2009 VCAA Mathematical Methods Exam 2 Solutions

SECTION 1: Multiple-choice questions

1	2	3	4	5	6	7	8	9	10	11
D	C	B	D	D	D	B	C	E	B	D

12	13	14	15	16	17	18	19	20	21	22
D	C	D	E	C	A	B	E	E	B	D

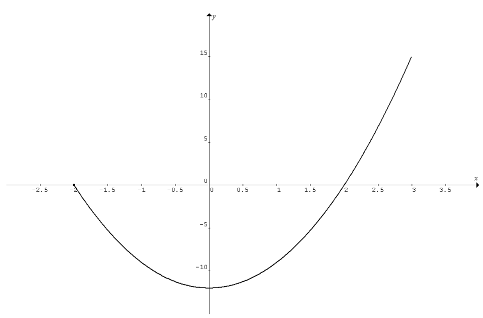
Q1 $(2,3)\rightarrow(2,6)\rightarrow(2,-6)$ D

Q2 C

Q3 $2x+1>0$, $x>-\frac{1}{2}$ B

Q4
$f(g(x))=\sin(2g(x))+1=\sin(2\log_e x)+1=\sin(\log_e x^2)+1$ D

Q5 D



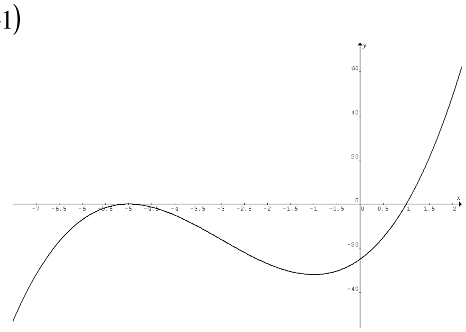
Q6
$\Pr(X>17)=\Pr\left(Z>\frac{X-\mu}{\sigma}\right)=\Pr(Z>1.5)=\Pr(Z<-1.5)$ D

Q7 $y=e^{2x}\cos 3x$,

$\frac{dy}{dx}=e^{2x}(-3\sin 3x)+(2e^{2x})\cos 3x=e^{2x}(-3\sin 3x+2\cos 3x)$

When $x=0$, $\frac{dy}{dx}=2$ B

Q8 $(-5,-1)$ C



Q9 $(3,8)$ is the image of $(1,5)$ after the translations.
$\therefore$ the tangent at $(3,8)$ has the same gradient as $y=3+2x$, i.e. 2.
$y-8=2(x-3)$, $\therefore y=2x+2$ E

Alternatively, make the same translations to the original tangent,
$y-3=3+2(x-2)$, $y=2x+2$

Q10 B

Q11 $\int_a^{0.5}\pi\sin 2\pi x\,dx=0.2$, $[-0.5\cos 2\pi x]_a^{0.5}=0.2$,
$-0.5\cos\pi+0.5\cos 2a\pi=0.2$, $0.5\cos 2a\pi=-0.3$,
$\cos 2a\pi=-0.6$, $2a\pi\approx 2.2143$, $a\approx 0.35$ D

Q12 $f(x)=x^3-e^x$

Average rate $=\frac{f(1)-f(0)}{1-0}=\frac{(1-e)-(-1)}{1}=2-e$ D

Q13 Binomial: $n=12$, $p=0.5$.
By calculator, $\Pr(X\le 4)\approx 0.1938$ C

Q14 $f(x)=\left|x^{\frac{3}{5}}\right|+2$, $f'(x)$ is undefined at $x=0$. D

Q15
$y=\sqrt{1-f(x)}$, $\frac{dy}{dx}=\frac{1}{2\sqrt{1-f(x)}}\times(-f'(x))=\frac{-f'(x)}{2\sqrt{1-f(x)}}$ E

Q16 Range of *f* is (e^3,∞) is the domain of f^{-1}.
Let $y=e^{2x+3}$, equation of f^{-1} is $x=e^{2y+3}$, $2y+3=\log_e x$,
$y=\frac{1}{2}\log_e x-\frac{3}{2}=\log_e\sqrt{x}-\frac{3}{2}$.
$f^{-1}(x)=\frac{1}{2}\log_e x-\frac{3}{2}=\log_e\sqrt{x}-\frac{3}{2}$ C

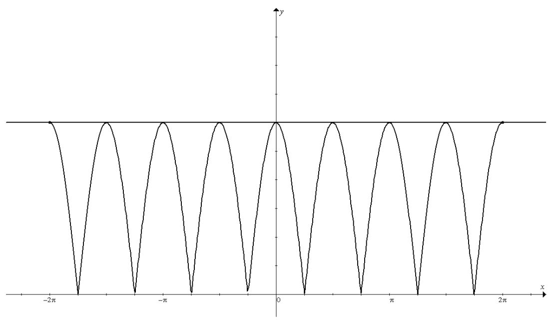
Q17 Let $X=\{1,3,5,7,9,11\}$ and $Y=\{1,4,7,10\}$
$\Pr(X\cap Y)=\Pr(\{1,7\})=\frac{2}{12}=\frac{1}{6}$
$\Pr(X)\Pr(Y)=\frac{6}{12}\times\frac{4}{12}=\frac{1}{6}$
$\therefore$ *X* and *Y* are independent. A

Q18 $\int_0^a\sec^2 2x\,dx=\frac{1}{2}$, $\left[\frac{1}{2}\tan 2x\right]_0^a=\frac{1}{2}$,

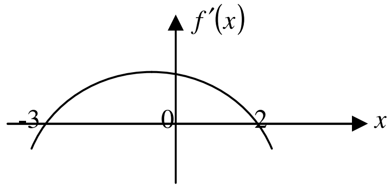
$\tan 2a=1$, $2a=\frac{\pi}{4}$, $a=\frac{\pi}{8}$ B

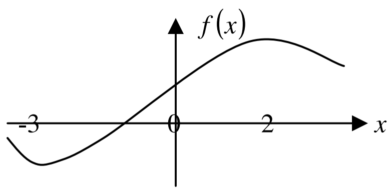
Q19 To obtain the graph of $1-f(2x)$, dilate $f(x)$ horizontally by a factor of $\frac{1}{2}$, then reflect in the x-axis, and then translate upwards by 1 unit. E

Q20 9 solutions E



Q21 $f'(x)=a(x-2)(x+3)$ is a quadratic function. For it to have a maximum value, $a<0$.





B

Q22 Inverse of $y=\log_e(x-1)$ is $y=e^x+1$

Area $=\int_0^3 (e^x+1)dx=\left[e^x+x\right]_0^3=(e^3+3)-(1)=e^3+2$ D

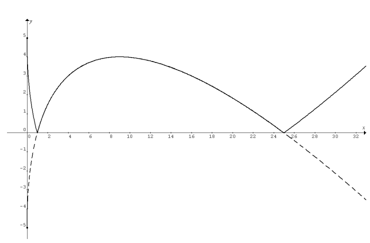
SECTION 2:

Q1a $f(x)=6\sqrt{x}-x-5$, $f'(x)=\frac{3}{\sqrt{x}}-1$.

Let $\frac{3}{\sqrt{x}}-1=0$, $\sqrt{x}=3$, $x=9$, $y=f(9)=4$.

Stationary point is $(9,4)$.

Q1b



Q1ci Area $=\int_1^{25}\left(6\sqrt{x}-x-5\right)dx=\left[4x^{\frac{3}{2}}-\frac{x^2}{2}-5x\right]_1^{25}$

$=(500-312.5-125)-(4-0.5-5)=64$ square units

Q1cii $AD=\frac{64}{24}=\frac{8}{3}$.

Q1di Gradient of chord $AB=m=\frac{0-3}{25-16}=-\frac{1}{3}$.

Q1dii $f'(x)=\frac{3}{\sqrt{x}}-1$, $f'(a)=\frac{3}{\sqrt{a}}-1=-\frac{1}{3}$, $\frac{3}{\sqrt{a}}=\frac{2}{3}$,

$\sqrt{a}=\frac{9}{2}$, $a=\frac{81}{4}$.

Q1ei $f(x)=6\sqrt{x}-x-5$, $g(x)=x^2$,

$f(g(x))=6\sqrt{g(x)}-g(x)-5=6\sqrt{x^2}-x^2-5$.

Q1eii $\frac{d}{dx}f(g(x))=\frac{df(g(x))}{dg(x)}\times\frac{dg(x)}{dx}$

$=\left(\frac{3}{\sqrt{g(x)}}-1\right)2x=\frac{6x}{\sqrt{x^2}}-2x=\frac{6x}{|x|}-2x$, $x\neq 0$.

For $x>0$, $\frac{d}{dx}f(g(x))=6-2x$.

For $x<0$, $\frac{d}{dx}f(g(x))=-6-2x$.

Alternatively,

$f(g(x))=6\sqrt{g(x)}-g(x)-5=6\sqrt{x^2}-x^2-5=6|x|-x^2-5$

$=\begin{cases}6x-x^2-5, & x>0\\ -6x-x^2-5, & x<0\end{cases}$

$\frac{d}{dx}f(g(x))=\begin{cases}6-2x, & x>0\\ -6-2x, & x<0\end{cases}$

Q2ai $y=\frac{1}{200}\left(x^3+bx^2+c\right)$, $\frac{dy}{dx}=\frac{1}{200}\left(3x^2+2bx\right)$.

Turning point at $x=4$, $\therefore \frac{1}{200}(48+8b)=0$ ……(1)

Passes through $(2,0)$, $\therefore \frac{1}{200}(8+4b+c)=0$ ……(2)

Q2aii From (1), $48+8b=0$, $\therefore b=-6$.
From (2), $8+4b+c=0$, $\therefore c=16$.

Q2bi $y=\frac{1}{200}\left(x^3-6x^2+16\right)=\frac{1}{200}(x-2)\left(x^2-4x-8\right)=0$,

$\therefore x^2-4x-8=0$, $x=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm 4\sqrt{3}}{2}=2\pm 2\sqrt{3}$.

M is $\left(2+2\sqrt{3},0\right)$ and P is $\left(2-2\sqrt{3},0\right)$.

Q2bii $y=\frac{1}{200}\left(x^3-6x^2+16\right)$. Use calc. to find the local minimum $(4,-0.08)$. Maximum depth $=0.08$ km $=80$ m.

Q2biii Use calc. to find the local maximum $(0,0.08)$. Maximum height $=0.08$ km $=80$ m.

Q2biv Length of tunnel $=NP=2-2+2\sqrt{3}=2\sqrt{3}$ km.

Q2c $PQ=6.2$ km. At P, $d=0$, $v=120$ km/h.

$v=k\log_e\frac{d+1}{7}$, $120=k\log_e\frac{1}{7}$, $120=-k\log_e 7$, $k=-\frac{120}{\log_e 7}$.

Q2d When $v=0$, $0=k\log_e\frac{d+1}{7}$, $\therefore \log_e\frac{d+1}{7}=0$,

$\frac{d+1}{7}=1$, $d=6$ km.

Distance $=6.2-6=0.2$ km $=200$ m.

Q3a $\Pr(X<68.5)=0.9332$ by calc.

Q3b $\Pr(65.6<X<68.4)=0.8385$ by calc.

Q3ci $\Pr(65.6<X<68.4 \mid X<68.5)$

$=\frac{\Pr(65.6<X<68.4)}{\Pr(X<68.5)}=\frac{0.8385}{0.9332}=0.8985$

Q3cii Binomial: $n=4$

Those in the tin outside $(65.6,68.4)$, $p=1-0.8985=0.1015$,

$q=0.8985$.

$\Pr(at\_least\_one)=1-\Pr(none)=1-0.8985^4=0.3483$.

Q3d $\Pr(65.6<X<68.4)=0.99$,

$\Pr\left(\frac{65.6-67}{\sigma}<Z<\frac{68.4-67}{\sigma}\right)=0.99$

$\Pr\left(\frac{-1.4}{\sigma}<Z<\frac{1.4}{\sigma}\right)=0.99$, $\therefore \Pr\left(Z<\frac{1.4}{\sigma}\right)=0.995$

By calc. $\frac{1.4}{\sigma}\approx 2.5758$, $\therefore \sigma\approx 0.54$ mm.

Q3e $\Pr(buy\_buy\_buy)=0.8\times 0.8\times 0.8=0.512$

Q3f

$\Pr(buy\_buy\_buy')+\Pr(buy\_buy'\_buy)+\Pr(buy'\_buy\_buy)$

$=0.8\times 0.8\times 0.2+0.8\times 0.2\times 0.15+0.2\times 0.15\times 0.8=0.176$

Q4ai $\frac{h}{r}=\frac{8}{4}$, $\therefore h=2r$.

Q4aii $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h=\frac{1}{12}\pi h^3$.

Q4b $\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}$, $\frac{dh}{dt}=\frac{\frac{dV}{dt}}{\frac{dV}{dh}}=\frac{\frac{9\pi}{4}}{\frac{\pi h^2}{4}}=\frac{9}{h^2}$ metres per hour.

Q4ci When $h=2$, $\frac{dh}{dt}=\frac{9}{2^2}=\frac{9}{4}$ metres per hour.

Q4cii When $\frac{dh}{dt}=\frac{1}{2}\times\frac{9}{4}=\frac{9}{8}$, $\frac{9}{h^2}=\frac{9}{8}$, $h=\sqrt{8}=2\sqrt{2}$ m.

Q4di $\frac{dh}{dt}=\frac{9}{h^2}$, $\frac{dt}{dh}=\frac{1}{\frac{dh}{dt}}=\frac{1}{\frac{9}{h^2}}$, $\therefore \frac{dt}{dh}=\frac{h^2}{9}$.

Q4dii $t=\int\frac{h^2}{9}dh=\frac{h^3}{27}+c$.

$h=0$ when $t=0$, $\therefore t=\frac{h^3}{27}$, $\therefore h=3t^{\frac{1}{3}}$.

Q4ei At time *t*, distance above ground level $=14-t$ metres.

Q4eii When the statue first touches the acid, $3t^{\frac{1}{3}}=14-t$.

By calc. $t=8$, i.e. 5.00 pm.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors