Further Mathematics

2012

Trial Examination 1

Core – Data analysis
Module 1 – Number patterns
Module 5 – Networks and decision mathematics
Module 6 – Matrices

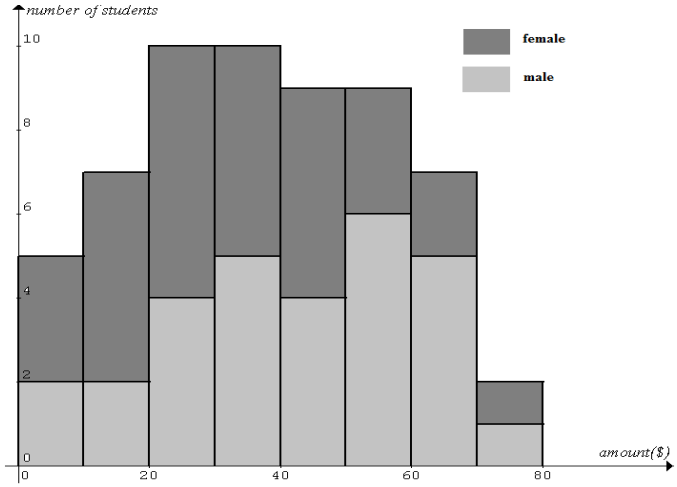
SECTION A Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1, 2 and 3

The results of a survey of the amount of money carried by a year 12 student at a public school on a particular day is shown below. Total number of year 12 students in the survey is 59.



Question 1

The number of female students in the survey is

A. 29

B. 30

C. 58

D. 59

E. not determinable from the graph

Question 2

The distribution of the amount of money carried by a year 12 female student is best described as

A. discrete

B. symmetric

C. negatively skewed

D. positively skewed

E. positively skewed with outliers

Question 3

The average amount of money ($) carried by a year 12 student is closest to

A. 34

B. 38

C. 41

D. 42

E. 43

The following information relates to Questions 4 and 5

The test results for a class of 26 students are displayed in the following back-to-back stem-and-leaf plot.

Boys					*Girls*				
		5	3	3 | 6				
			5	5 | 6	6	7	8	
	8	7	5	6 | 6	6	9	9	9
7	4	2	2	7 | 4	5			
		3	0	8 | 2	9			

Question 4

The range and the median of the test results are respectively

A. $[33,89]$ and 68

B. $(33,89)$ and 68.5

C. 56 and 68.5

D. 56 and 69

E. 89 and 69

Question 5

The number of outliers is

A. 0

B. 1

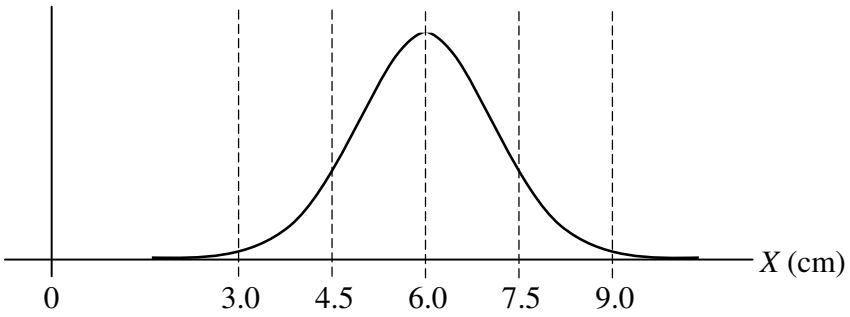
C. 2

D. 3

E. 4

The following information relates to Questions 6 and 7

A tradesman has accumulated a large quantity of nails of different sizes. The bell shaped distribution of the nail lengths (cm) can be modeled by the normal distribution shown below. 13.5% of the data are between values 3.0 and 4.5, and 34% are between values 4.5 and 6.0.



Question 6

Of the longer than 4.5 cm nails, the percentage longer than the mean length is closest to

A. 50

B. 55

C. 60

D. 70

E. 80

Question 7

The z-score of $X = 8.4$ is closest to

A. 1.67

B. 1.66

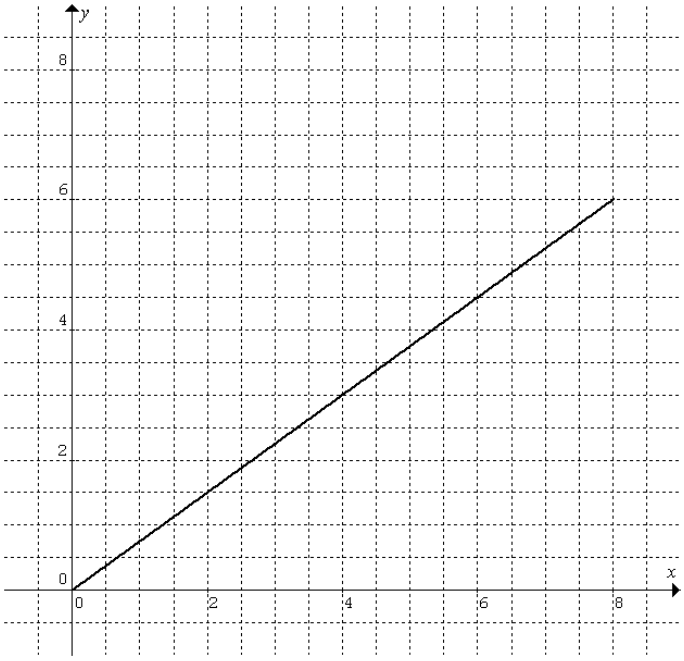
C. 1.65

D. 1.60

E. 1.55

The following information relates to Questions 8 and 9

The least squares regression line of a set of bivariate data is shown below. The mean of x is 3.7, and pearson's product-moment correlation coefficient r is 0.95.



Question 8

The mean of y is closest to

A. 4.0

B. 3.9

C. 3.8

D. 3.3

E. 2.8

Question 9

The value of the ratio $\dfrac{\text{standard deviation of } y}{\text{standard deviation of } x}$ is closest to

A. 3.4

B. 2.4

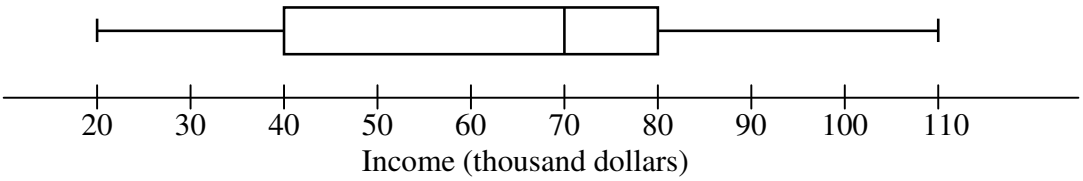
C. 1.4

D. 1.1

E. 0.8

Question 10

The following boxplot shows the distribution of incomes of households in a suburb in 2011.



Which one of the following statements is ***NOT*** true?

A. The average household income and the median household income are different.

B. The average household income is higher than the median household income.

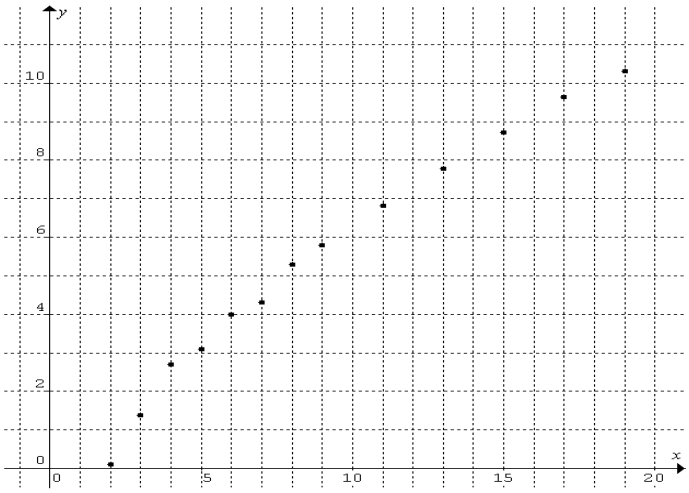
C. The average household income is lower than the median household income.

D. The greatest difference in household income in the middle 50% is $40000.

E. More than 25% of the households have more than $75000 income.

Question 11

A set of bivariate data is shown in the following scatterplot.



Which one of the following scatterplots is most likely to display linearity?

A. $\log_{10} y$ versus x

B. y versus $\log_{10} x$

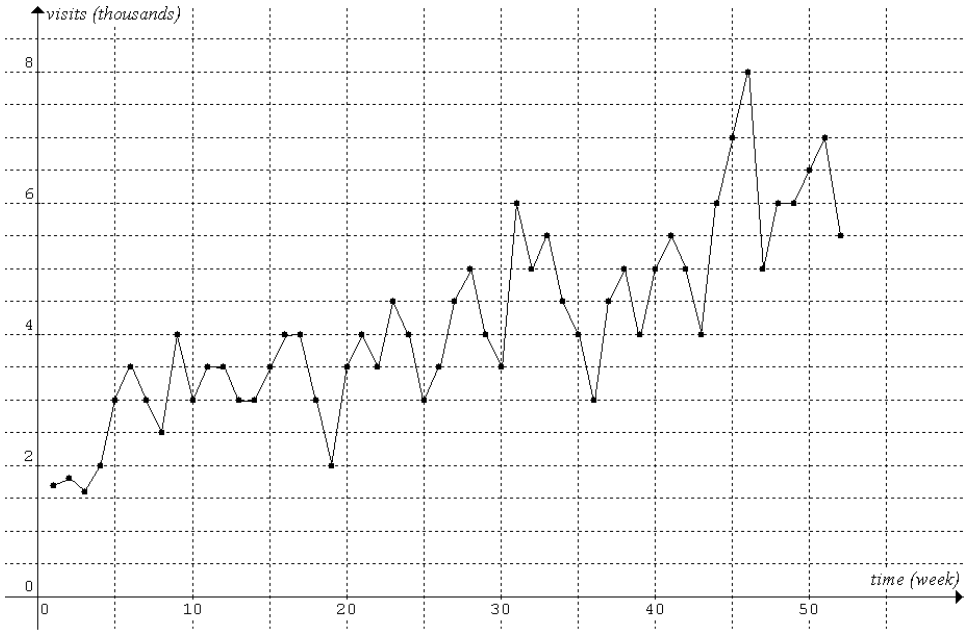
C. $\frac{1}{y}$ versus x

D. y versus x^2

E. y versus $\frac{1}{x}$

The following information relates to Questions 12 and 13

The number of visits N (in thousands) to a website over 52 weeks is shown in the following time series, where time t is in weeks.



A 3 median regression line is to be fitted to the time series.

Question 12

(t_L, N_L) and (t_U, N_U) are the two outer medians. The values of N_L and N_U are respectively

A. 3 and 5.5

B. 4 and 6

C. 3000 and 5500

D. 4000 and 5500

E. 4000 and 6000

Question 13

The equation of the 3-median regression line is closest to

A. $N = 65.9t + 2200$

B. $N = 71.4t + 2200$

C. $N = 0.0659t + 2.2$

D. $N = 0.0660t + 2.2$

E. $N = 0.0714t + 2.2$

SECTION B Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.

Module 1: Number patterns

Question 1

The 16^{th} term of the sequence 1, 1, 3, 4, 5, 7, 7, 10, 9, 13, 11, 16, …… is

A. 15

B. 17

C. 18

D. 20

E. 22

Question 2

Consider the following arithmetic sequence: $-2,\ 2.2,\ 6.4,$ ……
The two smallest ***whole number*** terms in the sequence are

A. 20 and 40

B. 20 and 38

C. 19 and 38

D. 19 and 40

E. 18 and 42

Question 3

An arithmetic sequence has 13 terms and a sum of 3.25. The value of $t_6 + t_7 + t_8$ equals

A. 1.50

B. 1.00

C. 0.75

D. 0.50

E. 0.25

Question 4

The sum of the 2 terms between 1.4 and -4.725 in the geometric sequence …, 1.4, …, …, -4.725, … is

A. -2.08

B. 0.95

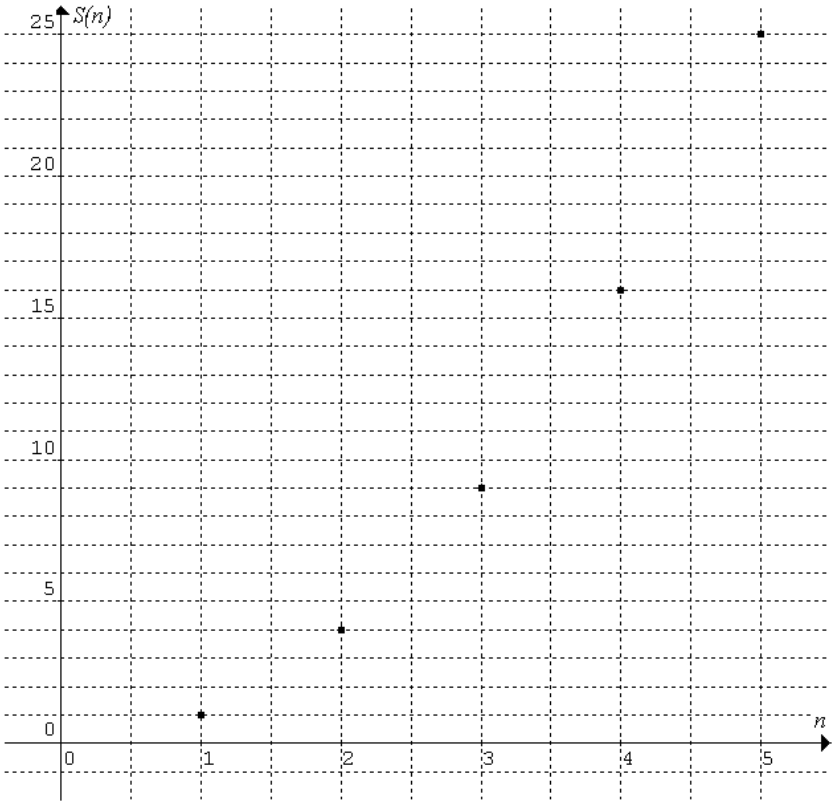
C. -1.05

D. 1.05

E. -0.95

Question 5

The following graph shows the ***sum*** of the first n terms, $S(n)$, of a sequence $t_1, t_2, t_3, t_4, t_5, \ldots\ldots$



The 7th term, t_7, of the sequence is

A. 49

B. 36

C. 13

D. 7

E. 6

Question 6

A weight loss program advertised a weekly reduction of 2.5% of the weight in the previous week. According to the advertisement, the number of weeks required to get below 50% of the initial weight is ***at least***

A. 20

B. 22

C. 24

D. 26

E. 28

Question 7

The terms in a sequence has the following properties: $t_2 - 2t_1 = 2$, $t_3 - 2t_2 = 6$, $t_4 - 2t_3 = 12$. If $t_1 = 3$, then $t_5 =$

A. 132

B. 120

C. 112

D. 108

E. 88

Question 8

-10, f_{10}, f_{11}, f_{12} and 12 are five consecutive terms of a Fibonacci sequence. f_{12} is closest to

A. 0.3

B. 11.3

C. -1.7

D. 12.7

E. -3

Question 9

\$500000 is invested at 5% p.a. interest. \$15000 is withdrawn after interest is paid to the account at the end of each year. Let $\$ A_n$ be the total amount of the investment at the beginning of the *n*th year. A difference equation to model the total amount of the investment at the beginning of the *n*th year is

A. $A_n = 0.05A_{n-1} - 15000,\ A_0 = 500000$

B. $A_{n+1} = 0.05A_n - 15000,\ A_1 = 500000$

C. $A_{n+1} - A_n = 0.05A_n - 15000,\ A_1 = 500000$

D. $A_{n+1} - A_n = 1.05A_n - 15000,\ A_1 = 500000$

E. $A_n - A_{n-1} = 1.05A_n - 500000,\ A_0 = 15000$

Module 5: Networks and decision mathematics

Question 1

A complete graph has 17 vertices. The graph has

A. 136 edges

B. 100 edges

C. 10 edges

D. 1 edge

E. 17 edges

Question 2

A graph has an odd number n of vertices. The degree of each vertex is at least 1. The number of edges is a minimum. The number of edges is

A. $\frac{n+1}{2}$

B. $n-1$

C. $\frac{n-1}{2}$

D. $n+1$

E. n

Question 3

A planar graph has an odd number of faces. The possible number of vertices and edges are respectively

A. 7 and 4

B. 7 and 7

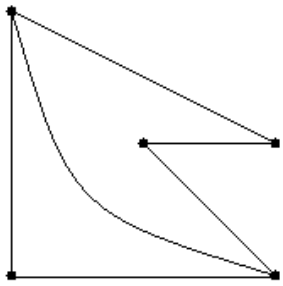
C. 8 and 9

D. 8 and 10

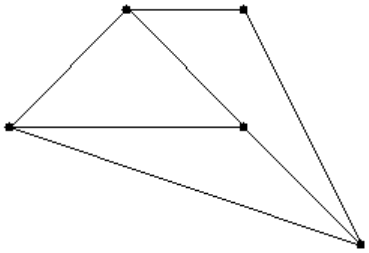
E. 10 and 8

Question 4

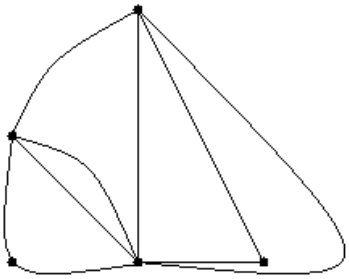
Graph I



Graph II



Graph III



An Euler path may start on any vertex for

A. Graph I only

B. Graph II only

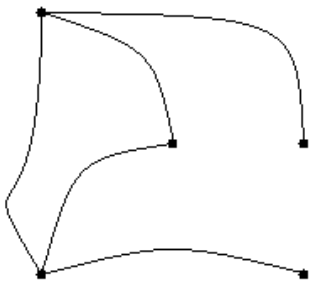
C. Graph III only

D. Graph I and Graph II only

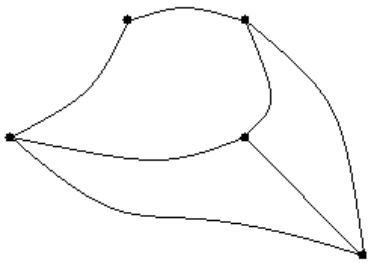
E. Graph I and Graph III only

Question 5

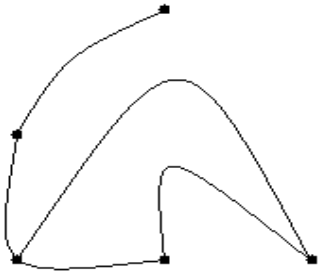
Graph I



Graph II



Graph III



A Hamiltonian path exists for

A. Graph I only

B. Graph II only

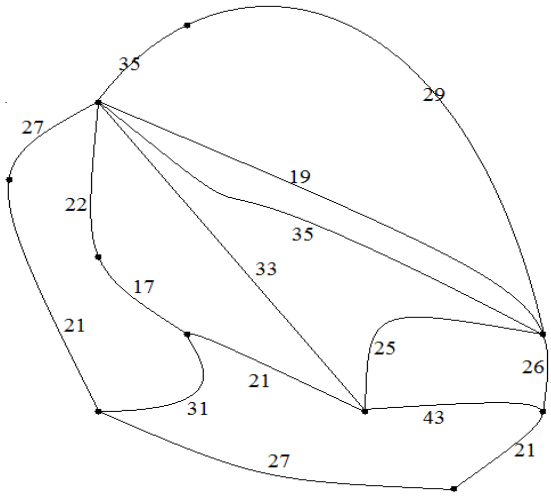
C. Graph III only

D. Graph I and Graph II only

E. all three graphs

Question 6

Ten towns are to be connected by freeways. The lengths in kilometres of the proposed freeways are shown in the following weighted graph.



The shortest total length of freeways required to connect the ten towns is

A. 175

B. 183

C. 197

D. 203

E. 211

Question 7

Four shops *P*, *Q*, *R* and *S*, are connected by one-way streets. Adjacency matrix *A* displays all of the possible one-stage routes from one shop to another by car.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} = A$$

The shops which can reach the other shops by car using one-stage or two-stage routes are

A. *P* and *Q* only

B. *Q* and *R* only

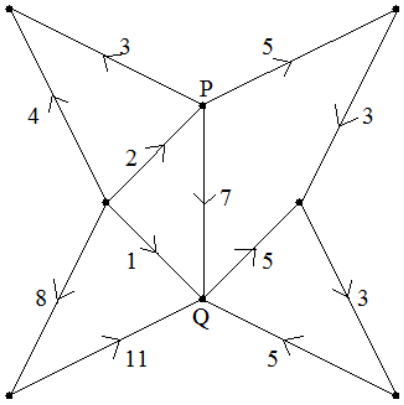
C. *R* and *S* only

D. *P*, *Q* and *S* only

E. *Q*, *R* and *S* only

Question 8

The following network shows the allocated maximum flow of data (in GB per hour) from one workstation to another.



The maximum amount of data (in GB per hour) originated from P reaching Q is

A. 15

B. 13

C. 12

D. 11

E. 10

Question 9

Four jobs *P*, *Q*, *R* and *S* must be completed within an 8-hour day by one or more of four workers *A*, *B*, *C* and *D*. The following table shows the time (hours) required by each worker to complete each job.

	P	*Q*	*R*	*S*
A	3	2	1	5
B	4	5	2	4
C	4	3	1	3
D	7	2	1	4

If all four workers can start work at the beginning of the working day, the shortest time to complete all four jobs is

A. 7

B. 6

C. 5

D. 4

E. 3

Module 6: Matrices

Question 1

Four towns A, B, C and D are connected by two-way roads. A matrix showing the number of possible routes from one town to another town is $\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & p & 2 \\ q & 0 & 1 & 0 \\ 3 & s & 1 & 1 \\ t & u & 1 & 0 \end{bmatrix} \end{array}$. The values of p, u and t are respectively

A. 1, 1 and 2

B. 1, 2 and 3

C. 3, 0 and 2

D. 3, 1 and 2

E. 2, 3 and 1

Question 2

$$\left(\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - 2\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\right)\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

A. $= -6$

B. $= \begin{bmatrix} -6 \end{bmatrix}$

C. $= \begin{bmatrix} -5 & 0 & 5 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

D. $= \begin{bmatrix} -5 & 2 & 1 \\ 0 & 0 & 0 \\ 5 & -2 & -1 \end{bmatrix}$

E. is undefined

Question 3

A, B and C are square matrices. They have the same order and they are not null matrices. Which one of the following statements is ***always false***?

A. $(A+B)+C=A+(B+C)$

B. $(AB)C=A(BC)$

C. $ABC=CBA$

D. $(A-B)-C=A-(B-C)$

E. $A(B-C)=BA-BC$

Question 4

An electrical discount store has three types of refrigerators selling for \$650, \$720 and \$1080, three types of washing machines selling for \$510, \$680 and \$800, and three types of LED HD televisions selling for \$340, \$490 and \$975. The owner of the store wishes to mark down the prices of refrigerators, washing machines and televisions by 12%, 15% and 18% respectively. The new prices of each of the nine items can be calculated by

A. $\begin{bmatrix} 650 & 510 & 340 \\ 720 & 680 & 490 \\ 1080 & 800 & 975 \end{bmatrix}\begin{bmatrix} 0.12 & 0.12 & 0.12 \\ 0.15 & 0.15 & 0.15 \\ 0.18 & 0.18 & 0.18 \end{bmatrix}$

B. $\begin{bmatrix} 0.12 & 0.12 & 0.12 \\ 0.15 & 0.15 & 0.15 \\ 0.18 & 0.18 & 0.18 \end{bmatrix}\begin{bmatrix} 650 & 510 & 340 \\ 720 & 680 & 490 \\ 1080 & 800 & 975 \end{bmatrix}$

C. $\begin{bmatrix} 0.88 & 0.88 & 0.88 \\ 0.85 & 0.85 & 0.85 \\ 0.82 & 0.82 & 0.82 \end{bmatrix}\begin{bmatrix} 650 & 510 & 340 \\ 720 & 680 & 490 \\ 1080 & 800 & 975 \end{bmatrix}$

D. $\begin{bmatrix} 650 & 510 & 340 \\ 720 & 680 & 490 \\ 1080 & 800 & 975 \end{bmatrix}\begin{bmatrix} 0.88 & 0 & 0 \\ 0 & 0.85 & 0 \\ 0 & 0 & 0.82 \end{bmatrix}$

E. $\begin{bmatrix} 0.88 & 0 & 0 \\ 0 & 0.85 & 0 \\ 0 & 0 & 0.82 \end{bmatrix}\begin{bmatrix} 650 & 510 & 340 \\ 720 & 680 & 490 \\ 1080 & 800 & 975 \end{bmatrix}$

Question 5

Given $x-2y=3$, $3y-4z=5$ and $5z-6x=7$, the solution to the simultaneous equations can be determined by

A. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{99}\begin{bmatrix} 45 & 30 & 24 \\ 72 & 15 & 12 \\ 54 & 36 & 9 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & -4 \\ 5 & -6 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 & -2 \\ -4 & 0 & 3 \\ 5 & -6 & 0 \end{bmatrix}\begin{bmatrix} z \\ x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$

D. $\begin{bmatrix} z \\ x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -4 & 0 & 3 \\ 5 & -6 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

E. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & -4 \\ 5 & -6 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

Question 6

Given the system of simultaneous equations $\begin{aligned} 2a+b-c-d &= 4 \\ a+2b-c-d &= 3 \\ a+b-2c-d &= 2 \\ a+b-c-2d &= 1 \end{aligned}$, the value of $a-b+c-d$ is closest to

A. 8

B. 6

C. 4

D. 2

E. 0

The following information relates to Questions 7, 8 and 9

In an Australian city 58% of commuters travel by car, 37% by public transports and 5% by bicycles in 2012. A survey was conducted to gauge a commuter's transportation preference in 2013. The following table summarises the results of the survey. For example, 95% of car users in 2012 will use car in 2013, 4% will switch to public transports and 1% to bicycles etc.

2013 \ 2012	Car	Public transports	Bicycles
Car	95%	1.5%	0.5%
Public transports	4%	98%	7.5%
Bicycles	1%	0.5%	92%

Question 7

The state matrix for the year 2012 showing the proportion of commuters travelling by car, by public transports and by bicycles is

A. $\begin{bmatrix} 0.015 \\ 0.980 \\ 0.005 \end{bmatrix}$

B. $\begin{bmatrix} 0.95 \\ 0.04 \\ 0.01 \end{bmatrix}$

C. $\begin{bmatrix} 0.005 \\ 0.075 \\ 0.920 \end{bmatrix}$

D. $\begin{bmatrix} 0.58 \\ 0.37 \\ 0.05 \end{bmatrix}$

E. $\begin{bmatrix} 0.015 & 0.980 & 0.005 \end{bmatrix}$

Question 8

Surveys done in the past three years showed the same trend shown in the table above. The percentage of commuters using public transports in 2010 was closest to

A. 90

B. 30

C. 22

D. 7

E. 2

Question 9

If the same trend continues into the future, the percentage of commuters using public transports will be approaching

A. 76

B. 71

C. 66

D. 61

E. 56

End of Exam 1