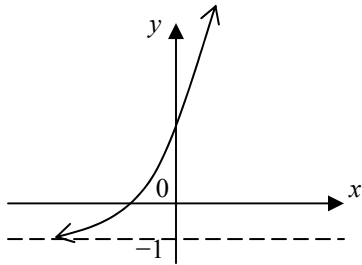


Q1 $f(x) = x^2 + 1$, $g(x) = 2x + 1$,
 $f(g(x)) = (g(x))^2 + 1 = (2x + 1)^2 + 1 = 4x^2 + 4x + 2$

Q2a Equation of the original function: $y = 3e^{2x} - 1$
 Equation of the inverse: $x = 3e^{2y} - 1$, $\therefore e^{2y} = \frac{x+1}{3}$,
 $\therefore y = \frac{1}{2} \log_e \left(\frac{x+1}{3} \right)$

Rule of the inverse function is $f^{-1}(x) = \frac{1}{2} \log_e \left(\frac{x+1}{3} \right)$.

Q2b Graph of f



Domain of f^{-1} is the range of f , i.e. $(-1, \infty)$.

Q3a $f(x) = e^{\cos x}$, a composite function.
 Apply the chain rule to find $f'(x) = (-\sin x)e^{\cos x}$

Q3b $y = x \tan x$, product of functions.

Apply the product rule to find $\frac{dy}{dx} = \tan x + x \sec^2 x$.

When $x = \frac{\pi}{6}$,

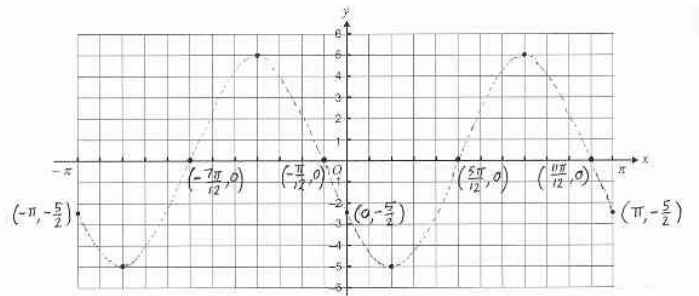
$$\frac{dy}{dx} = \tan \frac{\pi}{6} + \frac{\pi}{6} \times \frac{1}{\left(\cos \frac{\pi}{6}\right)^2} = \frac{1}{\sqrt{3}} + \frac{\pi}{6} \times \frac{4}{3} = \frac{3\sqrt{3} + 2\pi}{9}$$

Q4a Amplitude is 5, period = $\frac{2\pi}{2} = \pi$.

Q4b y -coordinate of end point at $x = \pi$:

$$y = 5 \cos 2\left(\pi + \frac{\pi}{3}\right) = 5 \cos \frac{8\pi}{3} = 5 \cos \frac{2\pi}{3} = -\frac{5}{2}$$

Same value for end point at $x = -\pi$.



Q5a $\Pr(X > 80) = \Pr\left(Z > \frac{80 - 72}{8}\right) = \Pr(Z > 1) = 1 - \Pr(Z < 1)$
 $= 1 - 0.84 = 0.16$

Q5b $\Pr(64 < X < 72) = \Pr(-1 < Z < 0)$
 $= \Pr(0 < Z < 1) = \Pr(Z < 1) - \Pr(Z < 0) = 0.84 - 0.50 = 0.34$

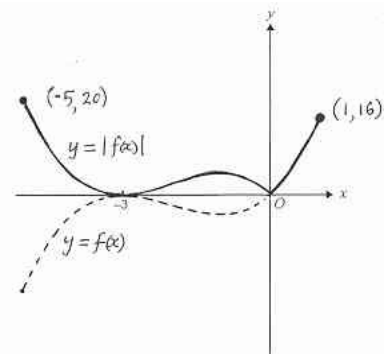
Q5c $\Pr((X < 64) | (X < 72)) = \frac{\Pr((X < 64) \cap (X < 72))}{\Pr(X < 72)}$
 $= \frac{\Pr(X < 64)}{\Pr(X < 72)} = \frac{\Pr(X > 80)}{\Pr(X < 72)} = \frac{0.16}{0.5} = 0.32$

Q6a $\Pr(X < 3) = \int_1^3 \frac{x}{12} dx = \left[\frac{x^2}{24} \right]_1^3 = \frac{9}{24} - \frac{1}{24} = \frac{1}{3}$

Q6b $\Pr(X \geq a) = \frac{5}{8}$, $\therefore \int_a^5 \frac{x}{12} dx = \frac{5}{8}$, $\therefore \left[\frac{x^2}{24} \right]_a^5 = \frac{5}{8}$,

$$\frac{25}{24} - \frac{a^2}{24} = \frac{5}{8}, a^2 = 10, \therefore a = \sqrt{10}$$

Q7a



Q7b When $x = -5$,
 $y = f(-5) = (-5)^3 + 6[-5]^2 + 9(-5) = -20$,
 $\therefore y = |f(-5)| = 20$.

When $x = 1$, $y = f(1) = (1)^3 + 6[1]^2 + 9(1) = 16$,
 $\therefore y = |f(1)| = 16$.

From graph, the range of $|f(x)|$ and domain $[-5, 1]$ is $[0, 20]$.

Q8 $y = \sqrt{x}$, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$.

Gradient of normal = -4 , \therefore gradient of tangent = $-\frac{1}{-4} = \frac{1}{4}$,

i.e. $\frac{1}{2\sqrt{x}} = \frac{1}{4}$, $\therefore x = 4$ and $y = 2$.

$y = -4x + a$ is the normal to the curve at $(4,2)$.

Hence $2 = -4(4) + a$, $\therefore a = 18$.

Q9a $A = (2a)b = 2ab$ where $a > 0$ and $b = 9 - 3a^2$,

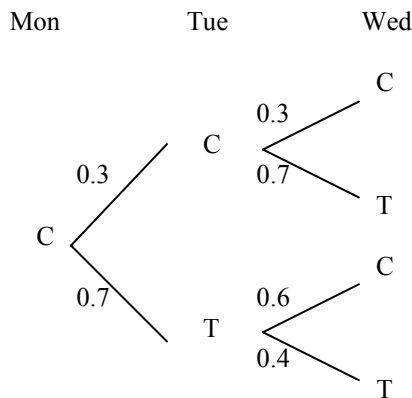
$\therefore A = 2a(9 - 3a^2) = 18a - 6a^3$

Q9b $\frac{dA}{da} = 18 - 18a^2$. At stationary points, $\frac{dA}{da} = 0$,

i.e. $18 - 18a^2 = 0$, $\therefore a = 1$.

At $a = 1$, $A_{\max} = 18(1) - 6(1)^3 = 12$ square units

Q10



$\text{Pr}(\text{tea on Wed}) = 0.3 \times 0.7 + 0.7 \times 0.4 = 0.49$

Q11 Shaded area = $\int_0^3 (-x^2 + ax + 12) dx = 45$,

$\therefore \left[-\frac{x^3}{3} + \frac{ax^2}{2} + 12x \right]_0^3 = 45$,

$\therefore -9 + \frac{9a}{2} + 36 = 45$, $\therefore a = 4$.

$\therefore f(x) = -x^2 + 4x + 12$.

To find the x-intercepts: Let $-x^2 + 4x + 12 = 0$

Factorise: $(-x - 2)(x - 6) = 0$, $\therefore x = -2$ or 6 .

Hence $a = 4$, $m = 6$ and $n = -2$.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors