ΔE_k of the occupant is constant under the same condition in a crash $\therefore F_{net} \propto \frac{1}{s}$

Example 4 In example 2, suppose the crumple zone is crushed by a distance of 1 m but the stopping time is unknown. Estimate the average net force exerted by the seatbelt on the dummy.

$$\Delta E_k = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = 0 - \frac{1}{2}\times60\times13.9^2 \approx -6000 \text{ J}$$

$$F_{net,av} = \frac{|\Delta E_k|}{s} = \frac{6000}{1} = 6\times10^3 \text{ N}$$

Example 5 The following graph shows the force on a 1 tonne car versus displacement of its centre of mass when it hits a barrier.
(a) Find the average force in stopping the car.
(b) Find the speed of the car just before it hits the barrier.



(a) Draw a horizontal dotted line representing F_{av} so that the area under it equals the area under the curve. Read from the vertical scale, $F_{av} \approx 4.7\times10^5$ N.

(b) $|\Delta E_k| = W$, $\left|0 - \frac{1}{2}mu^2\right| = F_{av}s$

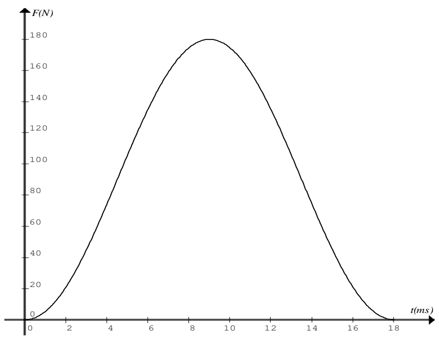
$\frac{1}{2}\times1000\times u^2 = 4.7\times10^5\times0.4$, $u \approx 19$ m s^{-1}

Questions: Next page

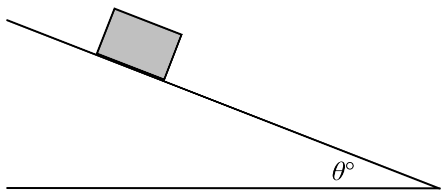
Q1 A 800 kg car travelling at 36 km h^{-1} comes to a stop in 0.50 s in an accident. (i) Determine the impulse on the car. (ii) Determine the average net force in stopping the car.

Q2 A 0.80 kg ball is dropped from a height of 2.0 m above the ground. It rebounds to a height of 1.6 m. The ball is in contact with the ground for 0.050 s. (i) Determine the impulse exerted by the ground (and gravity) on the ball in the 0.050 s. (ii) Determine the average net force on the ball in the 0.050 s.

Q3 The horizontal force exerted on a 0.30 kg ball by a bat is shown in the force-time graph. (i) Estimate the average horizontal force on the ball. (ii) Determine the change in horizontal momentum of the ball.



Q4 A 1 kg box slides down a frictionless inclined plane from rest. It attains a momentum of 2.0 kg m s^{-1} after sliding 2.0 m.
(i) Determine the net force on the box. (ii) Find $\theta°$.

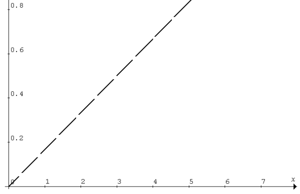


Q5 ***Ignore resistive forces in this question***. A 0.25 kg stone is dropped from a tall building. (i) Calculate the change in momentum in the 2nd second of its fall.
(ii) Calculate the change in momentum in the 2nd metre of its fall.

Q6 ***Ignore resistive forces in this question***. A truck travels at a speed of 2.0 m s^{-1} on a horizontal road while 800 kg of sand is poured vertically into it over a period of 3 seconds.
(i) Determine the average driving force needed over the 3 s period to keep the speed of the truck constant.
(ii) No more sand is poured into the truck after the 3 s period. What is the average driving force needed to keep the speed constant?
(iii) Now the sand runs out of the truck at 10 kg per second. What is the speed of the truck when it is empty of sand?

Q7 A 75 kg person pushes a 30 kg box with a force of 120 N along a horizontal floor for 3 metres. The force of friction between the box and the floor is 20 N. (i) Calculate the work done by the person. (ii) Calculate the work done against friction. (iii) Calculate work done on the box by the net force. (iv) Calculate the change in kinetic energy of the box.

Q8 The force of air resistance F N on a 0.10 kg marble dropped from a great height is shown in the following graph where x m is the distance fallen. (i) Calculate the work done by gravity on the marble after the first 5 metres of the fall.
(ii) Calculate the work done by air resistance in the 5 m fall.
(iii) Calculate the speed of the marble after falling the first 5 m.



Numerical, algebraic and worded answers: 1. (i) 8000 Ns opposite to the direction of motion (ii) 16000 N against motion. 2. (i) $I \approx 9.49$ Ns upward (ii) $F_{net,av} \approx 190$ N upward 3. (i) 90 N (ii) $\Delta p = 1.62$ kg m s^{-1} 4. (i) $F_{net} = 1.0$ N (ii) $\approx 5.9°$ 5. (i) $\Delta p = 2.45$ kg m s^{-1} (ii) $\Delta p \approx 0.46$ kg m s^{-1} 6. (i) $F_{av} \approx 530$ N (ii) Zero driving force (iii) Same speed 7. (i) $W_{push} = 360$ J (ii) $W_{friction} = 60$ J (iii) $W_{net.force} = 300$ J (iv) $\Delta E_k = W_{net.force} = 300$ J 8. (i) $W_{gravity} = 4.9$ J (ii) $W_{resist} = 2.0$ J (iii) $v \approx 7.6$ m s^{-1}