2012 VCAA Math. Methods (CAS) Exam 1 Solutions

Q1a $y=\left(x^2-5x\right)^4$, $\frac{dy}{dx}=4\left(x^2-5x\right)^3(2x-5)$

Q1b $f(x)=\frac{x}{\sin x}$, $f'(x)=\frac{\sin x-x\cos x}{\sin^2 x}$,

$f'\left(\frac{\pi}{2}\right)=\frac{\sin\frac{\pi}{2}-\frac{\pi}{2}\cos\frac{\pi}{2}}{\sin^2\frac{\pi}{2}}=1$

Q2 $\int\frac{1}{(2x-1)^3}dx=\int(2x-1)^{-3}dx=\frac{(2x-1)^{-2}}{-2\times 2}=-\frac{1}{4(2x-1)^2}$

Q3 $h(x)=2x^3+1$, equation of the inverse is $x=2y^3+1$,

.: $y=\left(\frac{x-1}{2}\right)^{\frac{1}{3}}$, .: $h^{-1}(x)=\left(\frac{x-1}{2}\right)^{\frac{1}{3}}$

Q4a $\overline{X}=E(X)=0\times 0.2+1\times 0.2+2\times 0.5+3\times 0.1=1.5$

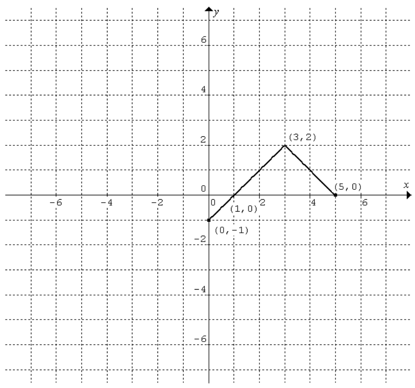
Q4b $\Pr(111)=0.2\times 0.2\times 0.2=0.008$

Q4c

$\Pr(total.of.four \mid calls.both.days)=\frac{\Pr(13)+\Pr(22)+\Pr(31)}{\Pr(0'0')}$

$=\frac{0.2\times 0.1+0.5\times 0.5+0.1\times 0.2}{0.8\times 0.8}=\frac{29}{64}$

Q5a



Q5bi $(3,2)\rightarrow(3,-2)\rightarrow(8,-2)$

Q5bii $y=-|x-3|+2$

$\rightarrow -y=-|x-3|+2\rightarrow -y=-|(x-5)-3|+2$ which can be simplified to $y=|x-8|-2$.

Q6a At $x=\frac{\pi}{3}$, $\cos x=a\sin x$, .: $\cos\frac{\pi}{3}=a\sin\frac{\pi}{3}$

.: $\frac{1}{2}=a\times\frac{\sqrt{3}}{2}$, $a=\frac{1}{\sqrt{3}}$

Q6b $\cos x=\frac{1}{\sqrt{3}}\sin x$, .: $\tan x=\sqrt{3}$, $x=\frac{\pi}{3}$ or $\frac{4\pi}{3}$

The other point of intersection is at $x=\frac{4\pi}{3}$.

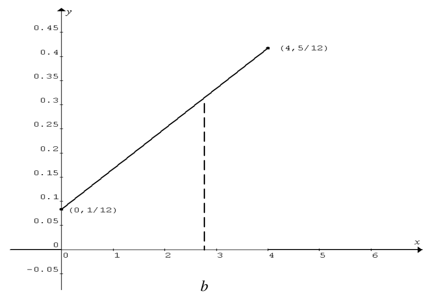
Q7 $2\log_e(x+2)-\log_e x=\log_e(2x+1)$, $x>0$

.: $\frac{(x+2)^2}{x}=2x+1$, $x^2-3x-4=0$, $(x+1)(x-4)=0$, .: $x=4$

Q8a Given $\overline{X}=100$ and $\Pr(X<106)=q$, then

$\Pr(94<X<100)=\Pr(100<X<106)=q-0.5$

Q8b



$f(b)=\frac{b+1}{12}$, $b>0$

Area of trapezium $=\frac{1}{2}\left(\frac{1}{12}+\frac{b+1}{12}\right)b=\frac{5}{8}$,

$b^2+2b-15=0$, $(b-3)(b+5)=0$, .: $b=3$

Q9a $f(x)=x\sin x$, $f'(x)=\sin x+x\cos x$

Q9b $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}f'(x)dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x dx+\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}x\cos x dx$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}x\cos x dx=\left[x\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}-\left[-\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}=\frac{5}{12}\pi-\frac{\sqrt{3}}{2}$

Q10ai $f(x)=e^{-mx}+3x$ for rational $m>0$

Let $f'(x)=-me^{-mx}+3=0$ to find the stationary point.

.: $e^{-mx}=\frac{3}{m}$, $e^{mx}=\frac{m}{3}$, $mx=\log_e\left(\frac{m}{3}\right)$, .: $x=\frac{1}{m}\log_e\left(\frac{m}{3}\right)$

Q10aii $x=\frac{1}{m}\log_e\left(\frac{m}{3}\right)$, x is positive when $\frac{m}{3}>1$, i.e. $m>3$

Q10b At $x=-6$, $y=e^{6m}-18$, $\frac{dy}{dx}=f'(-6)=-me^{6m}+3$

Tangent: $y-\left(e^{6m}-18\right)=\left(-me^{6m}+3\right)(x+6)$

The tangent passes through $(0,0)$, .: $-\left(e^{6m}-18\right)=6\left(-me^{6m}+3\right)$

.: $e^{6m}=6me^{6m}$, $6m=1$, $m=\frac{1}{6}$

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