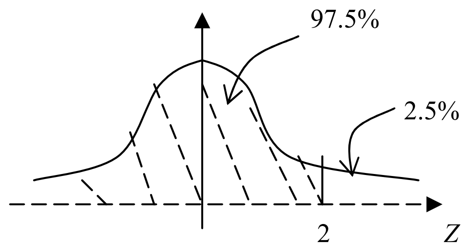
2006 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

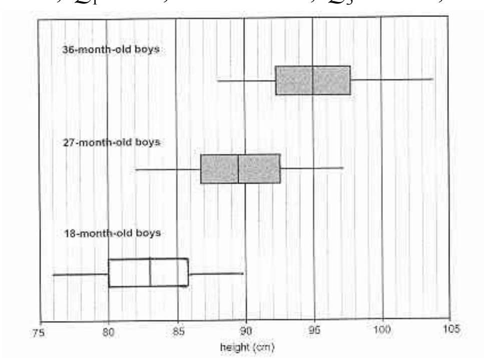
Q1a $S_X = 3.8$ (Graphics calculator)

Q1b $z = \dfrac{X - \overline{X}}{s_X} = \dfrac{83.1 - 89.3}{4.5} = -1.4$

Q1c



Q1d Min = 76, $Q_1 = 80$, median = 83, $Q_3 = 85.8$, max = 89.8



Q1e 89.5

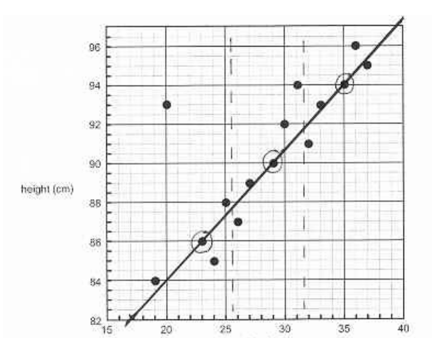
Q1f Median height increases as age increases.

Q2a 0.53

Q2bi $r^2 = 0.7541^2 = 0.569 = 56.9\%$

Q2bii The coefficient of determination 0.569 (56.9%) represents the proportion (percentage) of the variability in height with age that is explained by the least squares regression line.

Q3a



Q3b Slope $= \dfrac{94 - 86}{35 - 23} = \dfrac{2}{3} \approx 0.7$, $\therefore$ height $= c + \dfrac{2}{3} \times$ age,

$94 = c + \dfrac{2}{3} \times 35$, $\therefore$ vertical axis intercept $c \approx 70.7$.

$\therefore$ height $= 70.7 + 0.7 \times age$

Q3c The 'abnormal' height of the 20-month old boy has a much greater effect on the slope of the least squares regression line than on the slope of the three median line.

Module 1: Number patterns

Q1a $48000 - 3000 \times 2 = 42000$ kg

Q1b $d = -3000$

Q1c $48000 - 3000n = 0$, $n = 16$ days

Q2a $r = \dfrac{400}{500}$ or $\dfrac{500}{625} = 0.8$

Q2b Fourth month $400 \times 0.8 = 320$
Fifth month $320 \times 0.8 = 256$

Q2c $H_n = 625 \times 0.8^{n-1}$

Q2d $H_6 - H_7 = 625 \times 0.8^5 - 625 \times 0.8^6 \approx 41$ hours

Q2e $H_n < 100$, $625 \times 0.8^{n-1} < 100$, $0.8^{n-1} < 0.16$,

$(n-1)\log 0.8 < \log 0.16$, $\therefore n - 1 > \dfrac{\log 0.16}{\log 0.8}$, $n > 9.2$

$\therefore$ the tenth month.

Q2f $S_3 = 1525$, $S_{12} = \dfrac{625\left(1 - 0.8^{12}\right)}{1 - 0.8} = 2910.3$

Hours worked in the nine months $2910.3 - 1525 = 1385.3 \approx 1385$

Q3a $r = 90\% = 0.9$, $d = 2000$

Q3b $V_4 = 38225$ (Graphics calculator)

Q3c $n = 10$ (Graphics calculator)

Q3d $v_\infty = \dfrac{d}{1 - r} = \dfrac{2000}{0.1} = 20000$ litres

Module 2: Geometry and trigonometry

Q1a $\angle AXY = 180 - 90 - 45 = 45^0$

Q1b $XY = AY = 55$, $\therefore AX = \sqrt{55^2 + 55^2} = 77.8$ m

Q1c $78-45=33$, $\therefore 033^0$

Q1d $YC=142-55=87$, $XY=55$

$\therefore XC=\sqrt{87^2+55^2}\approx 102.9$ m

Q1e Area of $\Delta ABC=\frac{1}{2}\times 142\times 251\times \sin 45^0\approx 12601\text{ m}^2$

Q1fi $BC=\sqrt{142^2+251^2-2(142)(251)\cos 45^0}\approx 181\text{ m}$

Q1fii $\frac{\sin\angle ABC}{142}=\frac{\sin 45^0}{181}$, $\therefore \angle ABC\approx 33.7^0$

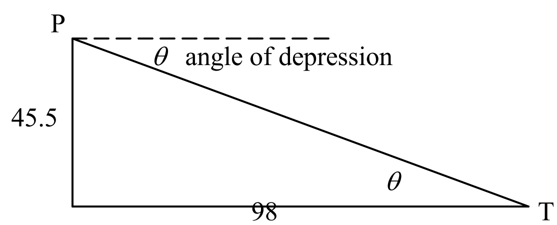
Q1g Let $MN=x$ m, $AN=MN=x$.

Area $=\frac{1}{2}\times MN\times AN$, $\therefore 3200=\frac{1}{2}x^2$, $x^2=6400$, $x=80$.

Hence $MN=80$ m

Q2a $\frac{PQ}{125}=\tan 20^0$, $\therefore PQ=125\tan 20^0\approx 45.5$ m

Q2b



$\tan\theta^0=\frac{45.5}{98}$, $\theta=24.9$

Q3a $r=\frac{3.5-0.25\times 2}{2}=1.5$ m

Q3b Internal height $h=2.4-0.25\times 2=1.9$ m

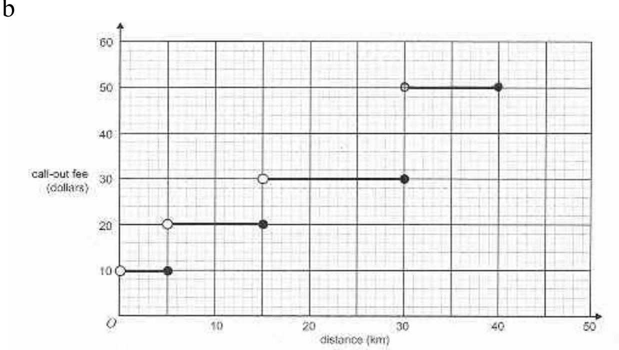
Maximum volume $=\pi r^2h=\pi(1.5)^2(1.9)=13\text{ m}^3$

Module 3: Graphs and relations

Q1ai \$30

Q1aii 5 km

Q1b



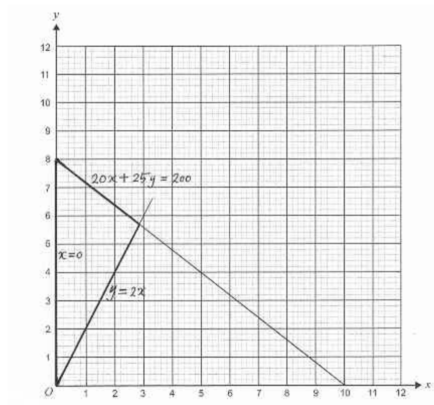
Q2a Slope $=-\frac{20}{160}=-0.125$. Let x be the distance in km and y the amount of fuel in litres left. $y=-0.125x+50$

Q2b The van uses 0.125 litres for each km.

Further distance $=\frac{30}{0.125}=240$ km

Q2c Capacity of the tank $=18\times 3.5+12=75$ litres

Q3a, 3ci



Q3b $y\geq 2x$

Q3cii Maximum number of dogs $=2$

Q3d $P=40x+30y$

Q3ei x and y must be whole numbers including zero. In the feasible region the point that produces maximum profit is $(2,6)$, i.e. 2 washes and 6 clips.

Q3eii Maximum $P=40(2)+30(6)=260$ dollars

Module 4: Business-related mathematics

Q1ai Annual depreciation $=10\%\times 60000=6000$

Q1aii Value (after 3 years) $=60000-6000\times 3=42000$

Q1aiii Number of years $=\frac{60000-12000}{6000}=8$

Q1bi $1-0.85=0.15=15\%$

Q1bii After 3 years, $V=60000\times(0.85)^3=36847.50$ dollars

Q1biii $60000\times(0.85)^n<12000$, $\therefore 0.85^n<0.2$,

$\therefore n>\frac{\log 0.2}{\log 0.85}$, $n>9.9$, $\therefore n=10$.

Q1c Sketch $y=60000-6000n$ and $y=60000\times(0.85)^n$. When $n=7$, $60000-6000n<60000\times(0.85)^n$.

Q2 Cost (in 8 years) $= 60000 \times \left(1 + \dfrac{2}{100}\right)^8 = 70300$ dollars

Q3a Total value A (in 8 years) = principal + interest

$= 7000 + \dfrac{7000 \times 6.25 \times 8}{100} = 10500$ dollars

Q3b Total value A (in 8 years)

$= 10000 \times \left(1 + \dfrac{6}{4 \times 100}\right)^{8\times 4} \approx 16103$ dollars

Q3c By graphics calculator TVM solver or by calculation,

$P = 500$, $Q = 200$, $R = 1 + \dfrac{6.5}{12 \times 100} = 1.005417$ and

$n = 8 \times 12 = 96$,

total value A (in 8 years) $= PR^n + \dfrac{Q(R^n - 1)}{R - 1}$

$= 500 \times (1.005417)^{96} + \dfrac{200(1.005417^{96} - 1)}{0.005417} \approx 25935$ dollars

Q4 By graphics calculator TVM solver or by calculation,

$P = 20000$, $R = 1 + \dfrac{10}{12 \times 100} = 1.00833333$, $n = 24$, $A = 0$,

$\therefore 20000 \times (1.00833333)^{24} - \dfrac{Q((1.00833333)^{24} - 1)}{1.00833333 - 1} = 0$

$\therefore Q = 922.90$ dollars

Total interest $= 922.90 \times 24 - 20000 \approx 2150$ dollars

Module 5: Networks and decision mathematics

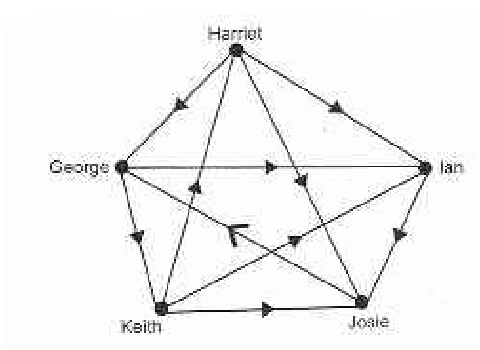
Q1a George must play guitar because Harriet can play drums only.

Q1b $\therefore$ Josie must fill the vocal position and Keith must play keyboards. Hence Ian must play saxophone.

Person	***Position***
Harriet	Drums
Ian	Saxophone
Keith	Keyboards

Q 2a Because one cannot compete against oneself.

Q3a Josie won against George.



Q2c George defeated Keith who defeated Ian.

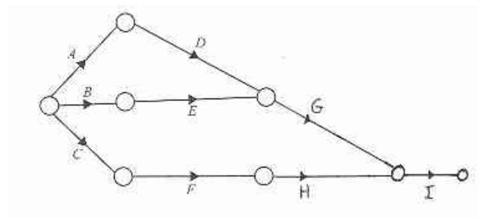
Q2d Keith defeated Harriet who defeated Josie. Also Keith defeated Ian who defeated Josie. $\therefore x = 2$

Q2e

$$D_1 + D_2 = \begin{bmatrix} 0 & 1 & 2 & 2 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 2 & 3 & 0 \end{bmatrix} \quad \begin{array}{c} \text{Dominance} \\ 6 \\ 7 \\ 2 \\ 3 \\ 8 \end{array}$$

First is Keith, last is Ian.

Q3a



Q3b The critical path could be one of the following paths: *ADGI*, *BEGI* and *CFHI*. Each path has four activities.
$\therefore$ there are $9 - 4 = 5$ non-critical activities.

Q3c Since activities A and C are non-critical, $\therefore$ the critical path is *BEGI*.

Q3d Duration of $I = 19 - 12 = 7$ hours

Q3e Max. combined duration for F and $H = 12 - 3 - 1 = 8$ hours

Module 6: Matrices

Q1a 2×3 because Q has 2 rows and 3 columns.

Q1bi $M = QP = \begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix} \begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix} = \begin{bmatrix} 75959.60 \\ 171848.50 \end{bmatrix}$

Q1bii \$75959.60 is the monthly takings of the outlet at Eastown and \$171848.50 is the monthly takings of the outlet at Noxland.

Q1c The number of columns in matrix P does not match up with the number of rows in matrix Q, $\therefore PQ$ is not defined.

Q2a

$$T = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} S & E & N & \\ \end{array}$$

$$T = \begin{array}{cc} \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} & \begin{array}{c} S \\ E \\ N \end{array} \end{array} \quad \text{(columns } S, E, N)$$

Q2b

$$K_0 = \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} \begin{matrix} S \\ E \\ N \end{matrix}$$

Q2c

$$TK_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 268800 \\ 136200 \\ 195000 \end{bmatrix}$$

Q2d

$$T^{35}K_0 = \begin{bmatrix} 194983.00 \\ 150514.40 \\ 254502.60 \end{bmatrix}, \quad T^{40}K_0 = \begin{bmatrix} 194982.90 \\ 150513.35 \\ 254503.75 \end{bmatrix}$$

$$T^{45}K_0 = \begin{bmatrix} 194982.89 \\ 150513.16 \\ 254503.95 \end{bmatrix}, \quad T^{50}K_0 = \begin{bmatrix} 194982.90 \\ 150513.12 \\ 254503.98 \end{bmatrix}$$

$$K \to \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

Q3a

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix}$$

Q3b Det = 1, $\therefore$ the equations have a unique solution.

Q3c By graphics calculator,

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & -1 \\ 2 & -4 & -3 \end{bmatrix}$$

Q3d

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & -1 \\ 2 & -4 & -3 \end{bmatrix} \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

$\therefore$ 3 bookshops, 4 sports shoe shops, 2 music stores.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors