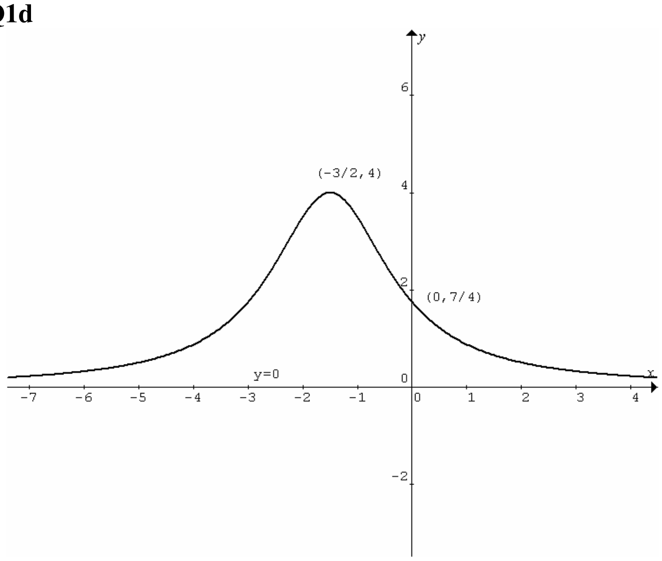
Q1a $f(x)=\dfrac{7}{x^2+3x+4}$, $y=f(0)=\dfrac{7}{4}$, $\left(0,\dfrac{7}{4}\right)$.

Q1b $f'(x)=\dfrac{-7(2x+3)}{\left(x^2+3x+4\right)^2}=0$, $x=-\dfrac{3}{2}$,

$f\left(-\dfrac{3}{2}\right)=\dfrac{7}{\left(-\frac{3}{2}\right)^2+3\left(-\frac{3}{2}\right)+4}=4$, $\left(-\dfrac{3}{2},4\right)$.

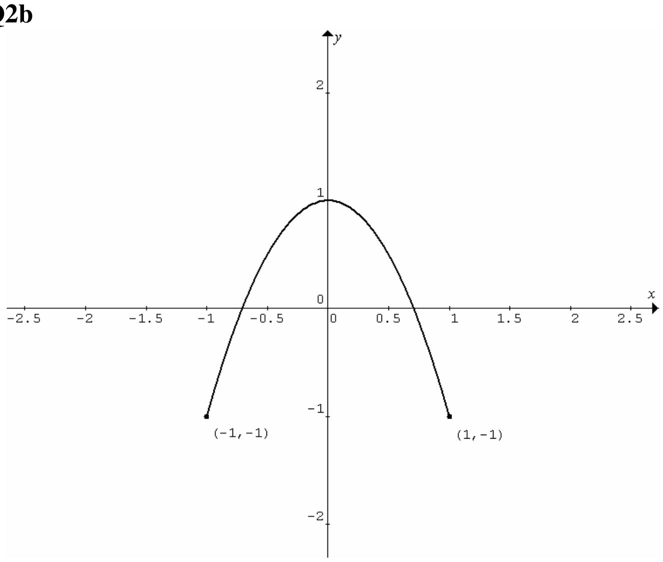
Q1c $y=0$.

Q1d



Q2a $\cos\left(2\sin^{-1}x\right)=1-2\left(\sin\left(\sin^{-1}x\right)\right)^2=1-2x^2$, $-1\le x\le 1$.

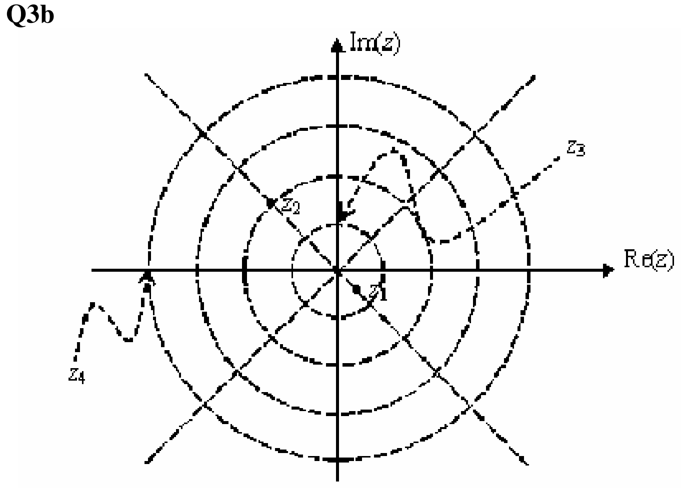
Q2b



Q3a $z_3=z_1z_2=\dfrac{1}{2}cis\left(-\dfrac{\pi}{4}\right)\times 2cis\left(\dfrac{3\pi}{4}\right)=cis\left(\dfrac{\pi}{2}\right)$.

$z_4=\dfrac{z_2}{z_1}=\dfrac{2cis\left(\frac{3\pi}{4}\right)}{\frac{1}{2}cis\left(-\frac{\pi}{4}\right)}=4cis(\pi)$.

Q3b



Q4a $z=-\dfrac{1}{\sqrt{2}}(1-i)$ is the other solution to $z^2+i=0$.

Q4b $z^8-1=\left(z^4-1\right)\left(z^4+1\right)=\left(z^2-1\right)\left(z^2+1\right)\left(z^2-i\right)\left(z^2+i\right)$

$=(z-1)(z+1)(z-i)(z+i)\left(z^2-i\right)\left(z-\dfrac{1}{\sqrt{2}}(1-i)\right)\left(z+\dfrac{1}{\sqrt{2}}(1-i)\right)$

Since z^8-1 has real coefficients, $\therefore$ the linear factors of z^2-i are complex conjugates of the factors of z^2+i.

Hence the solutions to $z^8-1=0$ are ±1 , $\pm i$, $\pm\dfrac{1}{\sqrt{2}}(1-i)$ or $\pm\dfrac{1}{\sqrt{2}}(1+i)$.

Q5a $F(3)=0$, where $F(x)=\int\dfrac{3}{2x^2+6}dx$.

$F(x)=\dfrac{\sqrt{3}}{2}\int\dfrac{\sqrt{3}}{3+x^2}dx=\dfrac{\sqrt{3}}{2}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)+c$.

$F(3)=\dfrac{\sqrt{3}}{2}\tan^{-1}\left(\sqrt{3}\right)+c=0$, $\therefore c=-\dfrac{\sqrt{3}}{2}\times\dfrac{\pi}{3}$.

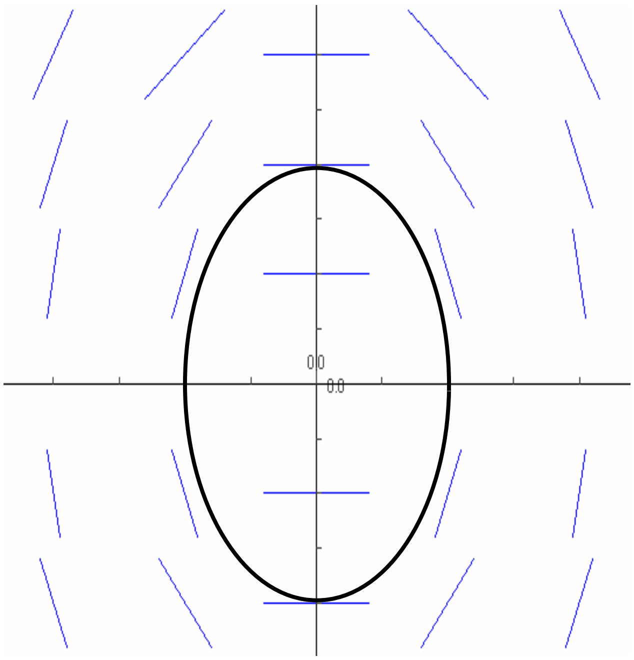
$\therefore F(x)=\dfrac{\sqrt{3}}{2}\left(\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)-\dfrac{\pi}{3}\right)$.

Q5b $\dfrac{d}{dx}\left(\sin^{-1}\sqrt{1-x^2}\right)=\dfrac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\times\dfrac{1}{2\sqrt{1-x^2}}\times(-2x)$

$=\dfrac{1}{x}\times\dfrac{-x}{\sqrt{1-x^2}}=-\dfrac{1}{\sqrt{1-x^2}}$, where $1-x^2>0$, i.e. $-1<x<1$.

Q6a $4x^2 + y^2 = 16$, $8x + 2y\dfrac{dy}{dx} = 0$, $\therefore \dfrac{dy}{dx} = -\dfrac{4x}{y}$.

Q6b and Q6c



Q7a $|3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k}| = \sqrt{3^2 + 4^2 + (-12)^2} = 13$

Unit vector $= \dfrac{1}{13}(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$.

Q7b $\cos\theta = -\dfrac{12}{13}$, $\theta = \cos^{-1}\left(-\dfrac{12}{13}\right) \approx 157.4°$.

Q7c Dependent if $n(p\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}) = 3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k}$,
i.e. $np\boldsymbol{i} - n\boldsymbol{j} + 3n\boldsymbol{k} = 3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k}$,
i.e. $n = -4$ and $np = 3$, $\therefore p = \dfrac{3}{n} = -\dfrac{3}{4}$.

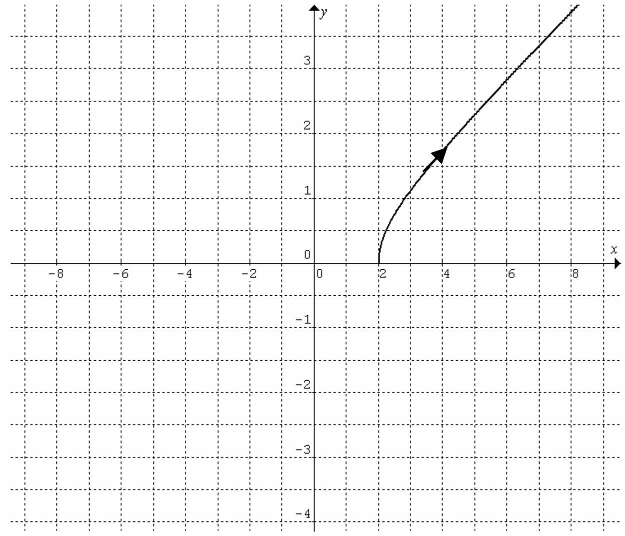
$\therefore$ independent if $p \neq -\dfrac{3}{4}$.

Q8a $\boldsymbol{r}(t) = 2\sec(t)\boldsymbol{i} + \tan(t)\boldsymbol{j}$.
$\boldsymbol{r}(0) = 2\sec(0)\boldsymbol{i} + \tan(0)\boldsymbol{j} = 2\,\boldsymbol{i}$.

Q8b $\boldsymbol{v} = \dfrac{d}{dt}\boldsymbol{r} = \dfrac{\sin(t)}{\cos^2(t)}\boldsymbol{i} + \sec^2(t)\boldsymbol{j} = \sec(t)\tan(t)\ \boldsymbol{i} + \sec^2(t)\boldsymbol{j}$.

Q8c $x = 2\sec(t)$ and $y = \tan(t)$, where $0 \le t < \dfrac{\pi}{2}$.

$1 + \tan^2(t) = \sec^2(t)$, $\therefore 1 + y^2 = \left(\dfrac{x}{2}\right)^2$, $\therefore \dfrac{x^2}{4} - y^2 = 1$.



Q9a $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = m\vec{a}$,
$3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + (-\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) + \overrightarrow{F_3} = 0.5\,(4\boldsymbol{i} + 2\boldsymbol{k})$,
$\therefore \overrightarrow{F_3} = -5\boldsymbol{j}$.

Q9b $|\vec{a}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$, $u = 0$, $t = 1$,
$\therefore s = ut + \dfrac{1}{2}at^2 = \sqrt{5}$. Distance $= \sqrt{5}$ m.

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