2012 VCAA Further Mathematics Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
E	B	E	D	D	B	D	A	B	D	C	A	E

SECTION B

Module 1: Number patterns

1	2	3	4	5	6	7	8	9
D	E	A	D	C	E	D	B	A

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
C	D	B	B	D	D	C	C	D

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
B	A	D	A	C	D	A	A	D

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
C	C	E	D	A	C	D	D	A

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
E	A	C	D	D	A	C	C	D

Module 6: Matrices

1	2	3	4	5	6	7	8	9
D	B	A	E	D	C	B	B	B

SECTION A Core: Data analysis

Q1 Frequency of north-west wind was the highest, $f = 41$. E

Q2 *Frequency of east or south-east wind* $= 10 + 25 = 35$

Percentage $= \frac{35}{214} \times 100\% \approx 16\%$ B

Q3 *The mean weight* $= \frac{1.53}{9} = 0.17 \text{ kg} = 170 \text{ g}$ E

Q4 Let *x* be the legal studies mark.

$z_{leg} = z_{bio}$, $\frac{x - 78}{5} = \frac{81 - 54}{15}$, $x = 87$ D

Q5 D

Q6 22.3% *of* $5170000 = \frac{22.3}{100} \times 5170000 \approx 1153000$ B

Q7 D

Q8 Temperature (*y*) is the dependent variable and wind speed the independent variable (*x*). *y*-intercept is 25.7 and the gradient is negative. A

Q9 The first and the last 2 points should be missing in the smoothed time series. .: either B or D is correct. B has the correct second point. B

Q10 Since $Q_U \geq Q_L$, .: $IQR = Q_U - Q_L \geq 0$ D

Q11 *Average of the 4 seasons*

$= \frac{52.0 + 54.5 + 48.8 + 61.3}{4} = 54.15$

Seasonal index for spring $= \frac{61.3}{54.15} \approx 1.13$ C

Q12 *Deseasonalised rainfall* $= \frac{actual}{index} = \frac{48.9}{1.01} \approx 48.4$ A

Q13 *Quarter 4 seasonal index* $= 4 - (1.2 + 0.7 + 0.8) = 1.3$

Deseasonalised sales $= 256000 + 15600 \times 4 = 318400$

Actual sales = deseasonalised sales $\times$ *seasonal index*

$= 318400 \times 1.3 = 413920$ E

SECTION B

Module 1: Number patterns

Q1 Fibonacci sequence: $t_6 = t_5 + t_4 = 29 + 19 = 48$ D

Q2 Geometric sequence: $r = 1 + \frac{10}{100} = 1.1$, $T_{n+1} = 1.1T_n$ E

Q3 Arithmetic sequence: $a = 500$, $d = 50$, $t_n = a + (n-1)d$,

$t_{20} = 500 + (20 - 1) \times 50 = 1450$ A

Q4 4 years = 48 months

$S_n = \frac{n}{2}[2a + (n-1)d]$, $S_{48} = \frac{48}{2}[2 \times 500 + 47 \times 50] = 80400$ D

Q5 Geometric series: $3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7 = 3279$

Alternatively,

$a = 3$, $r = 3$, $S_n = \frac{a(r^n - 1)}{r - 1}$, $S_7 = \frac{3(3^7 - 1)}{3 - 1} = 3279$ C

Q6 $r = \frac{160}{100} = 1.6$, $a = \frac{100}{1.6} = 62.5$,

$S_{10} = \frac{62.5(1.6^{10} - 1)}{1.6 - 1} \approx 11350$ E

Q7 The sum of the increases in speed each second forms an infinite geometric series: $S_\infty = 50 + 30 + 18 + ...$

$a = 50$, $r = \frac{30}{50} or \frac{18}{30} = 0.6$, $S_\infty = \frac{a}{1 - r} = \frac{50}{1 - 0.6} = 125$

The greatest speed $= 100 + 125 = 225$ D

Q8 The points form a curve, .: not arithmetic. All the terms are negative, .: *r* is positive.

The terms are approaching 0, .: $0 < r < 1$ B

Q9 $t_{n+1} = 1.5t_n - 3000$, $t_n = \frac{t_{n+1} + 3000}{1.5}$, given $t_3 = 12300$

.: $t_2 = \frac{12300 + 3000}{1.5} = 10200$, $t_1 = 8800$ A

Module 2: Geometry and trigonometry

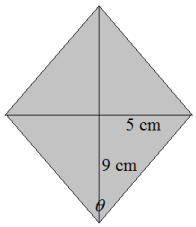
Q1 $x = 24 + 68 = 92$ C

Q2 *Shaded area = area of the square − area of triangle*
$= 4 \times 4 - \frac{1}{2} \times 3 \times 4 = 10$ D

Q3 *Circumference = length of rectangle*, $2\pi r = 50$,
$r = \frac{50}{2\pi} \approx 8.0$ B

Q4 *Average slope* $= \frac{rise}{run} = \frac{30-15}{1200} = 0.0125$ B

Q5

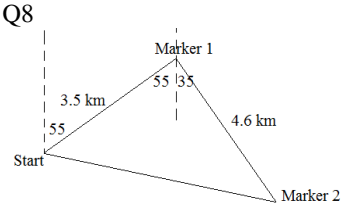


$\frac{\theta}{2} = \tan^{-1}\left(\frac{5}{9}\right) \approx 29°$
$\theta \approx 58°$ D

Q6 *Volume of cylinder* $= \pi r^2 h$, given $\pi R^2 H = V$ for a cylinder with radius R and height H
For a cylinder with radius $3R$ and height $3H$,
volume $= \pi(3R)^2(3H) = 27\pi R^2 H = 27V$ D

Q7 $\angle PQR = 180° - (37° + 42°) = 101°$
$\frac{y}{\sin 101°} = \frac{10}{\sin 37°}$, $\therefore\ y = 10 \times \frac{\sin 101°}{\sin 37°}$
$\frac{x}{\sin 42°} = \frac{10}{\sin 37°}$, $\therefore\ x = 10 \times \frac{\sin 42°}{\sin 37°}$ C

Q8



The vertex angle at Marker 1 is a right angle.
Distance between Marker 2 and Start $= \sqrt{3.5^2 + 4.6^2}$
≈ 5.8 km C

Q9 *Area of curved surface* $PQR = \frac{1}{8}\left(4\pi \times 15^2\right) \approx 353.4$
Area of sector OPQ = area of sector OQR = area of sector OPR
$= \frac{1}{4}\left(\pi \times 15^2\right) \approx 176.7$
Total area $\approx 353.4 + 3 \times 176.7 \approx 884$ D

Module 3: Graphs and relations

Q1 B

Q2 $2.40 \times 5 + 3.00 \times n = 24.00$, $n = 4$ A

Q3 *Minimum* $P = 2$ at $(3,4)$ D

Q4 Gradient $= \frac{0-700}{85-50} = -20$, $\therefore\ V = -20t + c$.
Given $(85,0)$, $\therefore\ 0 = -20 \times 85 + c$, $c = 1700$
$\therefore\ V = 1700 - 20t$ A

Q5 Volume first decreased by $1000 - 700 = 300$ litres
Time taken $= \frac{300}{15} = 20$ minutes
$\therefore$ Volume did not change for $50 - 20 = 30$ minutes C

Q6 *Average over the ten years* $= \frac{85-40}{10} = 4.50$ D

Q7 $Y = mX + c$
$m = \frac{5}{2}$, $Y = y$, $X = \frac{1}{x}$, $c = 0$, $\therefore\ y = \frac{5}{2} \times \frac{1}{x}$, $y = \frac{5}{2x}$ A

Q8 A

Q9 Let \$$x$ and \$$y$ be the costs for $0 < weight \le 120$ and $200 < weight \le 500$ respectively.
$x + 4 + y = 11$ and $2x + y = 8$, $\therefore\ x = 1$ and $y = 6$
$\therefore\ 4 + y = 4 + 6 = 10$ dollars D

Module 4: Business-related mathematics

Q1 $\$215 \times \left(1 - \frac{25}{100}\right) = \$161.25$ C

Q2 $\$3000 \times \frac{6.5}{100} \times 3 = \$585.00$ C

Q3 $21.99 = 5101.82 \times \frac{r}{100} \times \frac{1}{12}$, $r \approx 5.17$ E

Q4 *Salary after the second increase*
$= \$65000 \times \left(1 + \frac{2.8}{100}\right) \times \left(1 + \frac{3.5}{100}\right) \approx \$69160$ D

Q5 *Balance after the deposit* $= \$9000 - \$2500 = \$6500$,
interest rate $= 14.95\%\ pa$, 24 monthly repayments.
By CAS TVM Solver, *monthly repayment* $\approx \$315$ A

Q6 *Commission* $= 6500 + (380000 - 250000) \times \frac{1.6}{100} = 8580$ C

Q7 D

Q8 *Total value* $= \$15000 \times \left(1 + \frac{1}{12} \times \frac{6.1}{100}\right)^6 \times \left(1 + \frac{1}{12} \times \frac{6.25}{100}\right)^6$
$\approx \$15952.92$
Total interest $= \$15952.92 - \$15000 = \$952.92 \approx \$953$ D

Q9 Balance of the loan went up (interest added). A

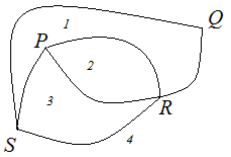
Module 5: Networks and decision mathematics

Q1 Sum of the degrees $= 1+3+2+2+2+2 = 12$ E

Q2 A

Q3 C

Q4 D



Q5 For an Euler circuit to exist, all vertices are of even degree.
.: the first graph does not have an Euler circuit. D

Q6 A

Q7 Line 1 does not separate the source from the sink.
Line 2: $110 + 240 = 350$
Line 3: $90 + 60 + 240 = 390$
Line 3: $90 + 280 = 370$
.: minimum cut is line 2. C

Q8 Reduce activities *A*, *B* and *F* by one hour each. C

Q9 Activities *D* and *E* are to be completed by Lisa only.
To achieve minimum number of days she starts activity *D* after activity *A* is completed by John, then she does activity *E*. By the time activity E is completed,
total number of days $= 3+7+6 = 16$
.: *total number of days to complete all activities* $= 16+5 = 21$ D

Module 6: Matrices

Q1 $2\times\begin{bmatrix}2 & 8\\ 4 & -1\\ 3 & 5\end{bmatrix}-\begin{bmatrix}3 & 7\\ 4 & 2\\ 2 & 3\end{bmatrix}=\begin{bmatrix}4 & 16\\ 8 & -2\\ 6 & 10\end{bmatrix}-\begin{bmatrix}3 & 7\\ 4 & 2\\ 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 9\\ 4 & -4\\ 4 & 7\end{bmatrix}$ D

Q2 24 is obtained by multiplying the second row of matrix *A* by the first column of matrix *B*. B

Q3 $\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ 8\\ 15\end{bmatrix}$

$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}^{-1}\begin{bmatrix}6\\ 8\\ 15\end{bmatrix}=\begin{bmatrix}-3 & -1 & 2\\ -2 & 0 & 1\\ 4 & 1 & -2\end{bmatrix}\begin{bmatrix}6\\ 8\\ 15\end{bmatrix}$ A

Q4 E

Q5 80% will remain purchasing from *B*, and 70% will remain purchasing from *F*. D

Q6 A matrix that shows the total number of students is a 1×1 matrix.
A. $M\times P\times F$ is undefined
B. $P\times G\times M$ is undefined
C. $F\times P\times N$ is a 1×1 matrix C
D. $P\times N\times F$ is a 4×4 matrix
E. $F\times N\times P$ is undefined

Q7 B

Q8 B

Q9 $\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}\times\begin{bmatrix}a\\ 3\end{bmatrix}=\begin{bmatrix}6 & 3\\ 2 & -1\end{bmatrix}\times\begin{bmatrix}2\\ b\end{bmatrix}$
.: $3a+12 = 12+3b$ and $a+6 = 4-b$
.: $a-b = 0$ and $a+b = -2$ B

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

