



2011 VCAA Further Mathematics Exam 1 Solutions

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SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
C	E	C	C	E	E	A	A	B	C	D	E	D

SECTION B

Module 1: Number patterns

1	2	3	4	5	6	7	8	9
C	A	D	C	D	B	E	A	D

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
D	A	C	C	D	D	B	D	D

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
B	C	A	A	B	D	E	C	E

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
E	D	E	E	E	D	B	E	B

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
B	C	B	A	E	A	D	D	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
E	A	C	B	C	E	A	D	C

SECTION A Core: Data analysis

Q1 The 'tail' of the distribution tends to the right. C

Q2 The first two columns on the left of the histogram add to 54. E

Q3 The first four columns on the left of the histogram add to 80, which is half of 160. The median percentage is 20%. C

Q4 C

Q5 E

Q6 By CAS/graphics calc., $\bar{x} = 125.9$, $S_x = 6.0$ E

Q7 The second column of data is y , the dependent variable. The third column is x , the independent variable. By CAS/graphics calc. $y = 70.3 + 0.790x$. A

Q8 $r^2 = 0.258$, $\therefore 25.8\%$ A

Q9 95% -interval: $\mu - 2\sigma = 4.8 - 2 \times 1.2 = 2.4$
 $\mu + 2\sigma = 4.8 + 2 \times 1.2 = 7.2$ B

Q10 $z = \frac{x - \mu}{\sigma}$, $-0.5 = \frac{x - 4.8}{1.2}$, $x = 4.2$ C

Q11 Since $r = -0.56$, there is a negative correlation between the time spent on computer games and fitness level. Choice B is incorrect because it implies fitness level is caused by the time playing computer games. D

Q12 $Seasonal_index = \frac{actual_figure}{deseasonalised_figure}$

$0.80 = \frac{actual_figure}{deseasonalised_figure}$

$\therefore Deseasonalised_figure = \frac{actual_figure}{0.80}$

$Deseasonalised_figure = 1.25 \times actual_figure$

\therefore increased by 25% E

Q13 D

2004		2005		2006		2007		2008
1012000		2016000		3900000		4830000		5140000
			2958000		4365000			
				3661500				

SECTION B

Module 1: Number patterns

Q1 Geometric sequence: $a = 1$, $r = 2$, $t_4 = 8$ C

Q2 Arithmetic sequence: $a = -3$, $d = -4$,
 $t_n = a + (n - 1)d = -3 + (n - 1)(-4) = 1 - 4n$ A

Q3 The sequence $8, -4, 2, -1, \frac{1}{2}$ is geometric with $r = -\frac{1}{2}$.

$A_{n+1} = -\frac{1}{2}A_n$ D

Q4 C

Month	1	2	3	4	5	6	7	8	9	10
Number	10	30	90	270	810	2430	7290	21870	65610	196830

Q5 $t_1 = a$, $t_2 = 7$, $t_{n+2} = t_{n+1} + t_n$, $\therefore t_3 = a + 7$,
 $t_4 = a + 7 + 7 = a + 14$
 $t_5 = a + 14 + a + 7 = 2a + 21 = 27$, $\therefore a = 3$ D

Q6 The geometric sequence $1, 2, 4, 8, \dots$ is divergent, i.e. $r > 1$, \therefore its infinite sum cannot be determined. B

Q7 E

Q8 $a = 1$, $d = 2$, $S_n = \frac{n}{2}(2a + (n - 1)d) = \frac{n}{2}(2 + (n - 1)2) = n^2$ A

Q9 $15, 17, 19, \dots$ is an arithmetic sequence with $a = 15$ and $d = 2$.

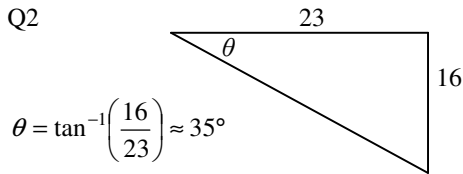
$S_n = \frac{n}{2}(2a + (n - 1)d)$, $735 = \frac{n}{2}(30 + (n - 1)2)$, $\therefore n = 21$

$t_{21} = 15 + (21 - 1) \times 2 = 55$ D

Module 2: Geometry and trigonometry

Q1 $x = 180 - 69 = 111$

Q2

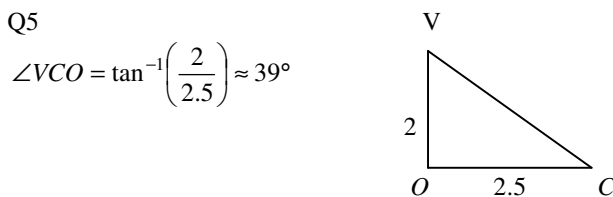


Q3 Let x be the side length of the square.

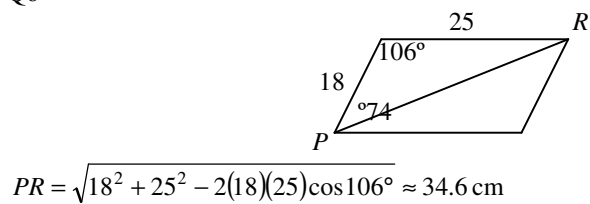
$$x^2 = \pi \times 6.5^2, \quad x = \sqrt{\pi \times 6.5^2} \approx 11.5$$

Q4 $\frac{AC}{\sin 80^\circ} = \frac{50}{\sin 40^\circ}, \quad AC = \frac{50 \sin 80^\circ}{\sin 40^\circ} \approx 77 \text{ m}$

Q5



Q6

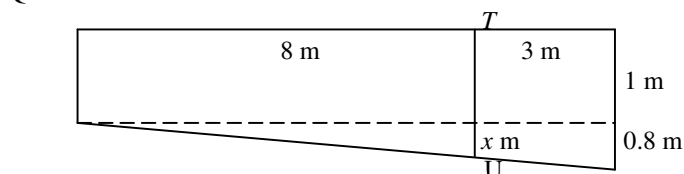


Q7 $BP = 2400 \tan 16^\circ \approx 688.189 \text{ mm}$

$QC = 2 \times 688.189 \approx 1376.38 \text{ mm}$

$QD = \sqrt{1376.38^2 + 2400^2} \approx 2767 \text{ mm}$

Q8



$\frac{x}{0.8} = \frac{8}{11}, \quad x = 0.5818 \text{ m}, \quad TU = 1 + 0.5818 \approx 1.58 \text{ m}$

Q9 Similar triangles: $\left(\frac{40}{24}\right)^2 \times 100 \approx 278$

Module 3: Graphs and relations

Q1

Q2 The charges are \$0.40, \$0.60, \$0.90 and \$1.50. No combination of 2 of them can give \$1.40.

Q3 (2,5) satisfies $3x + 4y = 26$

Q4 $F = a + bn$

Solve $18.20 = a + 20b$ and $25.70 = a + 30b$ to obtain $b = 0.75$.
D A

A

Q5 $C = 60 + 5x, \quad R = 15x$

Profit/loss = $R - C = 10x - 60$

$\therefore 10x - 60 = -20, \quad x = 4$

B

Q6

D

Q7 From the graph, the constraints defining the feasible region are $y \geq 1, \quad x \geq 2, \quad x + y \leq 9, \quad 2x + y \geq 6$ and $4x - y \leq 11$

C

Q8 $W = 13$ is the maximum at $C(3,1)$.

C

Q9 $M = 5x + 5y$ has the same gradient as AB.

E

D

Module 4: Business-related mathematics

Q1 $68 \times \left(1 + \frac{10}{100}\right) \times 3 = 224.40$

E

Q2 $A = PR^n = 22000 \times \left(1 + \frac{3.5}{100}\right)^3 = 24391.80$

D

Q3 $56000 - 0.42 \times 32000 = 42560$

E

Q4 Fees = $25 + 3.95 \times 48 = 214.60$

D

Percentage $\frac{214.60}{2500} \times 100\% \approx 8.6\%$

E

Q5 $P \times \frac{5.9}{100} = 500 \times 52, \quad P \approx 441000$

E

B

Q6 Amount owing = $3500 - 500 = 3000$

Total of repayments = $80 \times 60 = 4800$

Interest paid = $4800 - 3000 = 1800$

$I = \frac{PrT}{100}, \therefore 1800 = \frac{3000 \times r \times 5}{100}, \quad r = 12.0$

D

Q7 $R = 1 + \frac{r}{1200}, \quad A = PR^n,$

$I = A - P = PR^n - P = P(R^n - 1)$

Interest earned in the third year = total amount in the first three years - total amount in the first two years

$= 15000R^{36} - 15000R^{24}$

$= 15000 \left(1 + \frac{r}{1200}\right)^{36} - 15000 \left(1 + \frac{r}{1200}\right)^{24}$

B

B

Q8 $P = 120000, \quad r = 7.67$ compounded monthly, monthly repayment = 1430, $\therefore n = 120.64$ months, \therefore A is false, etc.

E

C

Q9 36 payments @ \$1446.75 - 18 payments @ \$2697.39 \approx \$3530

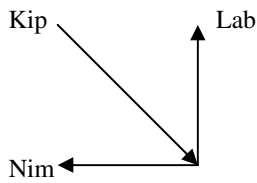
B

A

Module 5: Networks and decision mathematics

Q1

Q2



Q3 ACT is vertex *D*. Order (degree) of vertex *D* is 1.

Q4

Q5

Q6

Q7 It only takes Caleb 16 min to complete activity M, maximum time he can speak on the phone is $40 - 16 = 24$ min

Q8 The critical path is *ACFIK*.

Q9

Module 6: Matrices

Q1 \$101

Q2 $AB + 2C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

Q3 Only two with $\Delta \neq 0$.

Q4 The order of *BA* is 3×4 , the order of $2A$ is also 3×4 , $\therefore BA - 2A$ is defined.

Q5 $0.25 \times 5692 + 0.24 \times 3450 = 2251$

Q6

$$\begin{matrix} R & A \\ R & \begin{bmatrix} 0.75 & 0.24 \end{bmatrix}^{10} \begin{bmatrix} 5692 \\ 3450 \end{bmatrix} \approx \begin{bmatrix} 4479 \\ 4663 \end{bmatrix} \\ A & \begin{bmatrix} 0.25 & 0.76 \end{bmatrix} \end{matrix}$$

 $4663 - 4479 = 184$

Q7 $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, look for the one with $b > c$.

Q8

$A = A^{-1}, \begin{bmatrix} 3 & k \\ -4 & -3 \end{bmatrix} = \frac{1}{-9 - 4k} \begin{bmatrix} -3 & -k \\ 4 & 3 \end{bmatrix}$,
 $\therefore 3 = \frac{-3}{4k - 9}, 4k - 9 = -1, k = 2$

Q9

C

B

Two possibilities for *A*, the two non-zero elements are in the same row OR not in the same row.

C

Same row: $AB = \begin{bmatrix} a & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c & d \\ e & f \\ g & h \end{bmatrix} = \begin{bmatrix} not\ 0 & not\ 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, four zeros.

Not in the same row: $AB = \begin{bmatrix} a & 0 & 0 \\ b & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c & d \\ e & f \\ g & h \end{bmatrix} = \begin{bmatrix} not\ 0 & not\ 0 \\ not\ 0 & not\ 0 \\ 0 & 0 \end{bmatrix}$, two zeros.

\therefore the minimum number of zeros is 2.

B

A

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

E

A

D

D

B

E

A

C

B

C

E

A

D