2007 VCAA Further Mathematics Exam 2 Solutions
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Core – Data analysis

Q1a Positively skewed.

Q1bi 26

Q1bii $\frac{20}{103}\times 100\% = 19.4\%$

Q2a 1964

Q2bi For 2010, mean surface temp

$= -12.361 + 0.013\times 2010 = 13.77°C$

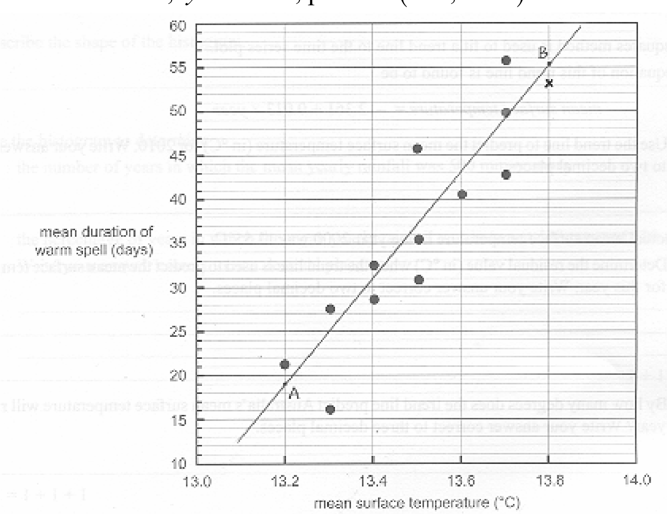
Q2bii For 2000, using the trend line, mean surface temp $= -12.361 + 0.013\times 2000 = 13.64°C$
Residual $= 13.55 - 13.64 = -0.09$

Q2biii Slope of the trend line $= 0.013°C$

Q3a See graph in part 3bii.

Q3bi
Mean duration of warm spell $= -776.9 + 60.3\times$ mean surface temperature

Q3bii When $x = 13.2$, $y = 19.06$, point A $(13.2, 19.06)$.
When $x = 13.8$, $y = 55.24$, point B $(13.8, 55.24)$



Scatterplot 1

Q3c The residual plot shows a random pattern.

Q3d $r^2 = 0.8288 \approx 83\%$

Q3e Strong positive linear relationship.

Module 1: Number patterns

Q1a 8700 kilojoules, obtained from graph.

Q1b Daily reduction $= \frac{8700 - 8100}{4} = 150$
Intake on day 6 $= 8100 - 150 = 7950$ kilojoules.

Q1c $a = 8850$ kilojoules

Q1d $6750 = 8850 - 150n$, $\therefore n = 14$, $\therefore$ the 14th day.

Q2a $r = 0.95 = 95\%$, $\therefore$ each day the intake reduced by 5%.

Q2b On day 3, intake $= 12000\times 0.95^2 = 10830$ kilojoules

Q2c On the nth day, intake $= 12000\times 0.95^{n-1}$.

Q2d
Difference $= 12000\times 0.95^8 - 12000\times 0.95^9 = 398$ kilojoules.

Q2e $S_{14} - S_8 = \frac{12000(1-0.95^{14})}{1-0.95} - \frac{12000(1-0.95^8)}{1-0.95}$
$= 42179$ kilojoules.

Q3a $M_2 = 20$, $M_3 = 0.75M_2 + 8 = 0.75\times 20 + 8 = 23$
$M_4 = 0.75M_3 + 8 = 0.75\times 23 + 8 = 25.25$ minutes.

Q3b The sequence is 20, 23, 25.25, ……
Since $23 - 20 \neq 25.25 - 23$, $\therefore$ not arithmetic.
Since $\frac{23}{20} \neq \frac{25.25}{23}$, $\therefore$ not geometric.

Q3c $M_2 = 0.75M_1 + 8$, $\therefore M_1 = \frac{M_2 - 8}{0.75} = \frac{20-8}{0.75} = 16$.

Q4 Swim: 100 150 200 …… $100 + (n-1)50$
Run: 500 500×1.02 500×1.02^2 …… $500\times 1.02^{n-1}$
$100 + (n-1)50 > 500\times 1.02^{n-1}$ when $n \geq 12$, i.e. on day 12.

Module 2: Geometry and trigonometry

Q1a $QW = \frac{1}{2}\times 24 = 12$ cm

Q1b $\tan\angle WAG = \frac{12}{32}$, $\angle WAG = \tan^{-1}\left(\frac{12}{32}\right) = 20.6°$ (20.56°)

Q1c $\angle AWB = 2\times\angle WAG = 41.1°$

Q1d Area of triangle $AWB = \frac{1}{2}$ of the area of rectangle $ABRQ$.

Q2a $AW = \sqrt{32^2 + 12^2} = 34.18 \approx 34$ cm

Q2b Area of rectangle $BCVW = 28 \times 34 = 952$
Area of base $ABCD = 24 \times 28 = 672$
Area of triangle $ABW = \frac{1}{2} \times 24 \times 32 = 384$
TSA $= 952 \times 2 + 672 + 384 \times 2 = 3344$ cm^2.

Q3a $V = \frac{1}{3} \times \text{base area} \times \text{height} = \frac{1}{3} \times 672 \times 32 = 7168$ cm^3.

Q3b $AC = \sqrt{24^2 + 28^2} = 36.8782$, $\frac{1}{2}AC = 18.4391$.
$\therefore AY = \sqrt{32^2 + 18.4391^2} = 36.93 \approx 37$ cm

Q3c $s = \frac{1}{2}(37 + 37 + 24) = 49$.
$A = \sqrt{49(49-37)(49-37)(49-24)} = 420$ cm^2.

Q4a Fraction of height removed $= \frac{24}{32} = \frac{3}{4}$.

Q4b Fraction of volume removed $= \left(\frac{3}{4}\right)^3 = \frac{27}{64}$.

Fraction of volume remained $= 1 - \frac{27}{64} = \frac{37}{64}$.

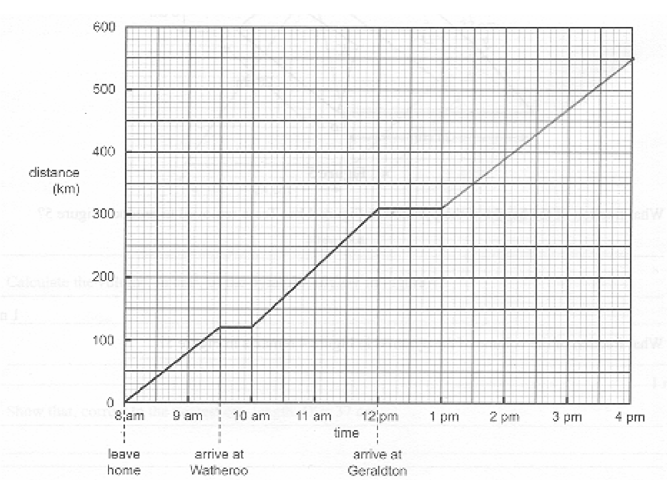
Module 3: Graphs and relations

Q1a 30 minutes

Q1b $310 - 120 = 190$ km

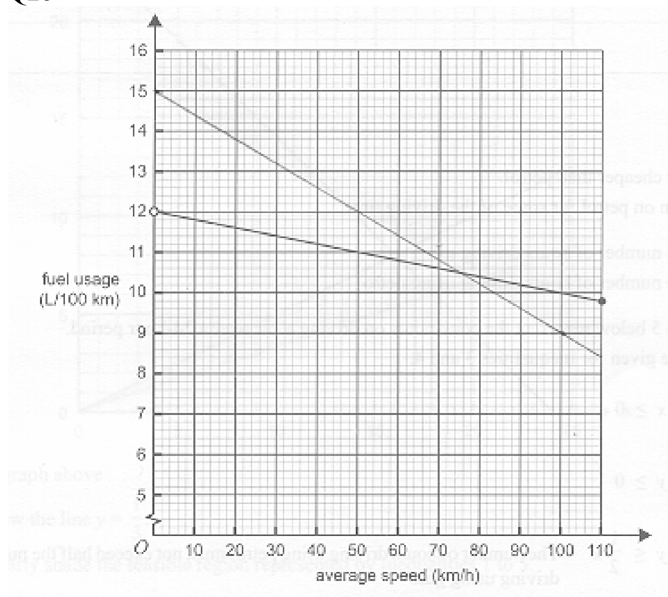
Q1c Average speed $= \frac{dist}{time} = \frac{190}{2} = 95$ kmh^{-1}

Q1d Distance from Geraldton to Hamelin $= 80 \times 3 = 240$ km
Total distance travelled since 8 am $310 + 240 = 550$ km



Q2a From graph, 10.8 L.
By calculation, $P = 12 - 0.02 \times 60 = 10.8$ L

Q2b

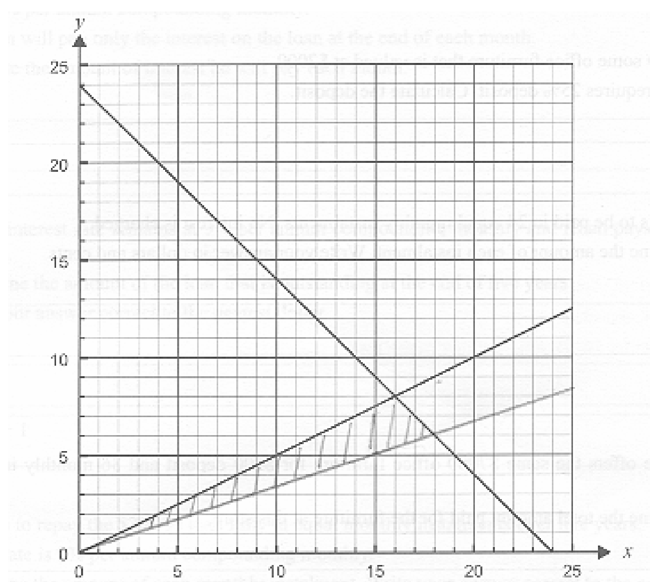


Q2c From graph above, average speed > 75 kmh^{-1}.
By calculation, $12 - 0.02s > 15 - 0.06s$, $0.04s > 3$, $s > 75$ kmh^{-1}.

Q2d $G = 15 - 0.06 \times 85 = 9.9$ L
Cost $= \$0.80 \times 9.9 = \$7.92$, i.e. 7 dollars 92 cents.

Q3a The number of hours driving using gas plus the number of hours driving using petrol must not exceed 24.

Q3bi and ii



Q3c When $x = 10$, $y = 15 - 10 = 5$. The point $(10,5)$ lies in the feasible region. (See graph above)

Q3d Additional constraints: $x \le 20$, $y \le 7$ and $x + y = 24$.
Maximum $x = 18$ hours
Minimum $x = 17$ hours.

Module 4: Business-related mathematics

Q1ai $\frac{25}{100} \times 7000 = \$1750$

Q1aii Balance $= 7000 - 1750 = 5250$
Instalment $= \frac{5250}{24} = 218.75$, i.e. 218 dollars 75 cents

Q1bi Total amount paid $= 500 + 220 \times 36 = \$8420$

Q1bii Interest $= 8420 - 7000 = 1420$.
Use $I = \frac{\Pr T}{100}$, $1420 = \frac{(7000 - 500) r \times 3}{100}$, $r \approx 7.3$.
Annual flat rate $\approx 7.3\%$.

Q1c After discount, $85\% \times 7000 = \$5950$.

Q2a $I = 30000 \times \frac{9}{100} \times \frac{1}{12} = \$225$

Q2b Use $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ or TVM Solver.
Amount $= \$16801$

Q2c Use $Q = \frac{PR^n (R-1)}{R^n - 1}$ or TVM Solver.
Instalment $= \$622.75$

Q3a $P\left(1 + \frac{10}{100}\right) = 900$, $P = \$818.18$.

Q3bi Flat rate depreciation:
Annual depreciation $= \frac{900 - 300}{5} = \$120$

Q3bii Unit cost depreciation:
Total number of faxes in five years $= 250 \times 5 = 1250$
Value after five years $= 900 - 0.46 \times 1250 = \$325$

Q4a Value after five years $= 10000\left(1 - \frac{12}{100}\right)^5 \approx \$5277$

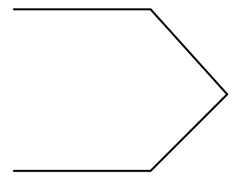
Q4b $10000\left(1 - \frac{r}{100}\right)^5 = 4000$, $\left(1 - \frac{r}{100}\right)^5 = 0.4$

$1 - \frac{r}{100} = 0.4^{\frac{1}{5}}$, $1 - \frac{r}{100} = 0.83255$, $r = 16.7$.
Rate is 16.7%.

Module 5: Networks and decision mathematics

Q1a Minimum number of edges $= 4$

Q1b



or other possibilities.

Q2a $4 + 2 + 5 + 2 + 4 + 4 + 3 = 24$

Q2bi G, or other possibilities.

Q2bii GABCAFGEFCEDC 2800 m

Q2c GABCDEFG or other possibilities.

Q3a $14 + 8 + 13 + 8 = 43$ from west to east.

Q3b $6 + 7 + 9 = 22$

Q3c The 10 children must be on the train with 11 available seats in order for them to set out from the West Terminal.
West 11, 13, 7, 7, 8 East. Maximum number of children is 7.

Q4a Critical path: BCFHI.
Minimum time $= 4 + 3 + 4 + 2 + 6 = 19$ weeks.

Q4b $LST - EST = 9 - 4 = 5$ weeks.

Q4c A, E, G.

Q4d Reduce C by 2, F by 2 and E by 1.
Minimum time is 15 weeks.

Q4e Total reduction in time $= 2 + 2 + 1 = 5$ weeks, which results in minimum additional cost of $\$5000 \times 5 = \$25000$.

Module 6: Matrices

Q1a $\begin{bmatrix} 1.2 & 20.1 & 4.2 \\ 6.7 & 0.4 & 0.6 \end{bmatrix}$

Q1bi $AB = [2 \times 531 + 2 \times 41 + 1 \times 534 + 1 \times 212] = [1890]$

Q1bii $B(4 \times 1) A(1 \times 4) = BA(4 \times 4)$

Q1biii It provides the total energy content in kilojoules of the peanut butter and honey sandwich.

Q1c

$$\begin{bmatrix} b \\ m \\ p \\ h \end{bmatrix} = \begin{bmatrix} 1.2 & 6.7 & 10.7 & 0 \\ 20.1 & 0.4 & 3.5 & 12.5 \\ 4.2 & 0.6 & 4.6 & 0.1 \\ 531 & 41 & 534 & 212 \end{bmatrix}^{-1} \begin{bmatrix} 53 \\ 101.5 \\ 28.5 \\ 3568 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$, by graphics calculator. $b = 4$, $m = 4$, $p = 2$ and $h = 1$.

Q2a $400 + 200 + 100 + 0 = 700$

Q2b 0.5

Q2ci $S_1 = TS_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$

Q2cii 280

Q2ciii $S_4 = T^4 S_0 = \begin{bmatrix} 10.24 \\ 56.32 \\ 312.96 \\ 320.48 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$

Number of live juveniles after 4 weeks $= 56$.

Q2civ $400 \times 0.4^n < 1$, $n > 6.5$.
After 7 weeks. (I think the question wants this answer)
In fact it is during the 7th week.

Q2cv $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix}$ because the adult insects have been sterilised.

Q2di

$$S_1 = TS_0 + BS_0 = \begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

Q2dii $S_2 = TS_1 + BS_1$
Number of live eggs $= 0.4 \times 190 + 0.3 \times 180 = 130$.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors