2008 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

Q1a

walked	10
sat or stood	7
ran	8

Q1b $\frac{8}{25}\times 100\% = 32\%$

Q2

	Year 6	Year 8	Year 10
ran	78%	40%	10%
sat or stood	2%	24%	68%

Note: Whole number %.

Yes. As the year level increases, % of girls who ran decreases, and the % of girls who sat or stood increases.

Q3a $\boxed{124}$ and $\boxed{148}$

Q3b The median arm span increases as the year level increases.

Q3c $Q_1 - 1.5\times IQR = 160 - 1.5\times 10 = 145$. Since $140 < 145$, $\therefore 140$ is still an outlier.

Q4a Height

Q4b Graphics calculator. $Arm\ span = 1.09\times\ height - 15.63$

Q4c Arm span increases by 1.09 cm for each cm increase in height.

Q5a Graphics calculator.

$$Homework\ hours = 102.90\times\frac{1}{television.hours} + 5.12$$

Q5b $Homework\ hours = 102.90\times\frac{1}{12} + 5.12 = 13.7$

Module 1: Number patterns

Q1a $1250 - 2\times 150 = 950$

Q1b $\boxed{^{-}150}$

Q1c $\frac{1250}{150} = 8.333...$ 8 full years.

Q2ai $C_1 = 1.08\times 1250 - 150 = 1200$

Q2aii Graphics calculator. $C_6 = 883$

Q2bi 8%

Q2bii Graphics calculator. 14 full years.

Q2c $1250 = 1.08\times 1250 - k$, $k = 100$

Q3a $32\times 1.5^3 = 108$

Q3b Graphics calculator or
$32\times 1.5^n > 820$, $n > 7.999624$. After 8 years.

Another possible answer (from comments received).
$32\times 1.5^n \geq 821$, $n \geq 8.00263$. After 9 years.

Q3c $32\times 1.5^5 - 32\times 1.5 = 195$

Q3d 32×1.5^n

Q3e $D_n = 1.5D_{n-1}$, $D_0 = 32$

Q4 Graphics calculator. 12 years

n	D_n	C_n
0	32	1250
1	35	1200
2	39	1146
3	44	1088
4	52	1025
5	63	957
6	78	883
7	99	804
8	128	718
9	170	626
10	227	526
11	308	418
12	422	301

Module 2: Geometry and trigonometry

Q1a $V = 6\times 10\times 0.2 = 12$

Q1bi 3 cm : 6 m, 3 cm : 600 cm, 1 : 200.
Scale down factor $= \frac{1}{200}$

Q1bii $Area = 60\times\left(\frac{1}{200}\right)^2 = 0.0015$ m^2 or 15 cm^2

Q2a Length of OM $= \sqrt{3.4^2 - 3^2} = 1.6$

Q2b Area $= 2.2 \times 6 + \frac{1}{2} \times 6 \times 1.6 = 18$

Q2c Volume $= 18 \times 10 = 180$

Q2di Area $= 2(18) + 6 \times 10 + 2(2.2 \times 10) + 2(3.4 \times 10) = 208$

Q2dii $\frac{208}{16} = 13$ litres

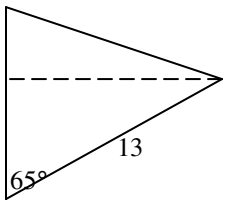
Q3a $\theta = \tan^{-1}\left(\frac{12}{13}\right) = 42.7°$

Q3b The cosine rule:
Distance NT $= \sqrt{10^2 + 13^2 - 2(10)(13)\cos 65°} = 12.6$

Q3c The sine rule: $\frac{\sin\theta°}{13} = \frac{\sin 65°}{12.6}$, $\theta° = 69°$

Q3d True bearing: $180° - 69° = 111°$

Q3e Possible.
$13\sin 65° = 11.78$ and $11.78 < 12$.



Module 3: Graphs and relations

Q1a Read from graph. Pulse rate = 110

Q1b Increase $= 150 - 70 = 80$

Q1ci *Maximum pulse rate* $= 220 - age$

Q1cii Max rate $= 220 - 20 = 200$
Between 60% of 200 and 75% of 200, i.e. between 120 and 150.

Q2a $R = 35x$

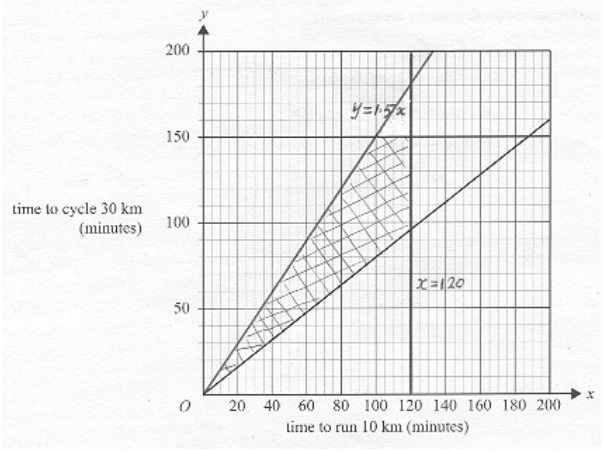
Q2b $C = 50625 + 12.50x$

Q2ci To break even, $R = C$, $35x = 50625 + 12.50x$,
$\therefore x = 2250$

Q2cii $P = R - C = 22.50x - 50625 = 22.50 \times 8670 - 50625$
$= \$144450$

Q3a The time taken to run 10 km is not more than 120 minutes.

Q3bi and ii



Q3c Read from graph. $80 \le y \le 150$

Q3di Solve $x + y = 90$ and $y = 1.5x$ to find $y = 54$

Q3dii Solve $x + y = 90$ and $y = 0.8x$ to find $x = 50$

Module 4: Business-related mathematics

Q1a $6870.67 - 6250.67 = 620$

Q1b Minimum balance = 6120.86
Interest $= 6120.86 \times \frac{3}{100} \times \frac{1}{12} = \$15.30$

Q2a $A = 3000\left(1 + \frac{4.1}{100}\right)^4 = \$3523.09$

Q2b $A = 3000\left(1 + \frac{4.1}{100}\right)^8 = \$4137.40$
Interest $= 4137.40 - 3000 = \$1137.40$

Q3a Value after 2 years $= 17000\left(1 - \frac{10}{100}\right)^2 = 13770$
Depreciation in the 3rd year $= 13770 \times \frac{10}{100} = \$1377$

Q3b $17000(0.9)^n < 7000$, $n > 8.4216$. After 9 years.

Q4a Annual depreciation $= \frac{17000 - 3500}{15} = \$900$

Q4b Flat rate $= \frac{900}{17000} \times 100\% = 5.3\%$ p.a.

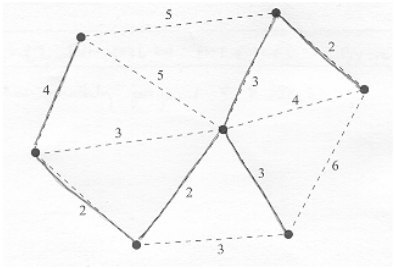
Q5a TVM Solver to find $N = 52.4225$. $\therefore$ 52 equal monthly payments of \$350.

Q5b $350 \times 0.4225 = \$148$

Q5c TVM Solver to find amount owing $FV = 12086.603$ after 1 year.
TVM Solver to find new monthly payment $PMT = \$388.30$

Module 5: Networks and decision mathematics

Q1a

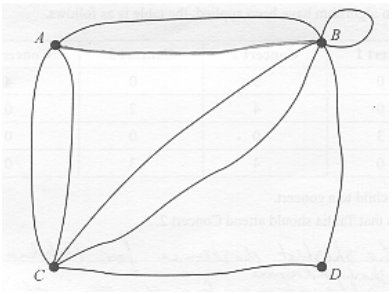


Q1b $4+2+2+3+3+2=16$

Q1c 2

Q2a 7

Q2bi



Q2bii C is an odd degree vertex, $\therefore$ not an Euler's circuit.

Q3a

	Concert 1	Concert 2	Concert 3	Concert 4
Tahlia	2	0	7	5

Q3b It would be the shortest distance for Tahlia without affecting the shortest distance allocations for the other three.

Q3c A possibility:

	Concert
James	3
Dante	4
Tahlia	2
Chanel	1

Q3d $18+15+13+10=56$

Q4a

$$D^1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 2 \\ 4 \\ 1 \end{array}$$

Barnaby and Cedric, Arnold and Edgar

Q4b Edgar

Q4c

$$D^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 2 \\ 2 \\ 3 \\ 6 \\ 1 \end{array}$$

$A \quad 1+2=3$
$B \quad 2+2=4$
$C \quad 2+3=5$
$D \quad 4+6=10$
$E \quad 1+1=2$

$\therefore$ 1st Darcy, 2nd Cedric, 3rd Barnaby, 4th Arnold, 5th Edgar.

Q4d

$$D^1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$D^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{array}$$

Total $= 4+4+4+4+4=20$

Module 6: Matrices

Q1a 1×5

Q1bi Graphics calculator.

$$R = NP = \begin{bmatrix} 23 & 57.5 & 80.5 & 207 & 92 \\ 18 & 45 & 63 & 162 & 72 \end{bmatrix}$$

Q1bii $R_{24} = 162$ is the number of Chemistry students awarded a D grade.

Q1ci

$$F = \begin{matrix} \text{Bio} & \text{Chem} \end{matrix} \begin{bmatrix} 110 & 95 \end{bmatrix}$$

Q1cii

$$L = FN = \begin{bmatrix} 110 & 95 \end{bmatrix} \begin{bmatrix} 460 \\ 360 \end{bmatrix} = \begin{bmatrix} 110\times 460 + 95\times 360 \end{bmatrix} = \begin{bmatrix} 84800 \end{bmatrix}$$

Total lab fees = \$84800

Q2ai $S_2 = TS_1 = \begin{bmatrix} 493.2 \\ 82.8 \end{bmatrix}$

Q2aii $S_5 = T^4 S_1 = \begin{bmatrix} 421.4556 \\ 154.5444 \end{bmatrix}$

Number of History students attending the fifth lecture is 421.

Q2b $S_n = T^{n-1} S_1$

Q2c The eighth lecture $S_8 = \begin{bmatrix} 396.847 \\ 179.153 \end{bmatrix}$

Q2d For large n, $T^n \to \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$.

Number of students attending $= \frac{2}{3}\times 540 + \frac{2}{3}\times 36 = 384$

Q3a $O_{2009} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix} \begin{bmatrix} 456 \\ 350 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \end{bmatrix} = \begin{bmatrix} 360 \\ 250 \end{bmatrix}$

Q3b $C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} = 0.8I$

$$O_{2009} = CO_{2008} - D = \begin{bmatrix} 360 \\ 250 \end{bmatrix}$$

$$O_{2010} = CO_{2009} - D = \begin{bmatrix} 248 \\ 162 \end{bmatrix}$$

248 Maths textbooks.

Q4

$$T = \begin{bmatrix} 0.88 & 0.52 & 0.65 \\ 0.10 & 0.44 & 0.10 \\ 0.02 & 0.04 & 0.25 \end{bmatrix} \qquad N_{2007} = \begin{bmatrix} 880 \\ 230 \\ 120 \end{bmatrix}$$

$$N_{2009} = T^2 N_{2007} = \begin{bmatrix} 997 \\ 191 \\ 42 \end{bmatrix}$$

42 will defer the 2009 academic year.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors