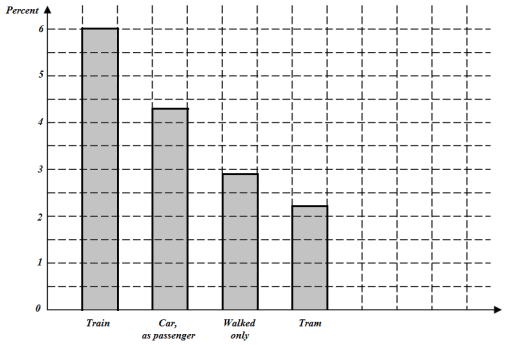
2013 Further Mathematics Trial Exam 2 Solutions

Core – Data analysis

Q1a $6.0\% + 2.2\% = 8.2\%$

Q1b Percentages do not total to 100% because some employed people use other means of transport not stated in the table.

Q1c



Q2a $a \approx 0.3$, $b \approx 19.0$ by CAS/graphics calculator

Q2b *Average annual increase* $\approx 0.3 \times 1000000 = 300000$

Q2c Year 2015 corresponds to $time = 15$.
Population $\approx 0.3 \times 15 + 19.0 = 23.5$ millions

Q3a $\frac{1}{N} = 1.286R - 1.098$ by CAS/graphics calculator

Q3b $r \approx 0.974$ by CAS/graphics calculator

Q3c $r^2 \approx 0.949$, $\therefore$ 95%

Q3d $\frac{1}{N} = 1.286 \times 0.88 - 1.098 \approx 0.03368$, $\therefore$ $N \approx 30$ thousands

Q4a

	summer	autumn	winter	spring	yr. av.	sp rain/yr av
2010	188	253	259	265	241.25	1.098
2011	207	249	252	259	241.75	1.071
2012	179	257	263	271	242.50	1.118

$$S.I. = \frac{1.098 + 1.071 + 1.118}{3} \approx 1.096$$

Q4b The rainfall in spring is about 10% higher than the yearly average over the period 2010 to 2012.

Q4c *Deseasonalised spring rainfall in 2011* $= \frac{259}{1.096} \approx 236$ mm

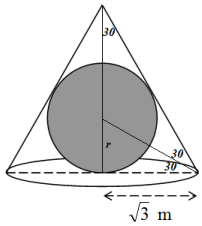
Module 2: Geometry and trigonometry

Q1a $AB \,//\, DC$, $\angle ABD = \angle BDC$ and $\angle BAC = \angle DCA$ because alternate angles are equal. Also, $\angle AOB = \angle COD = 90°$.
Three pairs of equal angles, $\therefore$ ΔOAB and ΔOCD are similar.

Q1bi Since $DC = 2 \times AB$, $\therefore$ $OC = 2 \times OA = 2a$ and $OD = 2 \times OB = 2b$
For ΔOAD, $a^2 + (2b)^2 = 139^2$, $\therefore$ $a^2 + 4b^2 = 19321$
For ΔOCB, $(2a)^2 + b^2 = 178^2$, $\therefore$ $4a^2 + b^2 = 31684$

Q1bii Add the 2 equations: $5a^2 + 5b^2 = 51005$
$\therefore$ $a^2 + b^2 = 10201$, $AB = \sqrt{a^2 + b^2} = \sqrt{10201} = 101$ m

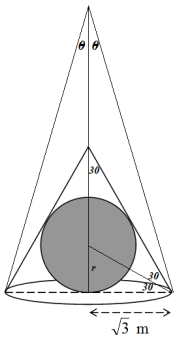
Q2a



$\frac{r}{\sqrt{3}} = \tan 30°$, $\frac{r}{\sqrt{3}} = \frac{1}{\sqrt{3}}$, $\therefore$ $r = 1$ m

Q2b All three dimensions are halved.
$\frac{V_{second}}{V_{first}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Q2c



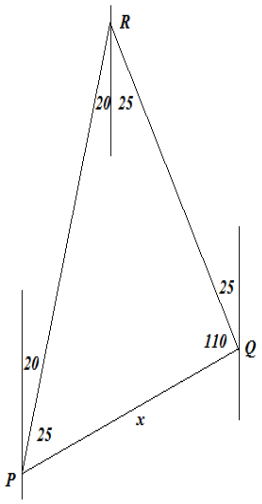
Original height $= \sqrt{3} \tan 60° = 3$, *new height* $= 3 \times 2 = 6$

$\tan\theta = \frac{\sqrt{3}}{6}$, $\theta \approx 16°$, $\therefore$ vertex angle $\approx 16° \times 2 = 32°$

Q3a *The third angle of the shade* $= 180° - 60° - 70° = 50°$
Let x be the length of the edge of the shade fastened to the wall.
$\frac{x}{\sin 50°} = \frac{6}{\sin 60°}$, $x \approx 5.31$
Area of the shade $\approx \frac{1}{2} \times 6 \times 5.31 \sin 70° \approx 15\,\text{m}^2$

Q3b *The length of the altitude from the vertex (* $50°$ *) to the opposite side* $= 6 \sin 70°$
Let ϕ be the angle between the shade and the wall.
$\cos\phi = \frac{2.5}{6 \sin 70°}$, $\phi \approx 64°$

Q4a



The sine rule: $\frac{QR}{\sin 25°} = \frac{x}{\sin 45°}$, $QR \approx 0.598x$

Q4b The sine rule: $\frac{PR}{\sin 110°} = \frac{x}{\sin 45°}$, $PR \approx 1.329x$

Let θ be the angle of elevation from P.

$\tan\theta = \frac{h}{PR} = \frac{QR\tan 31°}{PR} \approx \frac{0.598x\tan 31°}{1.329x} \approx 0.270$, $\therefore\ \theta \approx 15°$

Q4c $h \approx 0.598\times 30\tan 31° \approx 11\,\text{m}$

Module 3: Graphs and relations

Q1a $A = 18.2\times 1000 = 18200$ dollars

Q1b *Tax on each dollar* $= \frac{54.547 - 17.547}{180 - 80} = 0.37$, i.e. 37 cents

Q1c $T = 0.37(I - 80000) + 17547$ for $80000 \le I \le 180000$

$\therefore\ T = 0.37I - 12053$

Q1d When $I = 87000$,

$T = 0.37\times 87000 - 12053 = 20137$ dollars

Q2a $1608 - 263 = 1345$ m

Q2b Johnny passes Jenny's home at *time* ≈ 2.3 min

Johnny and Jenny are side by side at *time* $= 12$ min

Johnny is ahead of Jenny when $2.3 < time < 12$

Duration $\approx 12 - 2.3 = 9.7$, i.e. 10 min

Q2c The gradient turns steeper after *time* ≈ 12 min

Q2d *Johnny's average speed = the gradient of the line segment joining* $(0,0)$ *and* $(25,1608) = \frac{1608 - 0}{25 - 0} \approx 64$ m per min.

Q3a *Gradient* $= 1$, $\therefore\ I = x^{-2}$

Q3b $6 = x^{-2}$, $x^2 = \frac{1}{6}$, $x \approx 0.4$ m

Q4a $1.8x + 1.2y \le 38$, $1.4x + 1.6y \le 45$

Q4b $P = 35x + 33y$

Q4c The point giving maximum weekly profit is $(7,21)$.

$P_{\max} = 35\times 7 + 33\times 21 = 938$ dollars

Module 4: Business-related mathematics

Q1a $I = 25000\times\frac{3.25}{100}\times 0.5 = 406.25$,

$P = 25000 + 406.25 = 25406.25$ dollars

Q1b $I = 25406.25\times\frac{3.25}{100}\times 0.5 = 412.85$,

$P = 25406.25 + 412.85 = 25819.10$ dollars

Q2a *Monthly instalment* $= 350000\times\frac{6.50}{100}\times\frac{1}{12} = 1895.83$ dollars

Q2b *Monthly instalment* $= 350000\times\frac{6.00}{100}\times\frac{1}{12} = 1750.00$ dollars

Reduction $= 1895.83 - 1750.00 = 145.83$ dollars

Q2c By tvm solver, *number of monthly instalments* ≈ 243

Q3a By tvm solver, *monthly instalment* ≈ 601.87 dollars

Q3b By tvm solver, *number of months* ≈ 120 or 10 years, i.e. 60 years old.

Q4a *Loan establishment fee* $= 1200\times\frac{20}{100} = 240$ dollars

Six monthly loan fees $= 1200\times\frac{4}{100}\times 6 = 288$ dollars

Total owing $= 1200 + 240 + 288 = 1728$ dollars

Monthly repayment $= \frac{1728}{6} = 288$ dollars

Q4b *Total of fees* $= 240 + 288 = 528$ dollars

Let x% be the annual rate.

$1200\times\frac{x}{100}\times 0.5 = 528$, $x = 88$, $\therefore$ 88% p.a.

Q4c *Amount of extra fees* $= 35 + 7.00\times 7 = 84$ dollars

Q5a *% rise* $= \frac{102.8 - 100.4}{100.4}\times 100\% \approx 2.39\% \approx 2.4\%$

Q5b Let \$$x$ be the family's take home pay in June 2013.

$\frac{x - 2800}{2800}\times 100\% = 2.39\%$, $\therefore\ x \approx 2867$ dollars

Q6a *Depreciated value* $= 28000\times 0.80^3 = 14336$ dollars

Q6b Let x% be the flat rate of depreciation.

$28000\times\frac{x}{100}\times 3 = 14336$, $x \approx 17$, i.e. 17%

Q6c *Total distance* $= 20000\times 3 = 60000$ km

Depreciation $= \frac{14336}{60000} \approx 0.24$ dollars per km

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors