= Year 12 = Complex numbers = Worksheet 4

1. Find $\sqrt[3]{1+i}$. Write your answers in polar form.	2. Change your answers in Q1 to exact $x+yi$ form.
3. Find $\sqrt[8]{-1}$. Write your answers in polar form.	4. Change $cis\frac{\pi}{8}$ to exact $x+yi$ form.
5. Use the conjugate root theorem and the fundamental theorem of algebra to explain why az^3+bz^2+cz+d has at least one real root for $a,b,c,d\in R$.	6. Show that $z-1+i$ is a factor of z^3+2z^2-6z+8 . Find the other factors.
7. Find the roots of z^3+z^2+z .	8. Solve $2z^3-3z^2+4z-6=0$.
9. Given $z-1-i$ is a factor of $P(z)=z^3+pz+q$, find p and $q\in R$. Hence solve $P(z)=0$.	10. Consider $z=a+ib$, find a and b such that $z^2=i$. Hence solve $z^4=-1$.

Numerical, algebraic and worded answers.

1. $2^{1/6}cis(\pi/12)$, $2^{1/6}cis(3\pi/4)$, $2^{1/6}cis(-7\pi/12)$
2. $2^{-4/3}(1+\sqrt{3}) - 2^{-4/3}(1-\sqrt{3})i$, $-2^{-1/3} + i2^{-1/3}$, $2^{-4/3}(1-\sqrt{3}) - 2^{-4/3}(1+\sqrt{3})i$
3. $cis(-7\pi/8)$, $cis(-5\pi/8)$, $cis(-3\pi/8)$, $cis(-\pi/8)$, $cis(\pi/8)$, $cis(3\pi/8)$, $cis(5\pi/8)$, $cis(7\pi/8)$
4. $\sqrt{(2+\sqrt{2})}\,/2 + i\,/\sqrt{(4+2\sqrt{2})}$
5. Cubic polynomial has 3 roots (FTofA). For real coefficients, either all roots are real, or a pair of complex conjugate roots + 1 real root (CRT).
6. $z-1-i$, $z+4$
7. 0, $-1/2-i\sqrt{3}\,/2$, $-1/2+i\sqrt{3}\,/2$
8. $z = 3/2$, $i\sqrt{2}$, $-i\sqrt{2}$
9. $p = -2$, $q = 4$, $z = -2$, $1+i$, $1-i$
10. $a = \pm1/\sqrt{2}$ and $b = \pm1/\sqrt{2}$, $z = 1/\sqrt{2} + 1/\sqrt{2}\,i$, $-1/\sqrt{2} - 1/\sqrt{2}\,i$, $1/\sqrt{2} - 1/\sqrt{2}\,i$, $-1/\sqrt{2} + 1/\sqrt{2}\,i$