2010 VCAA Further Mathematics Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
D	D	B	A	C	C	B	B	B	A	E	D	B

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
B	D	E	B	D	E	B	A	D

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
C	B	C	D	A	D	C	C	B

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
B	C	E	E	C	D	C	A	B

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
E	D	E	B	C	A	D	B	D

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
A	C	B	E	C	C	B	D	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
C	A	E	B	D	C	D	E	B

SECTION A Core: Data analysis

Q1 21% between 179 and 180, 16% between 180 and 181.
$(21\%+16\%)\times 300 = 111$ D

Q2 $IQR \approx 181.5 - 178.8 = 2.7$ D

Q3 The 68% interval centred at the mean is $(178, 182.5)$ approximately. $\therefore\ 2\sigma \approx 182.5 - 178 = 4.5$, $\sigma \approx 2.3$ B

Q4 $88 - 74 = 14$ A

Q5 $z = \dfrac{23 - 24.2}{4.2} \approx -0.29$ C

Q6 20.0 cm is a standard deviation shorter than 24.2 cm.
$\therefore\ \dfrac{68\%}{2} = 34\%$ C

Q7 By inspection, $r \approx 0.7$. B

Q8 Pick two convenient points on the line, $(34, 185)$, $(20, 167)$.
Gradient $= \dfrac{185-167}{34-20} \approx 1.286$. Equation: $\dfrac{y-167}{x-20} \approx 1.286$,
$y \approx 1.286x + 141$, i.e. $height \approx 141 + 1.3 \times foot\ length$. B

Q9 B

Q10 $y = a + bx$ where $b = r\dfrac{s_y}{s_x} = -0.673464$ and
$a = \bar{y} - b\bar{x} = 9.16936$.
$\therefore\ y \approx 9.2 - 0.7x$ A

Q11

x	1	2	3	4	5	6	7	8
$1/x$	1	0.5	0.33	0.25	0.2	0.17	0.14	0.13
y	245	130	84	58	52	36	33	30

By CAS/calculator, slope of y versus $\dfrac{1}{x}$ is 249 approx. E

Q12 D

Q13 The sum of the 12 indices = 12. $\therefore$ the missing index is 1.41
$Deseasonalised\ figure = \dfrac{actual}{index} = \dfrac{104500}{1.41} = \$74113.48$ B

SECTION B

Module 1: Number patterns and applications

Q1 Common difference of 3 B

Q2 Common ratio of 2, $t_4 = 0.5 \times 2 = 1$ D

Q3 $t_8 = 20000 \times 0.95^7 \approx \$14000$ E

Q4 $S_{10} = \dfrac{a(1-r^n)}{1-r} = \dfrac{4(1-(-2)^{10})}{1-(-2)} = -1364$ B

Q5 $a = 100$, $d = 50$, $t_n = 800$
$t_n = a + (n-1)d$, $800 = 100 + (n-1)50$, $n = 15$ D

Q6 $t_n = t_{n-1} + t_{n-2}$, $90 = k + 10$, $k = 80$ E

Q7 B

Q8 The first term is a negative value. A

Q9 0.32, 0.24, 0.18, ... are the terms of a geometric sequence,
$a = 0.32$, $r = \dfrac{0.24}{0.32} = 0.75$.
$S_\infty = \dfrac{a}{1-r} = \dfrac{0.32}{0.25} = 1.28$.
$\therefore\ longest\ jump = 5.80 + 1.28 = 7.08$ m D

Module 2: Geometry and trigonometry

Q1 $x = 180 - 89 = 91$ C

Q2 $C = 2\pi r$, $r = \frac{C}{2\pi} = \frac{10}{2\pi} \approx 1.59$ cm B

Q3 *Area of cardboard remaining*
= area of triangle − area of circle
$= \frac{1}{2} \times 6 \times 6 \times \sin 60° - \pi \times 1^2 \approx 12.447 \text{ cm}^2$ C

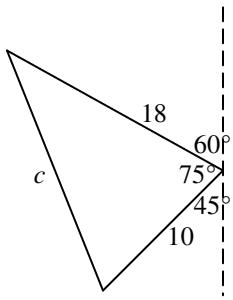
Q4 *Surface area of cube* $A = 256 \times 1.5^2 = 576 \text{ cm}^2$ D

Q5 $\overline{YC} = \frac{11.0 - 7.4}{2} = 1.8$, $\angle CBY = \tan^{-1}\left(\frac{1.8}{5.5}\right) \approx 18°$ A

Q6 $\overline{DT} = \sqrt{5.5^2 + 4.5^2} \approx 7.106$,
$\overline{TM} = \sqrt{5.5^2 + (5.5 - 4.5)^2} \approx 5.59$.
Total distance $= 7.106 + 5.59 \approx 12.7$ m D

Q7 $\frac{6}{\overline{BC}} = \sin 21.8°$, $\overline{BC} \approx 16.1565$.
$\angle CBD = \tan^{-1}\left(\frac{13}{16.1565}\right) \approx 38.8°$ C

Q8



$c = \sqrt{18^2 + 10^2 - 2(18)(10)\cos 75°} \approx 18.189$ km
Total distance $\approx 18.189 + 18 + 10 \approx 46$ km C

Q9 *Volume (cone)* $= \frac{1}{3}\pi(30^2)(24)$
Volume of water in the cylinder = volume of water in the cone
$\therefore \pi(20)^2 h = \frac{1}{3}\pi(30^2)(24)$
$\therefore h = \frac{900 \times 24}{400 \times 3} = 18$ cm B

Module 3: Graphs and relations

Q1 *Difference* $= 300 - 200 = 100$ B

Q2 *Rate = slope* $= \frac{450 - 300}{2} = 75$ C

Q3 $\frac{y-1}{x-10} = \frac{1-(-2)}{10-4}$, $\therefore x - 2y = 8$ E

Q4 E

Q5 80 mugs @ \$7.50, *revenue* $R = 80 \times 7.50 = \$600$,
cost $C = 150 + 6 \times 80 = \$630$. Loss of \$30. C

Q6 D

Q7 *John's time = Bruce's time*
$45 \times 6 + 15n = 45 \times 7 + 15 \times 8$, $n = 11$ C

Q8 A

Q9 PROBLEM with $0 \le \textit{length} < b$, the roof section with positive slope is undefined at $\textit{length} = b$. It is assumed that it is defined in equation (1) below.
Let h be the height of the roof in metres.
At $\textit{length} = b$, $h = \frac{1}{2} \times b + 3$ .....(1)
and $h - 1 = -\frac{1}{6} \times b + 5$ .....(2)
Solve the simultaneous equations for b, $b = 4.5$ m. B

Module 4: Business-related mathematics

Q1 $140 + 25.50 \times 24 = \$752$ E

Q2 $SI = \frac{6000 \times 3.5 \times 4}{100} = \$840$ D

Q3 *Amount* $= 1880 + 10\% \times 1880 = \$2068$ E

Q4 *Saturday's price* $= 160 \times 1.25 \times 0.90 \times 0.75 \times 1.20 = \$162$ B

Q5 *Value* $= 30000 \times 0.80^3 = \$15360$ C

Q6 *Minimum balance* $= 1450$
$SI = \frac{1450 \times 3.6 \times \frac{1}{12}}{100} = \$4.35$ A

Q7 By CAS/calculator. D

Q8 *Depreciation* $= 40000 - 22000 = 18000$
$\frac{40000 \times r \times 3}{100} = 18000$, $r = 15$ B

Q9 *First quarter interest* $= \dfrac{4000\times 5\times \frac{1}{4}}{100} = 50$

Value at the beginning of the second quarter
$= 4000 + 50 + 800 = 4850$

Second quarter interest $= \dfrac{4850\times 5\times \frac{1}{4}}{100} = 60.625$

Value at the beginning of the third quarter
$= 4850 + 60.625 + 800 = \$5710.625$ D

Module 5: Networks and decision mathematics

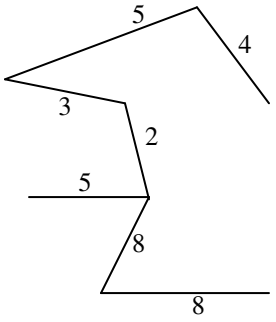
Q1 A

Q2 C

Q3 B

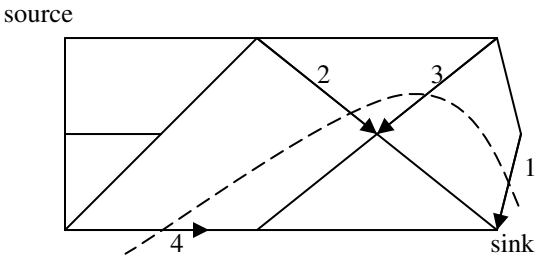
Q4 E

Q5 $4+5+3+2+5+8+8=35$ C



Q6 $11+5+7=23$ C

Q7 $4+2+3+1=10$ B



Q8 *CEGIL*: $4+8+0+9+9+5=35$ D

Q9 $9+5+6+8=28$ B

	U	*V*	*W*	*X*
Aiden	3	2	9	9
Bing	5	6	12	12
Callum	9	6	12	14
Dee	8	3	8	12

Module 6: Matrices

Q1 C

Q2 A

Q3

$1x + 3y = 9$

$2x + 5y = 16$

$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}9\\ 16\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}^{-1}\begin{bmatrix}9\\ 16\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-5 & 3\\ 2 & -1\end{bmatrix}\begin{bmatrix}9\\ 16\end{bmatrix}$ E

Q4 B

Q5 D

Q6

$UT = \begin{bmatrix}5 & 3 & 2\\ 4 & 4 & 3\\ 6 & 1 & 2\end{bmatrix}\begin{bmatrix}12 & 15 & 14\\ 8 & 7 & 8\\ 20 & 19 & 17\end{bmatrix} = \begin{bmatrix}124 & 134 & 128\\ 140 & 145 & 139\\ 120 & 135 & 126\end{bmatrix}$ C

Q7 Choices A and B are true.

Also $\begin{bmatrix}0.4 & 0.3\\ 0.6 & 0.7\end{bmatrix}^n \to \begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ \frac{2}{3} & \frac{2}{3}\end{bmatrix}$ as $n \to \infty$, i.e. in the long term.

.: choices C and E are true. D

Q8 E

Q9 Suppose Robbie randomly guessed alternative B as the answer to the first question, the state matrix is $\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

Second question state matrix is $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\\0&1&0&0\end{bmatrix}\begin{bmatrix}0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

Third question state matrix is $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\\0&1&0&0\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

Fourth question state matrix is $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\\0&1&0&0\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$ B

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

