2006 VCAA Further Mathematics Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
D	B	E	C	D	B	A	C	A	C	D	C	A

SECTION B
Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
D	C	C	A	D	E	B	D	E

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
B	D	A	C	C	D	E	A	D

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
D	E	C	A	B	C	E	A	C

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
B	C	E	B	E	C	D	A	B

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
B	B	D	D	E	D	C	D	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
B	D	A	B	D	E	C	E	B

SECTION A Core: Data analysis

Q1 Temperature-numerical; town-categorical.

Q2 The maximum temperatures vary between 15 and 38.
Range $= 38 - 15 = 23$

Q3 18 and 19 are outliers because they are less than $Q_1 - 1.5IQR$.

Q4 $\mu = 49.5$, $\sigma = 1.5$, $\Pr(48 < X < 51) = 0.68$,
$\therefore \Pr(X < 48) = \frac{1}{2}(1 - 0.68) = 0.16$
Number $0.16 \times 400 = 64$

Q5 $7 + 11 + 14 + 16 + 18 + 12 = 78$

Q6 Count rectangles upwards from either end. The 50th and 51st rectangles lie in the 40-45 interval.

Q7 $b = r\frac{s_y}{s_x} = -1.5177$, $a = \bar{y} - b\bar{x} = 30.85$

Q8 Residual = actual – predicted
$= 67 - (-20 + 1.11 \times 80) \approx -2$

Q9 Calculator $y = 7.147 + 2.9387x^2$

Q10 $\frac{35 + 99 + 75}{3} = 69.7$

Month	Mar	Apr	May
Rainfall	35	99	75
3-mean		69.7	

Q11 For 12 months, the sum of the indices = 12. The missing one = 12 – (sum of the 11 given indices) = 0.98

Q12 Deseasonalised number $= \frac{330}{0.94} = 351$

Q13 Deseasonalised number $= 373.3 - 3.38 \times 6 = 353.02$
Actual number $= 353.02 \times 0.86 = 303.6$

SECTION B
Module 1: Number patterns and applications

Q1 Only sequence D has a common difference of 3.

Q2 Common ratio, $\frac{x}{6} = \frac{54}{x}$, $x^2 = 54 \times 6$, $x = 18$.

Q3 $84\% \times 50 = 42$.
Number at the start of second year $= 50 + 42 - 40 = 52$.

Q4 Assuming 40 sheep are to be sold at the end of each year and the % increase remains the same, (the question is not clear about these)
$S_{n+1} = (1 + 0.84)S_n - 40 = 1.84S_n - 40$, $S_1 = 50$.

Q5 $f_{n+1} = f_n + 5$, $f_1 = -1$, $f_2 = -1 + 5 = 4$, $f_3 = 4 + 5 = 9$.

Q6 $12.0 \times (1 + 0.03)^{14} = 18.15$

Q7 Common difference does not exist, $\therefore$ not A, C and D.
Only B gives $t_2 = 9$.

Q8 $t_n = t_{n-1} + t_{n-2}$, $t_1 = 1$, $t_2 = 2$ $\therefore t_3 = 2 + 1 = 3$,
$t_4 = 3 + 2 = 5$, $t_5 = 5 + 3 = 8$.
Total $= 1 + 2 + 3 + 5 + 8 = 19$

Q9 0.400, 0.380, 0.361, ……form a geometric sequence. The common ratio is $\frac{0.380}{0.400}$ or $\frac{0.361}{0.380}$, i.e. 0.95.
$S_\infty = \frac{a}{1 - r} = \frac{0.400}{1 - 0.95} = 8$
Eventual body weight $= 73.4 + 8 = 81.4$ kg.

Module 2: Geometry and trigonometry

Q1 $\theta = \sin^{-1}\left(\frac{142}{215}\right) = 41.3^o$.

Q2 The sine rule: $\frac{RT}{\sin 108^o} = \frac{45}{\sin 54^o}$,

$RT = \frac{45\sin 108^o}{\sin 54^o} = 52.9$.

Q3 $300 - 200 = 100$ m

Q4 Cross-sectional area (front):

$A = \left(4\times 8 + \frac{1}{2}\times 8\times(6-4)\right) = 40$ m^2.

Volume $V = AL = 40\times 24 = 960$ m^3.

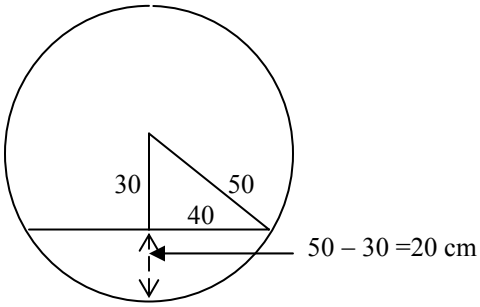
Q5 $s = \frac{36+58+42}{2} = 68$

Q6 Height $h = \frac{V}{A} = \frac{6}{1.5\times 2} = 2$ m.

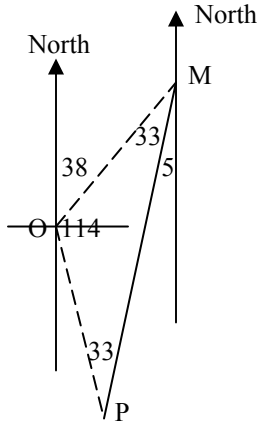
TSA $= 2(1.5\times 2) + 2(1.5\times 2) + 2(2\times 2) = 20$ m^2.

Q7 Similar triangles, same ratio for pairs of corresponding sides. $\frac{DE}{24} = \frac{27}{27+9}$, $DE = 18$ cm.

Q8



Q9



The bearing of P from M is 180 + 5 = 185

Module 3: Graphs and relations

Q1 $y = 2$

Q2 A horizontal line at depth = 10 m cuts the curve at four points.

Q3 Constant $M = 220$ from $t = 4.5$ onwards.

Q4 Slope (gradient) $= \frac{220 - 332.5}{4.5 - 0} = -25$ for $0 \le t \le 4.5$

$M = 220$ for $4.5 < t \le 6$.

Q5 Slope $= \frac{12}{4} = 3$.

Q6 $(2,-2)$ satisfies the equation $2x + 2y = 0$.

Q7 The sloping line is $x + y \le 80$, i.e. the total number of cows and sheep cannot exceed 80.

Q8 The selling price per pan must be greater than \$50. $\therefore$ either A or C.
For 10 pans, cost $= 400 + 50\times 10 = 900$,
revenue $= 90\times 10 = 900$, the same.

Q9 $(6,2)$ satisfies all the inequalities (constraints).

Module 4: Business-related mathematics

Q1 5% of $4000 = \frac{5}{100}\times 4000 = 200$.

Q2 Minimum balance is 473.92

$0.15\%\times 473.92 = \frac{0.15}{100}\times 473.92 = 0.71$

Q3 Perpetuity: interest earned = pension

$584 = \frac{P\times 6.2\times \frac{1}{12}}{100}$, $\therefore P = 113032.26$

Q4 Price (with GST)
= price (without GST) + 10% of price (without GST)
Price (with GST) = 1.1 × price (without GST)
$825 = 1.1\times$ price (without GST)

Price (without GST) $= \frac{825}{1.1} = 750$

GST $= 825 - 750 = 75$

Q5 Depreciation $= 48000 - 21000 = 27000$

Number of copies $= \frac{27000}{0.04} = 675000$

Q6 Amount borrowed = $2000 - 200 = 1800$
Total amount of 36 monthly instalments = $68 \times 36 = 2448$
Interest = $2448 - 1800 = 648$
$\therefore r = \dfrac{100I}{Pt} = \dfrac{100 \times 648}{1800 \times 3} = 12$

Q7 After the 20% discount, the price is 80% of the original price P.
$\therefore 0.80P - 80 = 368$, $P = 560$

Q8 The balance stays constant within a year for the six years, and increases at the start of each year, $\therefore$ interest is credited annually. The interest increases annually, $\therefore$ interest is compounded annually.

Q9 $P = 18000$, $R = 1 + \dfrac{9.2}{12 \times 100} = 1.0076667$, $n = 60$, $A = 0$
Use $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$ to find Q. $Q = 375.40$.
After $n = 10$, $A = 15542.41$.
Jenny has paid off $18000 - 15542.41 = 2457.59$

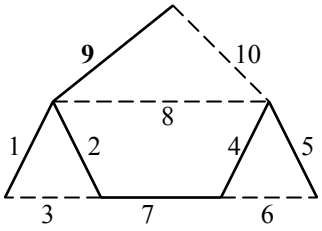
Module 5: Networks and decision mathematics

Q1 2

Q2 Intersection 5

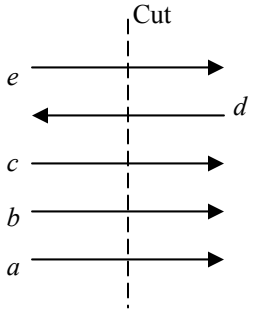
Q3 Two vertices (B and G) have odd degree, the other vertices have even degree. At least two Eulerian paths exist.

Q4



Q5 E

Q6



Capacity of the cut = $a + b + c - d + e$

Q7 C

Q8 For D, $v + f = 5 + 4 = 9$, $e + 2 = 10 + 2 = 12$,
$\therefore v + f \neq e + 2$.

Q9 $B + G + I = 16$, $A + F = 13$, $A + C + E + H = 14$, $\therefore$ the earliest start time for J is 16. If BGI is crashed, the earliest start time for J becomes 14. Maximum reduction in completion time is $16 - 14 = 2$ hours.

Module 6: Matrices

Q1 $\begin{bmatrix} 12 & 36 \\ 0 & 24 \end{bmatrix} = \begin{bmatrix} 12 \times 1 & 12 \times 3 \\ 12 \times 0 & 12 \times 2 \end{bmatrix} = 12 \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

Q2 $BC = \begin{bmatrix} 0 & 9 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix}$ is not defined because the number of elements in the row of the first matrix does not equal to the number of elements in the column of the second matrix.

Q3 $A^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B - C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$,
$A^3(B - C) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Q4 B

Q5 $MP = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.35 \end{bmatrix} \begin{bmatrix} 145 & 210 & 350 \\ 185 & 270 & 410 \end{bmatrix}$
$= \begin{bmatrix} 174 & 252 & 420 \\ 249.75 & 364.50 & 553.50 \end{bmatrix}$

Q6 $X \begin{bmatrix} 1 & 3 \\ 6 & 4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \\ 3 & 5 \end{bmatrix}$
(3×3) (3×2) (3×2)

Q7 The following three sets have unique solution .
$x = 0$, $x + y = 6$;
$x - y = 3$, $x + y = 3$;
$x = 8$, $y = 2$.

Q8 E

Q9 The transition matrix shows that 20% of birds at location A moves to location B, and 0% of birds at location B moves to location A every night. Hence the number of birds at location A will gradually decrease to zero.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors