2011 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

Q1ai The lowest average age $= 25$ years

Q1aii The median average age $= 28.2$ years

Q1b $Q_L = 29.9$, $Q_U = 31.0$,
$IQR = Q_U - Q_L = 31.0 - 29.9 = 1.1$ years

Q1c $Q_L - 1.5 \times IQR = 29.9 - 1.5 \times 1.1 = 28.25$
$26.0 < 28.25$, .: 26.0 is an outlier.

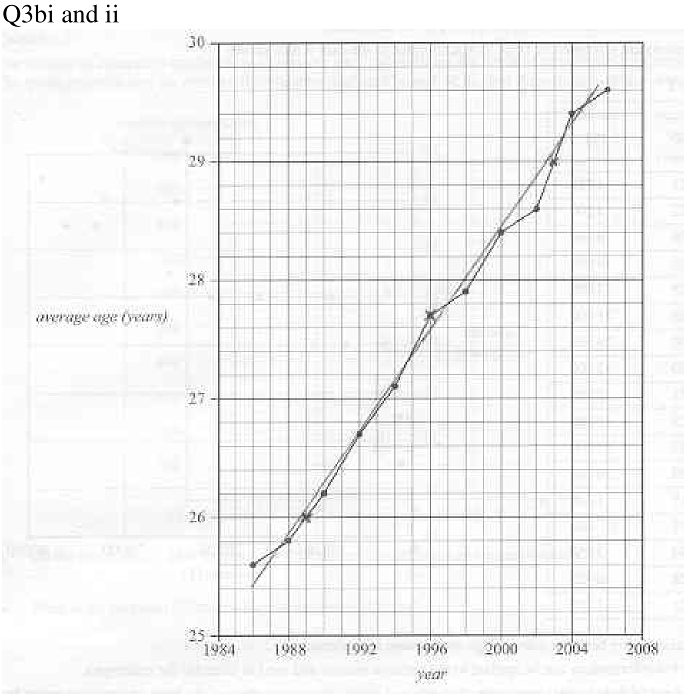
Q2a $42.1\% + 23.4\% = 65.5\%$

Q2b Yes. From 1986 to 2006, the age of women at first marriage has a negative correlation with year of marriage. Consider the 19 years and under group:

1986	1996	2006
8.5%	3.7%	2.0%

Q3a The average age of women at first marriage was fairly constant from 1915 to 1935. It had a decreasing trend from 1935 and tapered off towards 1970.

Q3bi and ii

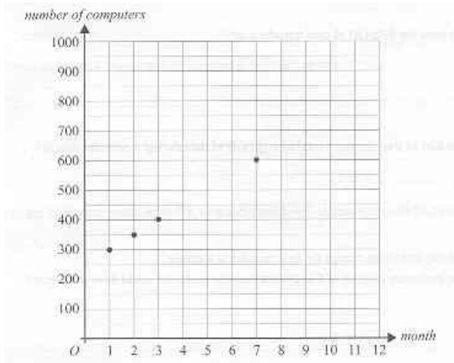


Q4a $average\ age = 2.39 + 5.89 \times \log(income)$

Q4b The average age of women at first marriage with an average yearly income of $20000 is 27.7 years.

Module 1: Number patterns

Q1a



Q1b $d = 50$

Q1c $850 - 700 = 150$ or $50 \times 3 = 150$

Q2a Arithmetic sequence: $a = 4.00$ $d = -0.20$,
$t_5 = 4.00 + 4(-0.20) = 3.20$, .: $3.20

Q2b Arithmetic series: $S_8 = \frac{8}{2}(2(4.00) + 7(-0.20)) = 26.4$,
.: $26.40

Q2c It takes 20 GB to reduce the charge to $0.20 per GB.
.: 20 GB

Q2di $D_{n+1} = 1.5D_n - 1$
$D_1 = 6$, $D_2 = 1.5D_1 - 1 = 8$, $D_3 = 1.5D_2 - 1 = 11$, .: 11 GB

Q2dii 11 GB in his the third month,
.: $S_{11} = \frac{11}{2}(2(4.00) + 10(-0.20)) = 33.00$, .: $33.00

Q2e $D_9 \approx 104.5$ GB, .: the 9th month

Q3a Charge for the second month:
$95\% \times 50 = 0.95 \times 50 = 47.50$, .: $47.50

Q3b Geometric sequence: $a = 50$, $r = 0.95$,
$t_6 = 50 \times 0.95^5 = 38.689$, $difference = 50 - 38.69 = 11.31$,
.: $11.31

Q3c Geometric series: $S_{12} = \frac{50(1 - 0.95^{12})}{1 - 0.95} \approx 459.64$, .: $460

Q4a Start of January: 200000
Start of February: $0.99 \times 200000 + 100 = 198100$

Q4b Given January is month 1,
.: $C_{n+1} = 0.99C_n + 100$ and $C_1 = 200000$

Q4c $C_3 = 0.99C_2 + 100 = 0.99(198100) + 100 = 196219$
$Difference = 200000 - 196219 = 3781$, .: 3781 more customers

Module 2: Geometry and trigonometry

Q1a Scale 1 cm : 30 m. From point *A* to the lighthouse the measured distance is 5 cm on the contour map, .: the actual distance is $30 \times 5 = 150$ m.

Q1b Average slope $= \dfrac{15}{75} = 0.2$

Q2ai $x° = 180° - 28° = 152°$

Q2aii $y° = 360° - 130° - 152° = 78°$

Q2b Bearing of the lighthouse from Ship A is S28ºW (208ºT).

Q2c The triangle is isosceles, .: $\angle ALB = \dfrac{180° - 78°}{2} = 51°$

.: bearing of Ship B from the lighthouse: $28° + 51° = 79°$,

.: N79ºE (079ºT)

Q3a $\theta = \dfrac{360°}{8} = 45°$

Q3b *Area of* $\Delta POQ = \dfrac{1}{2} \times 2^2 \times \sin 45° = \sqrt{2} \approx 1.4 \text{ m}^2$

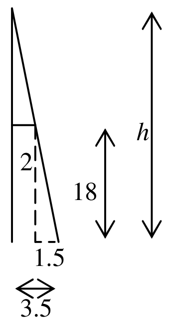
Q3c *Minimum distance* $= \dfrac{6.4 - 4}{2} = 1.2 \text{ m}$

Q3d *Shaded area* $= \pi \times 3.2^2 - 8 \times \sqrt{2} \approx 21 \text{ m}^2$

Q4a $\tan \alpha° = \dfrac{18}{1.5}$, $\alpha° = \tan^{-1}\left(\dfrac{18}{1.5}\right) \approx 85.2°$

Q4bi

$\dfrac{h}{3.5} = \dfrac{18}{1.5}$, .: $h = 42 \text{ m}$



Q4bii *Volume* $= \dfrac{1}{3}\pi(3.5^2)42 - \dfrac{1}{3}\pi(2^2)(42 - 18) \approx 438 \text{ m}^3$

Module 3: Graphs and relations

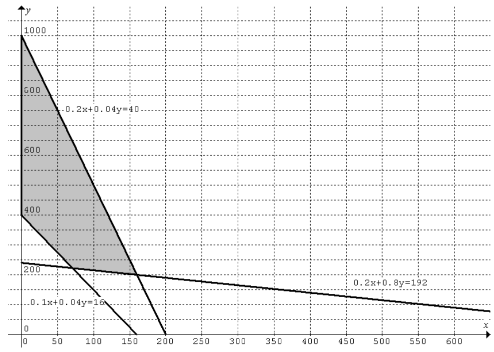
Q1a Weight of carbohydrate $= 0.2 \times 180 + 0.8 \times 250 = 236$ g

Q1b Let *n* g of raisins be added to the trail mix.

$0.2 \times 320 + 0.04n = 72$, .: $n = 200$, .: 200 g of raisins

Q1c $0.1x + 0.04y \geq 16$

Q1di and ii



Q1e $(0,1000)$ satisfies Michael's dietary requirements and gives maximum weight ($0 + 1000 = 1000$ g) of trail mix.

Q1f Solve $x + y = 500$ and $0.2x + 0.04y = 40$ for *x*.

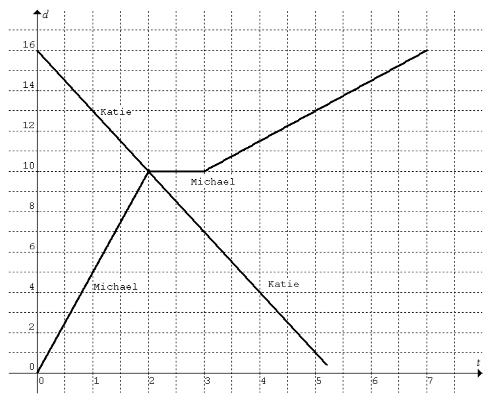
$x = 125$, .: 125 g of almonds

Q2a *Average speed* $= \dfrac{16}{7} \approx 2.3 \text{ km h}^{-1}$

Q2b *Gradient* $a = \dfrac{16 - 10}{7 - 3} = \dfrac{6}{4} = 1.5$.: $d = 1.5t + b$

When $t = 3$, $d = 10$, .: $b = 5.5$

Q2c



When they pass each other they are the same distance from the national park office, i.e. $d = 10$ when $t = 2$, .: 2 hours

Q2d $d_{Katie} = -3t + 16$, $t \geq 0$

$d_{Michael} = 5t$, $0 \leq t \leq 2$

$d_{Katie} - d_{Michael} \leq 3$, .: $-8t + 16 \leq 3$, $-8t \leq -13$, .: $1.625 \leq t \leq 2$

From the graph, $d_{Michael} - d_{Katie} \leq 3$ for $2 \leq t \leq 3$

.: they are not more than 3 km apart when $1.625 \leq t \leq 3$.

.: duration $= 3 - 1.625 = 1.375 \approx 1.38$ hours

Module 4: Business-related mathematics

Q1a Let $\$x$ be the cost without *GST*.

$\left(1+\frac{10}{100}\right)x = 3630$, .: $x = 3300$,

.: $GST = \$3630 - \$3300 = \$330$

Q1b $Total\ cost = \$82 + \$220 + \$160 = \$462$

Q1c Let $r\%$ be the percentage discount.

$220\times\left(1-\frac{r}{100}\right) = 202.40$, .: $r = 8$, .: 8% discount.

Q2a End of 1st month,

$total\ amount = \$5600\times\left(1+\frac{7.2}{100}\right) + \$200 = \$5833.60$

Q2b $Value\ of\ investment = 8000\times\left(1+\frac{7.2}{100}\right)$ dollars at the end of the 1st year.

Q2c End of 12 months, Pattie's account: $8576.00
Tom's account: $8497.58 by CAS.
.: Pattie has $\$8576.00 - \$8497.58 = \$78.42$ more than Tom.

Q2d $8000\times\left(1+\frac{r}{100}\right) = 9000$, $r = 12.5$, .: annual compounding rate is 12.5%.

Q3a *Stamp duty*

$= \$2870 + (\$300000 - \$130000)\times\frac{6}{100} = \$13070$

Q3b *House value*

$= \$300000\times\left(1+\frac{3.17}{100}\right)^5 = \$350661.76 \approx \$351000$

Q3c $\$300000\times\left(1+\frac{3.17}{100}\right)^n = \$450000$, .: $n \approx 12.9924$
Close to the end of 2023 the house value will reach $450000.
.: At the start of 2024 the house value will first exceed $450000.

Q4ai By CAS $n \approx 299.57$, .: 300 monthly repayments.

Q4aii Amount owing after the 1st year: by CAS $261305.75
Principal paid off: $\$265000 - \$261305.75 = \$3694.25$

Q4b Number of monthly repayments: $12\times 19 = 228$
Amount owing: $261305.75
Rate: 8.2%
By CAS new monthly repayment: $2265.04

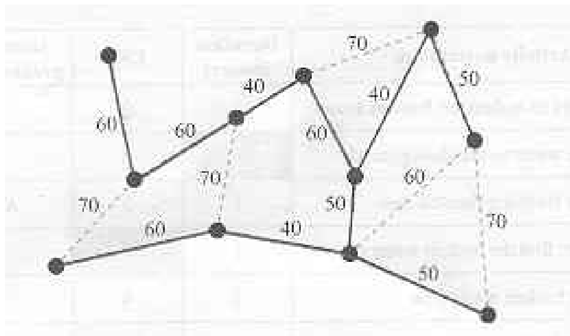
Module 5: Networks and decision mathematics

Q1a $60 + 140 = 200$, .: 200 km

Q1b From Farnham to Carrie: 6 different ways.

Q1c A possible path is DFEABCDEB and will finish at Bredon.
Q1d $130 + 110 = 240$, .: 240 km

Q2a



Q2b $60 + 60 + 40 + 60 + 40 + 50 + 50 + 50 + 40 + 60 = 510$
.: 510 km

Q3a $Duration\ of\ activity\ B = 2$ hours

Q3b $EST\ for\ D = 3$ hours

Q3c Activities *F* and *H*

Q3d Shortest time: $2 + 1 + 1 + 4 + 4 + 1 = 13$, .: 13 hours

Q3e Shortest time: $2 + 1 + 1 + 4 + 5 + 1 = 14$, .: 14 hours

Q4a Storm water from Source | 2 | cannot reach Outlet | 1 |

Q4b Outlet 1: 700 kL per min
Outlet 2: 700 kL per min

Q4c 300 kL per min

Module 6: Matrices

Q1ai birds eat lizards

Q1aii insects, birds or lizards do not eat birds

Q1b

$$\begin{array}{cc} & \begin{array}{cccc} I & B & L & F \end{array} \\ Z = & \begin{bmatrix} & & & 1 \\ & & & 0 \\ & & & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{array}{c} I \\ B \\ L \\ F \end{array} \end{array}$$

Q2a

$$K = \begin{bmatrix} 100000 & 400 & 1000 & 800 \end{bmatrix} \begin{bmatrix} 0.995 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.025 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$$

$$= \begin{bmatrix} 99500 & 20 & 25 & 240 \end{bmatrix}$$

Q2b 20

Q2c $M = \begin{bmatrix} 99500 & 20 & 25 & 240 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 285 \end{bmatrix}$

Q2d If you are given $M = \begin{bmatrix} 285 \end{bmatrix}$ only, 285 is the total number of creatures killed. If you know $M = KF$ as well, then 285 is the total number of birds, lizards and frogs killed.

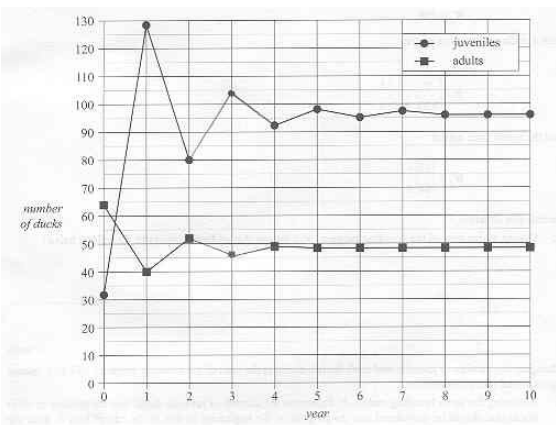
Q3a $32 + 64 = 96$, .: 96 female ducks

Q3b $W_1 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 32 \\ 64 \end{bmatrix} = \begin{bmatrix} 128 \\ 40 \end{bmatrix}$

Q3ci $W_2 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 128 \\ 40 \end{bmatrix} = \begin{bmatrix} 80 \\ 52 \end{bmatrix}$

$W_3 = \begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 80 \\ 52 \end{bmatrix} = \begin{bmatrix} 104 \\ 46 \end{bmatrix}$

.: 104 juvenile female ducks and 46 adult female ducks



Q3cii $\begin{bmatrix} 0 & 2 \\ 0.25 & 0.5 \end{bmatrix}^n \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 96 \\ 48 \end{bmatrix}$ as n is a large number.

$96 + 48 = 144$, .: 144 female ducks

Q3d $W_0 = \begin{bmatrix} 32 \\ 64 \end{bmatrix}$, total is 96

After 4 years, $W_4 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^4 \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 28 \\ 23 \end{bmatrix}$, total is 51

After 5 years, $W_5 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^5 \begin{bmatrix} 32 \\ 64 \end{bmatrix} \approx \begin{bmatrix} 23 \\ 18.5 \end{bmatrix}$, total is 41.5

.: during the 5th year

Q3e $\begin{bmatrix} 0 & 1 \\ 0.25 & 0.5 \end{bmatrix}^2 \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$, .: $\begin{bmatrix} 0.25 & 0.5 \\ 0.125 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$

$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.25 & 0.5 \\ 0.125 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 400 \\ 0 \end{bmatrix}$

.: 400 juvenile female ducks and no adult female duck

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

