2010 VCAA Further Mathematics Exam 2 Solutions
© Copyright 2010 itute.com Free download from www.itute.com

Core

Q1a 11 out of 22, $\frac{11}{22}=\frac{1}{2}$ or 0.5 or 50%

Q1b $Median=\frac{32+24}{2}=28$, $range=56-0=56$, $IQR=38-21=17$

Q1c 1|2 4 ***6***

Q1d If we take the 'centre' to mean the middle value of a set of data arranged in order, then the median is the perfect measure of the centre of *any* distribution because the median divides the data set into 2 halves, 50% above and below the value. The mean indicates the average of the values in the data set. If the distribution of the data set is close to symmetric, like in this case, then the mean is close to and a good indicator of the centre.

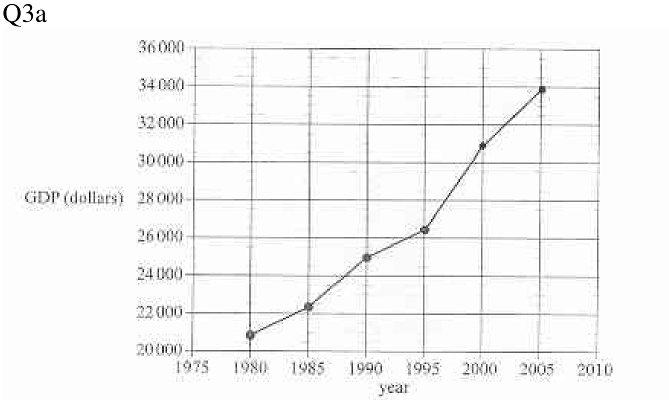
Q2a Male income

Q2b $Increase\ in\ female\ income=0.35\times1000=\$350$

Q2ci *Average annual female income* $=13000+0.35\times15000=\$18250$

Q2cii The data set is for the 16 countries where the male income is $40000 or over (except 1), .: it is not reliable to use the regression line to predict the average annual female income for a country with a much lower average annual male income of $15000.

Q3a



Q3b Increasing trend

Q3c Use CAS/calculator: $GDP=20000+524\times time$

Q3d For year 2007, $time=27$,
predicted $GDP=20000+524\times27=34148$.
$Error=34900-34148=752$ below the actual *GDP*.

Module 1: Number patterns

Q1ai $7.90-6.20=1.70$, $6.20-4.50=1.70$, .: $d=\$1.70$

Q1aii $A_5=4.50+(5-1)1.70=\$11.30$

Q1aiii $A_n=4.50+(n-1)1.70=16.40$, .: $n=8$

Q1aiv $S_{15}=\frac{15}{2}(2\times4.50+(15-1)1.70)=\$246$

Q1av Compare $A_{n+1}=mA_n+k$ with $A_{n+1}=A_n+d$ for an arithmetic sequence, $m=1$ and $k=d=1.70$

Q1bi The charge is $5.00 for travelling along one section of road in a single trip on the tollway.

Q1bii $B_{n+1}=0.9B_n+3$, $B_1=5$
$B_2=0.9B_1+3=0.9\times5+3=7.50$
$B_3=0.9B_2+3=0.9\times7.50+3=9.75$ dollars

Q1biii Use CAS to list the terms. The terms approach 30 as the limit. .: *maximum charge* $=\$30$

Q1c Use CAS to list the terms for both difference equations:

n	A_n	B_n
10	19.8	20.31
11	21.5	21.28

For 10 or less sections, it is cheaper to use pass A.
For 11 or more sections, it is cheaper to use pass B.

Q2a $S_n=\frac{a(r^n-1)}{r-1}$, $r=1.05$, $n=6$
.: $100=\frac{a(1.05^6-1)}{0.05}$, .: the first section $a\approx14.7$ km

Q2b $L_{n+1}=1.05L_n$ where $L_1=14.7$ and $1\le n\le5$

Module 2: Geometry and trigonometry

Q1a $x°=180°-60°=120°$

Q1b Bearing of the entry gate from the canoeing activity: $360°-120°=240°$

Q1c $40\cos60°=20$ m north of the entry gate

Q1di $Area=\frac{1}{2}\times40\times90\times\sin120°\approx1558.8\text{ m}^2$

Q1dii $\overline{GW}=\sqrt{40^2+90^2-2(40)(90)\cos 120°}\approx 115.3\text{ m}$

Q1ei $\angle CKW = 180° - 2\times 10° = 160°$

Q1eii $\dfrac{\overline{CK}}{\sin 10°}=\dfrac{90}{\sin 160°}$, $\overline{CK}\approx 45.7\text{ m}$

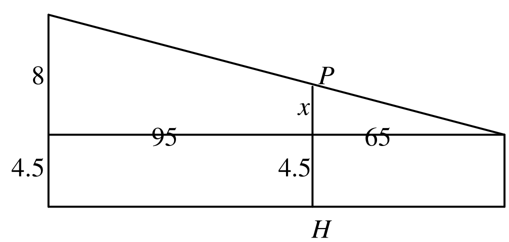
Q2a *Horizontal distance* $\overline{DF}=4.5\times 2 = 9.0\text{ m}$

Q2b $\theta=\tan^{-1}\left(\dfrac{10}{9}\right)\approx 48° > 45°$, unsafe

Q2c Let x m be the horizontal distance from E to F.
$\dfrac{4}{x}=0.8$, $x=5$.: on the map $\overline{EF}=2.5\text{ cm}$

Q3 $V=\dfrac{1}{3}x^2h$, $1.8=\dfrac{1}{3}x^2\times 2.5$, $x\approx 1.47$

Q4



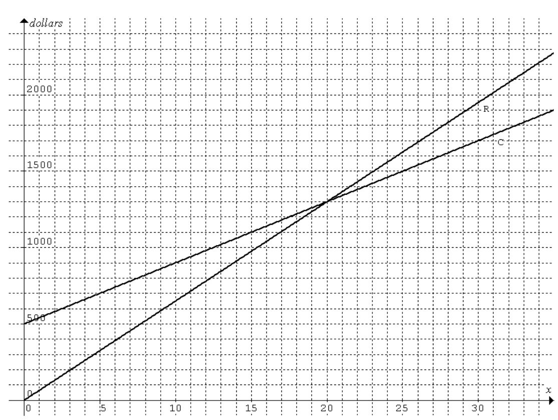
$\dfrac{x}{65}=\dfrac{8}{95+65}$, $x=3.25$, .: $\overline{PH}=3.25+4.5=7.75\text{ m}$

Module 3: Graphs and relations

Q1a $R=65x$

Q1b $C=500+40x = 500+40\times 30=1700$ dollars

Q1c



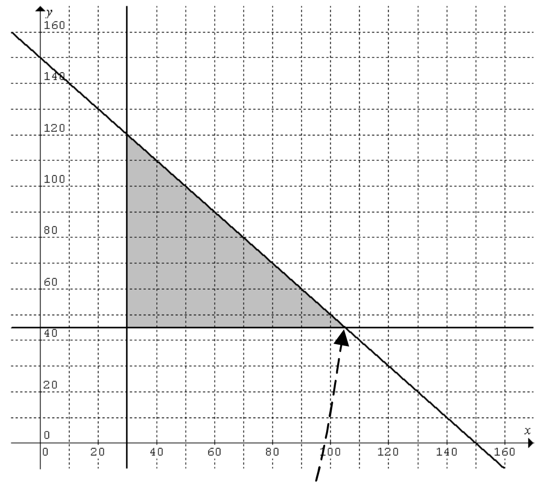
Q1d Read from graph, 20.

Q2 $R=65\times 35+50m=4275$, $m=40$

Q3a The total number of pillows that can be sold each week does not exceed 150.

Q3b Read from graph, $k=45$.

Q3ci and ii

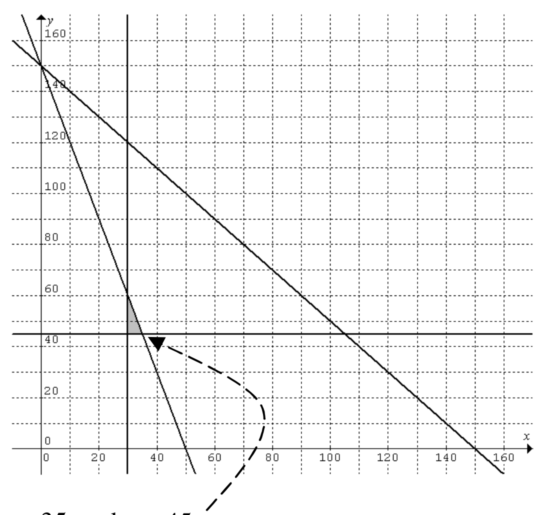


Q3d $R=65x+50y$
When $x=105$ and $y=45$,
Maximum $R=65\times 105+50\times 45=9075$ dollars

Q3e $2x+x+y\le 150$, .: $3x+y\le 150$

Q3f $R=65x+50y+55(2x)$, .: $R=175x+50y$

Q3g



When $x=35$ and $y=45$,
Maximum $R=175\times 35+50\times 45=8375$ dollars

Module 4: Business-related mathematics

Q1ai *Total amount* $= 55 \times 48 = \$2640$

Q1aii *Total interest* $= 2640 - 2000 = \$640$

Q1aiii $I = \dfrac{\Pr T}{100}$, $640 = \dfrac{2000 \times r \times 4}{100}$, $r = 8$, i.e. 8%

Q1b *Cash price* $= 2000 \times \left(1 + \dfrac{2.5}{100}\right)\left(1 + \dfrac{2}{100}\right) = \$2091$

Q2a *Monthly payment* $= \dfrac{I}{12} = \dfrac{360000 \times 5.2}{100 \times 12} = \$1560$

Q2b \$360000

Q3a The same calculation is used to find the interest in the first year for both investments. Simple Saver pays higher annual interest amount than Growth Plus does. .: the Simple Saver's rate is higher than that of Growth Plus.

Q3b *Amount of annual interest* $= \dfrac{21800 - 8000}{15} = \$920$

Q3ci $A = P\left(1 + \dfrac{r}{100}\right)^n$, $24000 = 8000\left(1 + \dfrac{r}{100}\right)^{15}$

Q3cii $r \approx 7.6$, i.e. 7.6%

Q4ai TVM Solver: *Monthly repayment* $\approx \$1827.32$

Q4aii *Total interest amount*
$= 1827.32 \times 240 - 250000 \approx \$188557$

Q4b TVM Solver: *Outstanding principal* $\approx \$213118$

Q4c TVM Solver: *Number of months to repay the \$100000 loan* ≈ 104.
.: *number of months to repay the \$250000 loan*
$= 12 \times 9 + 104 = 212$ months

Module 5: Networks and decision mathematics

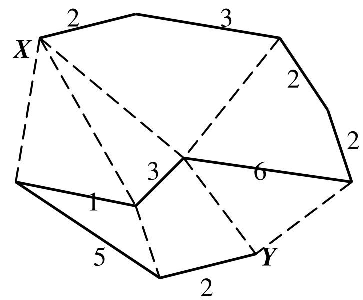
Q1a No direct communication

Q1b $f = 1$, $g = 0$

Q2a From X to Y, $4 + 3 + 4 = 11$ minutes

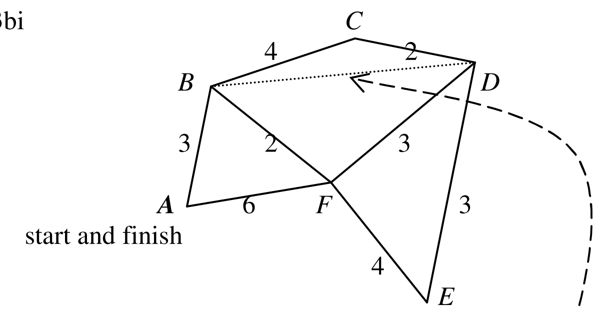
Q2bi Hamiltonian path

Q2bii



Q3a D

Q3bi

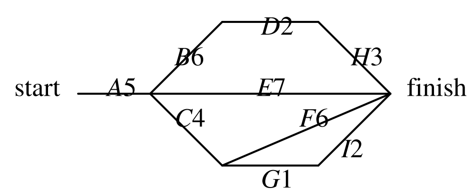


A path: *AFDEFBCDFBA*, .: *B*, *D* and *F*

Q3bii $6 + 3 + 3 + 4 + 2 + 4 + 2 + 3 + 2 + 3 = 32$ km

Q3c Euler's circuit: all vertices are even. .: bush path connects *B* and *D*.

Q4 Use the immediate predecessors to construct the following diagram:



Q4a 2

Q4b *Earliest start time for F* $= 5 + 4 = 9$ minutes

Q4c *A* and *C*

Q4d *Float time for G* $= 13 - 9 = 4$ minutes

Q4e *Shortest completion time* $= 5 + 6 + 2 + 3 = 16$ minutes

Q4f The critical path is *A-B-D-H*.

Module 6: Matrices

Q1a $N = \begin{bmatrix} 4 & 8 & 2 \end{bmatrix}$

Q1b $P = N \times G = \begin{bmatrix} 4 & 8 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 26 \end{bmatrix}$

Q1c The total number of points scored by Oscar in the game was 26.

Q2a The percentage of players in ***H*** training is changed to ***M*** training from week to week.

Q2b

$$\begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.5 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 90 \\ 150 \\ 60 \end{bmatrix} = \begin{bmatrix} 66 \\ 138 \\ 96 \end{bmatrix}$$

66 players

Q2c

$$\begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.5 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 66 \\ 138 \\ 96 \end{bmatrix} = \begin{bmatrix} ..... \\ 144 \\ ..... \end{bmatrix}$$

$150 - 144 = 6$ fewer players

Q2d After repeated applications of transition matrix *T*,

S_6, S_7, S_8, …… $= \begin{bmatrix} 50 \\ 150 \\ 100 \end{bmatrix}$, i.e. the number of players in each type of training becomes stable after five weeks.

.: the number of players in each type of training becomes stable after seven weeks.

Q3a

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} p \\ r \\ a \end{bmatrix} = \begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix}$$

Q3b

$$\begin{bmatrix} 7 & -1 & -4 \\ -1 & 0 & 1 \\ x & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 0 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

.: $x = -5$

Q3c

$$\begin{bmatrix} p \\ r \\ a \end{bmatrix} = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 0 & 1 \\ -5 & 1 & 3 \end{bmatrix} \begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix} = \begin{bmatrix} ... \\ 10 \\ ... \end{bmatrix}$$

$r = 10$

Q4ai

$$A_2 = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 2100 \\ 1100 \end{bmatrix}$$

Q4aii $2100 + 1100 = 3200$ caps

Q4b

$$A_{10} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^9 \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} \approx \begin{bmatrix} 2613 \\ 1613 \end{bmatrix}$$

Q4c

$$A_{80} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^{79} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 3000 \\ 2000 \end{bmatrix}$$

$$A_{81} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^{80} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} = \begin{bmatrix} 3000 \\ 2000 \end{bmatrix}$$

The number of people attending the Dinosaurs' game is expected to increase gradually from 2000 and become constant at 3000.

Q4d

$$A_2 = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 1860 \\ 1660 \end{bmatrix}$$

$$A_{10} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^9 \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 1142 \\ 942 \end{bmatrix}$$

$$A_{80} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^{79} \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 600 \\ 400 \end{bmatrix}$$

$$A_{81} = \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix}^{80} \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2000 \\ 1800 \end{bmatrix} = \begin{bmatrix} 600 \\ 400 \end{bmatrix}$$

The number of people attending the Dinosaurs' game is expected to decrease gradually from 2000 and become constant at 600.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

© Copyright 2010 itute.com