2009 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

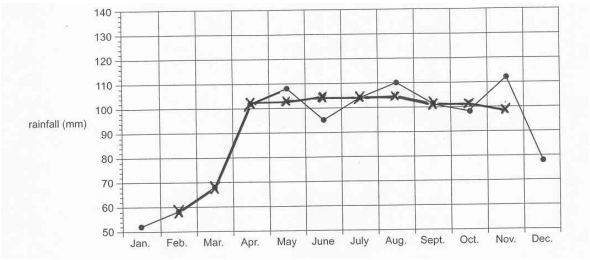
Q1a The dot above 14

Q1bi $\frac{15+16}{2}=15.5$

Q1bii $\frac{11}{12}\times 100\% \approx 92\%$

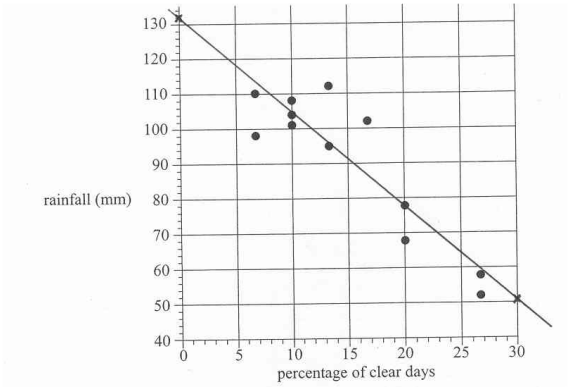
Q2a November

Q2b



Q2c Increases from Feb to April, and stays constant at April's rainfall up to Nov.

Q3a



Q3b rainfall $=131-2.68\times 35=37.2$ mm

Q3ci About 81% of the variation in monthly rainfall can be explained by the variation in the monthly percentage of clear days in 2008.

Q3cii $r=-\sqrt{0.8081}=-0.899$

Q4a $4-(0.78+1.05+1.07)=1.10$

Q4b Deseasonalised value $=\frac{188}{0.78}\approx 241$ mm

Q4c Autumn rainfall is higher than the likely rainfall if there is no seasonal fluctuations by 5%.

Module 1: Number patterns

Q1a AP: $a=28$, $d=1$, $t_{10}=28+9=37$ seats.

Q1b AP: $70=28+(n-1)1$, $n=43$ rows.

Q1c $S_{20}=\frac{20}{2}(2\times 28+19)=750$ seats.

Q1di GP: $a=2800$, $r=1.01$, $t_2=2800\times 1.01=\$2828.00$

Q1dii $t_9=2800\times 1.01^8=\$3032.00$

Q1e $S_{30}=\frac{2800(1.01^{30}-1)}{1.01-1}\approx \$97398$

Q1f AP: $a=28$, $d=1$, $t_{26}=28+25=53$ seats

GP: $a=2800$, $r=1.01$, $t_{26}=2800\times 1.01^{25}=\$3590.81$

Seat price in row $26=\frac{3590.81}{53}=\$67.75$

Q2a 2%

Q2b $R_{n+1}=1.02R_n$, $R_n=\frac{R_{n+1}}{1.02}$.

$R_6=2601$, $R_5=\frac{2601}{1.02}=2550$, $R_4=\frac{2550}{1.02}=\$2500$

Q3a $T_{n+1}=0.8T_n+1000$

$T_1=12000$

$T_2=0.8\times 12000+1000=10600$

$T_3=0.8\times 10600+1000=9480$

Q3b $T_2-T_1=-1400$, $T_3-T_2=-1120$. The sequence has no common difference, it is not arithmetic.

Q3c $T_{10}=5939.5$, week 10.

Q3d $t_n=ar^{n-1}+d\left(\frac{1-r^{n-1}}{1-r}\right)$

$t_n=12000\times 0.8^{n-1}+1000\left(\frac{1-0.8^{n-1}}{0.2}\right)$, which can be simplified to $7000\times 0.8^{n-1}+5000$, and it is always > 5000.
The performance season will continue.

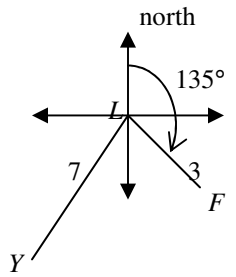
Module 2: Geometry and trigonometry

Q1a Gradient $= \dfrac{50}{400} = 0.125$

Q1b Angle of elevation $= \tan^{-1}(0.125) \approx 7.1°$

Q1c $FC = \sqrt{400^2 + 50^2} \approx 403.1\,\text{m}$

Q2a



Q2b $\angle FLY = 210 - 135 = 75°$

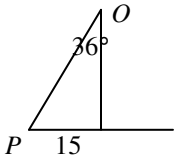
Q2c $FY = \sqrt{7^2 + 3^2 - 2\times 7\times 3\cos 75°} \approx 6.87\,\text{km}$

Q2d 315°T

Q3a $\angle POQ = \dfrac{360}{5} = 72°$

Q3b

$OP = \dfrac{15}{\sin 36°} \approx 25.52\,\text{cm}$



Q3c For each triangle, $s = \dfrac{30 + 25.52 + 25.52}{2} = 40.52$.

Area of pentagon

$= 5\times\sqrt{40.52(40.52-30)(40.52-25.52)(40.52-25.52)} \approx 1548\,\text{cm}^2$

Q4a $V = \pi r^2 h + \dfrac{1}{2}\times\dfrac{4}{3}\pi r^3 = \pi 12^2\times 30 + \dfrac{1}{2}\times\dfrac{4}{3}\pi 12^3 \approx 17191\,\text{cm}^3$

Q4b Fraction of oil remaining $= \left(\dfrac{20}{50}\right)^3 = \dfrac{64}{1000}$.

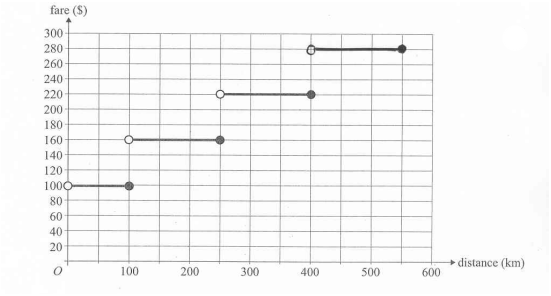
Fraction of oil removed $= 1 - \dfrac{64}{1000} = \dfrac{936}{1000}$.

% removed $= \dfrac{936}{1000}\times 100\% = 93.6\%$

Module 3: Graphs and relations

Q1a 250 km

Q1b



Q1c $fare = 40 + 0.5\times 300 = 190$, saving $220 - 190 = \$30$

Q1d $40 + 0.5\times distance = 220$, $distance = 360$ km

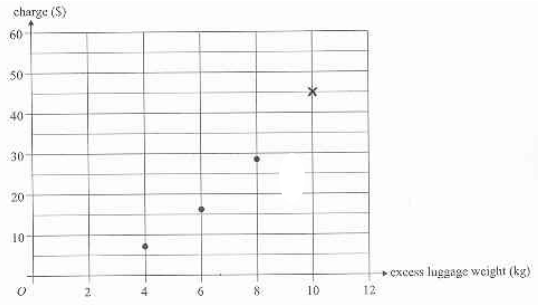
Q1e Pick two convenient points: $(100,100)$ and $(400,220)$.

$b = gradient = \dfrac{220-100}{400-100} = 0.4$

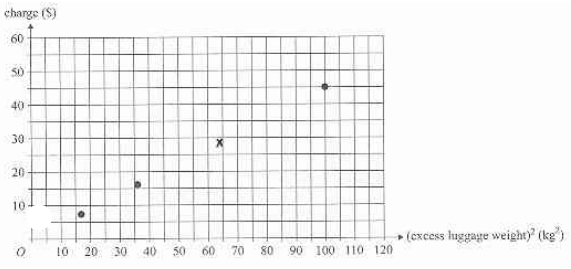
$fare = a + 0.4\times maximum\ distance$

$100 = a + 0.4\times 100$, $a = 60$

Q2a



Q2b 64



Q2c $45.00 = k\times 100$, $k = 0.45$

Q2d $charge = 0.45\times 12^2 = \$64.80$

Q3 $fare = 20 + 0.47\times 450 = 231.50$.

$charge = 299 - 231.50 = 67.50$.

$\therefore 67.50 = m\times 15^2$, $m = 0.3$

Q4 $profit = 1300x + 2100y$

At $(2,7)$, $maximum\ total\ profit = 1300\times 2 + 2100\times 7 = \$17300$.

Module 4: Business-related mathematics

Q1a $500 - 120 = \$380$

Q1b $\frac{120}{500} \times 100\% = 24\%$

Q2a $250 \times \frac{1.5}{100} = \$3.75$

Q2b $\frac{6 \times 12}{250} \times 100\% = 28.8\%$

Q3a $\frac{4.4\%}{4} = 1.1\%$

Q3b $A = PR^n = 3400 \times 1.011^{12} = \$3876.97$

Q3c $A = 3400 \times 1.011^{24} = 4420.86$
Interest $= 4420.86 - 3400 = \$1020.86$

Q4a Depreciation $= \frac{22000 \times 12 \times 4}{100} = 10560$
Depreciated value $= 22000 - 10560 = \$11440$

Q4b Depreciated value $= 22000 \times 0.84^4 = \$10953.17$

Q4c Reducing balance method will give a lower depreciated value than that of the flat rate method. ∴ reducing balance method will give a greater depreciation amount of $22000 - 10953.17 = \$11046.83$

Q5a TVM Solver: \$151133.38

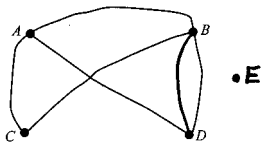
Q5b Total of monthly payments $= 1500 \times 60 = 90000$
Reduction in loan amount $= 200000 - 151133.38 = 48866.62$
Interest $= 90000 - 48866.62 = \$41133.38$

Q5c TVM Solver: \$1825.03

Module 5: Networks and decision mathematics

Q1a *E* is isolated from the other suburbs by water.

Q1bi and ii



Q2a *F*, *K*

Q2b *KJH* *KMJH* *KFJH*

Q3a 4

Q3bi *P*, because there are only 2 odd-degree vertices, *N* and *P*.

Q3bii 5

Q3ci *SRQPONUT*

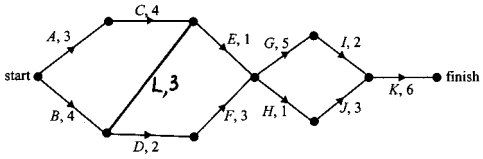
Q3cii *SRQPONTU* *SRQPUTNO* *SRUTNOPQ*

Q4a 7

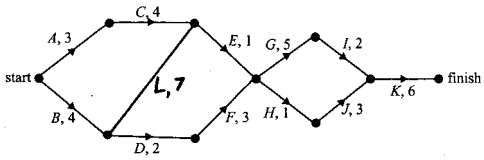
Q4b *BDFGIK*

Q4c *A* 1 week, *C* 1 week, *E* 1 week, *H* 3 weeks, *J* 3 weeks.
Longest float time is 3 weeks.

Q4d



Q4e



Now the critical path is *BLEGIK*.
Total overall time to complete $= 4 + 7 + 1 + 5 + 2 + 6 = 25$ weeks

Module 6: Matrices

Q1a 2×3

Q1bi

$$W = PQ = \begin{bmatrix} 6.80 & 5.30 & 6.20 \\ 7.30 & 4.90 & 6.15 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \\ 3 \end{bmatrix} = \begin{bmatrix} 131.30 \\ 130.75 \end{bmatrix}$$

Q1bii Safeworth

Q2a

35 2

Q2b

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 283 & 28 & 5 \\ 35 & 4 & 2 \\ 84 & 3 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix} = \begin{bmatrix} 27 \\ 32 \\ 35 \end{bmatrix}$$

Cost of a teacher ticket = \$32

Q3a $160 + 120 + 220 = 500$ students

Q3bi

25

Q3bii 5%

Q3biii $0.85\times 160 + 0.40\times 120 + 0.10\times 220 = 206$ students

Q3c

$$S_1 = \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix} \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix} = \begin{bmatrix} 310 \\ 130 \\ 60 \end{bmatrix}$$

Q3d

$$S_3 = \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix}^3 \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix} = \begin{bmatrix} 361 \\ 91.1 \\ 47.9 \end{bmatrix}$$

361 'yes' students

Q4a

$$L_2 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 68 \end{bmatrix}$$

68 students attending the extra singing rehearsals in week 3.

Q4b

$$L_4 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 100 \\ 68 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 97 \\ 59 \end{bmatrix}$$

Number of students not expected to return $= (100 + 68) - (97 + 59) = 12$.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors