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Specialist Mathematics

2009

Trial Examination 2

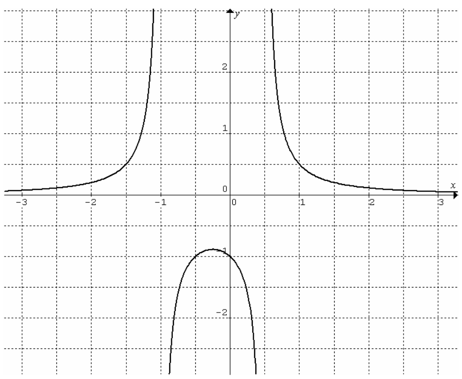
SECTION 1 Multiple-choice questions

Instructions for Section 1

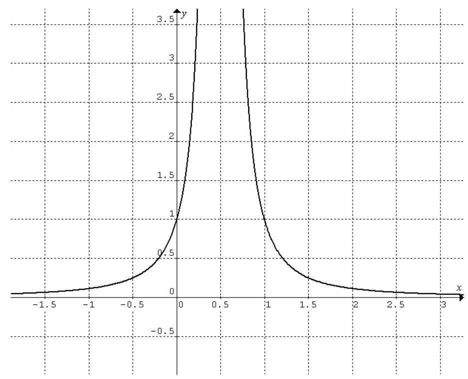
Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 Which one of the following graphs is **NOT** that of $f(x) = \dfrac{1}{ax^2 + bx + c}$, where $a \neq 0$?

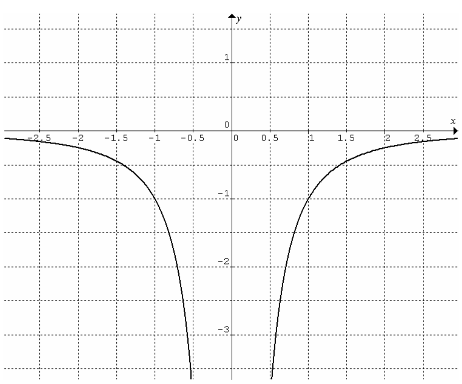
A.



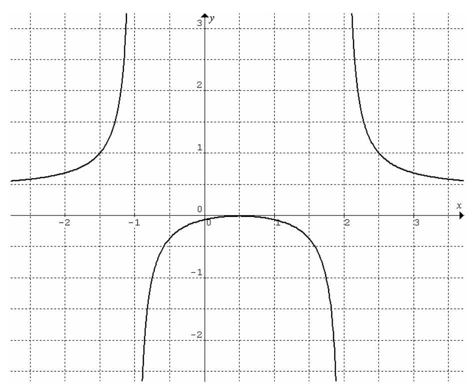
B.



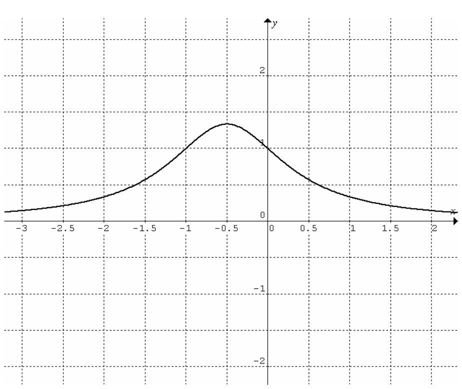
C.



D.



E.



Question 2

$(\sec(x+y)-\tan(x+y))(\operatorname{cosec}(x+y)+\cot(x+y))(\sec(x+y)+\tan(x+y))(\operatorname{cosec}(x+y)-\cot(x+y))$ is equal to

A. $\cos(x+y)+\sin(x+y)$

B. $\cos(x+y)-\sin(x+y)$

C. $\cos^2(x+y)+\sin^2(x+y)$

D. $\cos^2(x+y)-\sin^2(x+y)$

E. $\cos^4(x+y)+\sin^4(x+y)$

Question 3 If $f(x)=\frac{k\pi}{2}-\tan^{-1}x$ and $k\in R$, then $f^{-1}(x)=$

A. $\tan x$, where $x\in(0,\pi)$

B. $\cot x$, where $x\in(0,\pi)$

C. $\tan x$, where $x\in\left(0,\frac{k\pi}{2}\right)$

D. $\cot\left(\frac{k\pi}{2}-x\right)$, where $x\in\left(\frac{(k-1)\pi}{2},\frac{(k+1)\pi}{2}\right)$

E. $\tan\left(\frac{k\pi}{2}-x\right)$, where $x\in\left(\frac{(k-1)\pi}{2},\frac{(k+1)\pi}{2}\right)$

Question 4 The domain and range of $\cos^{-1}\left(\tan\left(x+\frac{\pi}{4}\right)\right)$ are respectively

A. $\left[-\frac{\pi}{2},0\right]$ and $[0,\pi]$

B. $\left[0,\frac{\pi}{2}\right]$ and $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

C. $\left(0,\frac{\pi}{2}\right)$ and $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

D. $\left\{x:\frac{(1\pm 2n)\pi}{2}\le x\le(1\pm n)\pi, n=0,1,2,......\right\}$ and $[0,\pi]$

E. $\left\{x:\frac{(1\pm 2n)\pi}{2}< x<(1\pm n)\pi, n=0,1,2,......\right\}$ and $(0,\pi)$

Question 5 $\dfrac{3x^3-4x^2-x-4}{6x^2-12x+6}=ax+b+\dfrac{c}{1-x}+\dfrac{d}{(1-x)^2}$ when

A. $a=\frac{1}{2}$, $b=\frac{1}{2}$, $c=1$ and $d=1$

B. $a=\frac{1}{2}$, $b=\frac{1}{3}$, $c=0$ and $d=-1$

C. $a=\frac{1}{3}$, $b=\frac{1}{2}$, $c=1$ and $d=-1$

D. $a=\frac{1}{3}$, $b=-\frac{1}{2}$, $c=0$ and $d=1$

E. $a=\frac{1}{2}$, $b=-\frac{1}{3}$, $c=1$ and $d=-1$

Question 6 If $z=i\left(\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}+\theta\right)\right)$ and $0<\theta<\frac{\pi}{2}$, $Arg\left(\frac{1}{z}\right)$ is

A. $\frac{\pi}{2}-\theta$ B. $\frac{\pi}{2}+\theta$ C. $\pi-\theta$ D. $\pi+\theta$ E. $\theta-\pi$

Question 7 $1-2i$ is a root of $z^3-(1-2i)z^2+3z-3+6i=0$.

A. There are two more roots and both are real.

B. There are two more roots and both are complex.

C. There are two more roots, one is real and the other is complex.

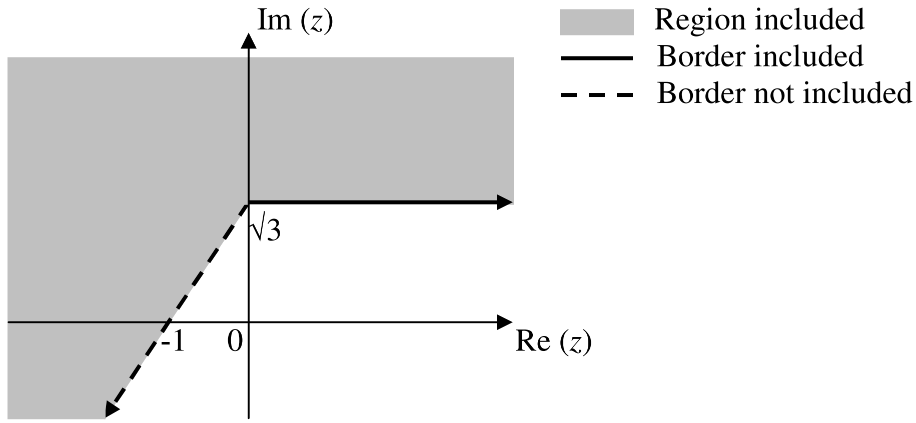
D. $1+2i$ is another root.

E. $-1+2i$ is another root.

Question 8 $cis\left(-\frac{7\pi}{12}\right)-cis\left(\frac{\pi}{12}\right)=$

A. $2cis\left(\frac{\pi}{3}\right)$ B. $-2cis\left(\frac{\pi}{6}\right)$ C. $-\sqrt{3}cis\left(\frac{\pi}{4}\right)$ D. $\sqrt{3}cis\left(\frac{\pi}{4}\right)$ E. $2cis\left(\frac{5\pi}{6}\right)$

Question 9



The shaded region in the complex plane C represents

A. $C \setminus \left\{ z : -\frac{2\pi}{3} \le Arg\left(z - i\sqrt{3}\right) < 0 \right\}$

B. $C \setminus \left\{ z : -\frac{2\pi}{3} < Arg\left(z - i\sqrt{3}\right) \le 0 \right\}$

C. $\left\{ z : 0 \le Arg\left(z - i\sqrt{3}\right) < \pi \right\} \cup \left\{ z : -\pi \le Arg\left(z - i\sqrt{3}\right) < -\frac{2\pi}{3} \right\}$

D. $\left\{ z : 0 \le Arg\left(z + i\sqrt{3}\right) < \pi \right\} \cup \left\{ z : -\pi \le Arg\left(z + i\sqrt{3}\right) < -\frac{\pi}{3} \right\}$

E. $\left\{ z : 0 \le \arg\left(z + i\sqrt{3}\right) < \frac{4\pi}{3} \right\}$

Question 10 The area of the region bounded by the y-axis, the x-axis, $y = 2\pi$ and $y = 2\cos^{-1}(2x)$ is

A. 4 B. 3 C. 2 D. 1 E. 0

Question 11 An anti-derivative of $\frac{e^{2x}-1}{e^{2x}+1} - \frac{e^{2x}+1}{e^{2x}-1}$ is

A. $\log_e\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$ B. $\log_e\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$ C. $\log_e\left(\frac{e^{2x}}{e^{2x}-1}\right)$ D. $\log_e\left(\frac{e^{2x}}{e^{2x}+1}\right)$ E. $\log_e\left(\frac{e^{4x}}{e^{4x}-1}\right)$

Question 12 $P(x, y)$ is a point on the ellipse $\frac{x^2}{2}+y^2=1$, and $\frac{x^2}{2}+y=c$. The maximum value of c is

A. 1 B. $\frac{6}{5}$ C. $\frac{5}{4}$ D. $\frac{4}{3}$ E. $\frac{3}{2}$

Question 13 An anti-derivative of $\frac{1}{2}\sin(2x)\sqrt{1-\cos x}$ is

A. $\frac{2}{3}(1-\cos x)^{1.5}+\frac{2}{5}(1-\cos x)^{2.5}$

B. $\frac{2}{3}(1-\cos x)^{1.5}-\frac{2}{5}(1-\cos x)^{2.5}$

C. $\frac{2}{5}(1-\cos x)^{2.5}-\frac{2}{3}(1-\cos x)^{1.5}$

D. $\frac{4}{5}(1-\cos x)^{2.5}-\frac{4}{3}(1-\cos x)^{1.5}$

E. $\frac{4}{3}(1-\cos x)^{1.5}+\frac{4}{5}(1-\cos x)^{2.5}$

Question 14 Given $f(x)=\tan^{-1}(x)$, $f'(x)=f''(x)$ when $x=$

A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

Question 15 Let $\frac{dy}{dx}=\tan^{-1}(x^2)$ and $y=c$ when $x=-1$. When $x=-2$, the value of y is closest to

A. $c+1.12$ B. $1.12-c$ C. $c-1.12$ D. $1.12c$ E. $\frac{c}{1.12}$

Question 16 A solution of $25x-4(y-2)\frac{dy}{dx}+25=0$ is

A. $y=2+\frac{5}{2}\sqrt{(x-3)(x+1)}$

B. $y=2-\frac{5}{2}\sqrt{(x-3)(x+1)}$

C. $y=2-\frac{5}{2}\sqrt{(x+3)(x-1)}$

D. $y=2+\frac{5}{2}\sqrt{(x+3)(x+1)}$

E. $y=2+\frac{5}{2}\sqrt{(x-3)(x-1)}$

Question 17 $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are non-zero vectors independent of each other. If $\tilde{a}+\tilde{b}$ is perpendicular to $\tilde{c}+\tilde{d}$, and $\tilde{b}+\tilde{c}$ is perpendicular $\tilde{d}+\tilde{a}$, then

A. $\tilde{b}+\tilde{d}$ is perpendicular to $\tilde{c}-\tilde{a}$

B. $\tilde{b}-\tilde{d}$ is perpendicular to $\tilde{c}+\tilde{a}$

C. $\tilde{b}+\tilde{d}$ is perpendicular to $\tilde{c}+\tilde{a}$

D. $\tilde{b}-\tilde{d}$ is perpendicular to $\tilde{c}-\tilde{a}$

E. $\tilde{a}-\tilde{b}+\tilde{c}$ is perpendicular to $\tilde{b}-\tilde{c}+\tilde{d}$

Question 18 The acceleration of a particle is given by $a=-2(x-3)^3$ at displacement x from the origin O. At $x=3+\sqrt{2}$, the speed is $v=0$. The minimum displacement $x_{\min}$ from O and the maximum speed $v_{\max}$ are respectively

A. $3-\sqrt{2}$, 2 B. $3+\sqrt{2}$, 2 C. $-\sqrt{2}$, 4 D. $\sqrt{2}$, 2 E. $\sqrt[3]{3}$, 4

Question 19 The position of a particle is given by $\tilde{r} = (\cos t)\tilde{i} + (\sin t)\tilde{j} + 4\tilde{k}$ at time t. The closest distance of the particle from the fixed position $\tilde{r} = 2\sqrt{3}\tilde{i} - 2\tilde{j}$ is

A. $4\sqrt{2}$ B. $3\sqrt{3}$ C. 5 D. 4 E. 3

Question 20 A vector independent of $\tilde{i} - 2\tilde{j}$ and $-\tilde{j} + 2\tilde{k}$ is

A. $\tilde{i} - 2\tilde{j} + 2\tilde{k}$ B. $2\tilde{i} - \tilde{j} - 6\tilde{k}$ C. $3\tilde{i} - 4\tilde{j} - 4\tilde{k}$ D. $-\tilde{i} + \tilde{j} + 2\tilde{k}$ E. $\tilde{i} - 4\tilde{k}$

Question 21 A particle slides **up** an inclined plane, which makes a 30° angle with the horizontal. The forces on the particle are its weight $W = 14.7$ N and the reaction $R = 14.7$ N of the plane on it. The magnitude of the particle's acceleration in ms^{-2} is

A. 9.8 B. 4.9 C. 2.5 D. 1.2 E. 0.60

Question 22 A particle moves with **constant** acceleration a ms^{-2} in a straight line. Its velocity changes from 15.0 ms^{-1} forward to 10.0 ms^{-1} in the opposite direction after travelling 65.0 m. The value of $|a|$ is

A. 3.96 B. 3.50 C. 2.96 D. 2.50 E. 0.96

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

a i. Given $f(x) = \log_e \sqrt[4]{\dfrac{x^2+x+1}{x^2-x+1}}$, show that $f'(x) = \dfrac{1-x^2}{2(x^2+x+1)(x^2-x+1)}$. 2 marks

a ii. Given $g(x) = \tan^{-1}\left(\dfrac{2x+1}{\sqrt{3}}\right) + \tan^{-1}\left(\dfrac{2x-1}{\sqrt{3}}\right)$, find $g'(x)$. 2 marks

a iii. Hence show that $\dfrac{dy}{dx} = \dfrac{1}{x^4+x^2+1}$, where $y = f(x) + \dfrac{1}{2\sqrt{3}} g(x)$. 1 mark

b. Hence or otherwise find the **exact** area of the region bounded by $h(x) = \dfrac{1}{x^4+x^2+1}$, the axes and $x = 1$.
2 marks

c. Find the coordinates (round to 4 decimal places) of the inflection points of $h(x) = \dfrac{1}{x^4+x^2+1}$. 2 marks

Total 9 marks

Question 2 Consider $y = \frac{1}{\sqrt{1+x^2}}$.

a. Use the substitution $x = \tan\theta$ to show that $\int \frac{dx}{\sqrt{1+x^2}} = \int \sec\theta d\theta$ 1 mark

b i. Show full working that $\int \sec\theta \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} d\theta = \log_e|\sec\theta + \tan\theta| + c$. 2 marks

b ii. Hence show that the area of the region bounded by $y = \frac{1}{\sqrt{1+x^2}}$, the axes and $x = 2\sqrt{2}$ is $\log_e\left(3 + 2\sqrt{2}\right)$.

2 marks

c. The region referred to in part **b ii.** is rotated in the *x*-axis. Show that the volume of revolution of the solid is $\pi \tan^{-1}\left(2\sqrt{2}\right)$. 1 mark

d. The same region is rotated in the *y*-axis. Find the exact volume of revolution of the solid. 2 marks

Now the function $y=\frac{1}{\sqrt{1+x^2}}$ is transformed to $y=\frac{1}{\sqrt{1+x^2}}+1$.

e. The region bounded by $y=\frac{1}{\sqrt{1+x^2}}+1$, the axes and $x=2\sqrt{2}$ is rotated in the x-axis. Use the results obtained previously to find the exact volume of revolution of the solid. 2 marks

f. The region bounded by $y=\frac{1}{\sqrt{1+x^2}}+1$, the axes and $x=2\sqrt{2}$ is rotated in the y-axis. Find the exact difference between the volume of revolution of this solid and that found in part **d**. 1 mark

Total 11 marks

Question 3 The velocity of a particle is given by $\tilde{v}=\left(xe^{-t}\right)\tilde{i}-(5t-1)\tilde{j}$, where $x>0$ is the x-coordinate of the particle's position at time $t\geq 0$. The position vector of the particle at $t=0$ is $\tilde{r}(0)=\tilde{i}+2\tilde{j}+3\tilde{k}$. Time is measured in seconds and distance in metres.

a Use Euler's method and step size of 0.1 to find the first order approximation (2 decimal places) of the x-coordinate of the particle's position at time $t=0.2$ s.

3 marks

The x-coordinate of the particle's position at time $t \geq 0$ is $x = e^{1-e^{-t}}$.

b i. Find the position vector of the particle at time t. 2 marks

b ii. Hence find the time (2 decimal places) when the particle is closest to the origin. 2 marks

c. Find the speed (2 decimal places) of the particle at $t = 0.3$ s. 1 mark

d. Find the acceleration (2 decimal places) of the particle at $t = 0.3$ s. 2 marks

e. The velocity of a **second** particle is given by $\tilde{v} = (\log_e(10t-1))\tilde{i} + (\tan^{-1}(10t-1))\tilde{j} + \frac{3}{\sqrt{t}}\tilde{k}$. The position vector of the particle at $t = 0$ is $\tilde{r}(0) = 2\tilde{i} + \tilde{j}$. Find the time when the second particle arrives at the plane that the first particle moves in.

2 marks

Total 12 marks

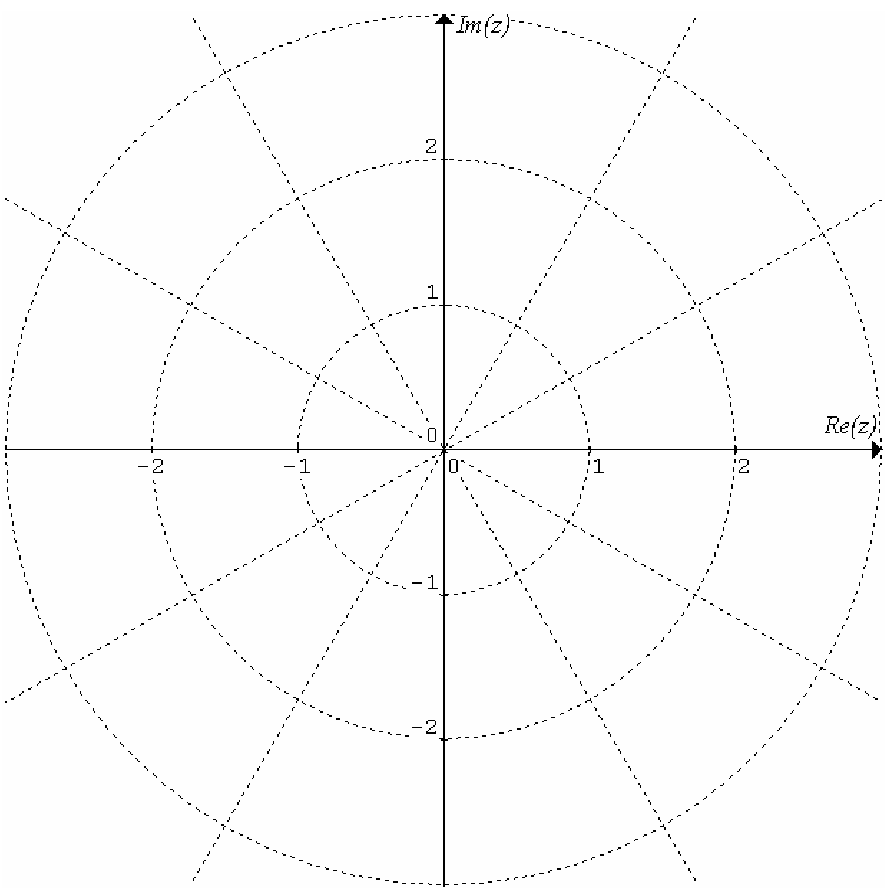
Question 4 Let $z \in C$.

a i. Given $z^4 + z^2 + 1 = (z^2 + h)^2 - kz^2$, and $h, k \in R^+$, find the values of h and k. 1 mark

a ii. Hence find the roots of $z^4 + z^2 + 1 = 0$. 1 mark

b. Find the roots of $z^4 - z^2 + 1 = 0$. Show working. 2 marks

c. Accurately plot the roots of $z^8 + z^4 + 1 = 0$. 2 marks



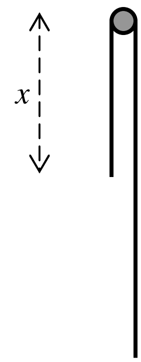
d. Write down the value of the product of the roots in part **c**. 1 mark

e i. Describe the subset of the complex plane represented by $\{z : |\text{Im}(z-2i)| \leq \sqrt{2}|z+2i|\}$. 3 marks

e ii. How many of the roots found in part **c** belong to $\{z : |\text{Im}(z-2i)| \leq \sqrt{2}|z+2i|\}$? Explain your answer by means of a diagram and/or calculation. 3 marks

Total 13 marks

Question 5 A 5-metre long uniform rope has a linear density of 0.50 kg per metre. It passes over a frictionless pulley of negligible radius and mass. Let x metres be the length of the shorter side at time t seconds from the time when both sides have the same length and are moving at 0.20 ms^{-1}.



a. Show that the acceleration of the rope is given by $(1-0.4x)g$ ms^{-2}. 2 marks

b i. Set up a differential equation involving the speed v ms^{-1} of the rope and x. 1 mark

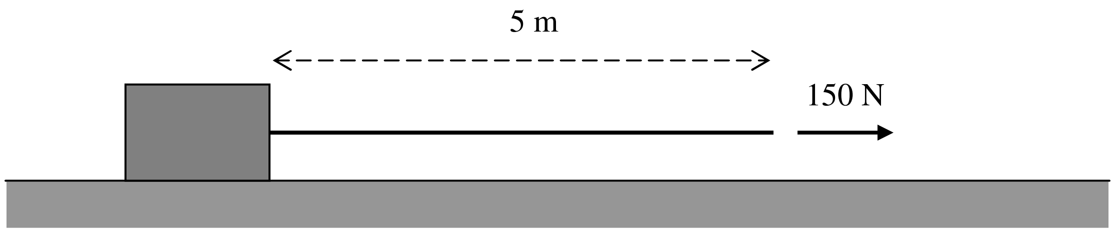
b ii. Find the speed (2 decimal places) of the rope when it starts to fall freely under gravity. 2 marks

b iii. Find the time taken (2 decimal places) for the rope to pass over the pulley completely. 2 marks

b iv. Find the magnitude of the change in momentum (2 decimal places) of the rope in the time interval found in part **b iii**.

1 mark

Now the same rope is used to pull from **rest** a 15-kg box on a horizontal floor. The coefficient of friction between the floor and the box is 0.90. The force pulling the rope is 150 N. Assume that the rope does not contribute to the friction between the floor and the box.



c i. Calculate the acceleration (2 decimal places) of the box. 2 marks

c ii. Calculate the distance (2 decimal places) travelled by the box if the average speed over the distance is 1.0 ms^{-1}.

1 mark

d. Calculate the difference (2 decimal places) in tension between the two ends of the 5-m rope. 1 mark

e. If the pulling force is maintained and the box is at rest initially, what is the minimum **additional** mass to the box that will prevent it from sliding along the same floor?

1 mark

Total 13 marks

End of Exam 2

