2013 NSW BOS General Mathematics Solutions

Section I

1	2	3	4	5	6	7	8	9
C	A	D	B	A	D	B	C	A

10	11	12	13	14	15	16	17	18
A	C/D	B	B	C	C	B	C	D

19	20	21	22	23	24	25	-	-
B	A	A	D	A	C	D		

Q1 $\Pr(H)=\frac{1}{2}$; $\Pr(3)=\frac{1}{6}$; $\Pr(G)=\frac{1}{26}$; $\Pr(first.prize)=\frac{4}{100}=\frac{1}{25}$ C

Q2 A

Q3 $Pay=\$24.80(14+1.5\times4+2\times3.5)+\$50=\$719.60$ D

Q4 $\theta=\tan^{-1}\left(\frac{81}{108}\right)\approx37°$ B

Q5 $\frac{12T^2}{3T\times2W}=\frac{(6T)(2T)}{(6T)W}=\frac{2T}{W}$ A

Q6 D

Q7 Each number is expected to occur $np=72\times\frac{1}{6}=12$ times. B

Q8 C

Q9 $I=P\times r\times n=\$1000\times\frac{0.0375}{12}\times15=\$46.875$

$A=\$1000+\$46.875\approx\$1046.88$ A

Q10 $\Pr(male\cap with.parents)=\frac{155}{505}\approx0.31=31\%$ A

Q11 $Gross\ annual\ salary=\$45000\times(1+0.06)+\$7500=\$55200$

$Gross\ monthly\ pay=\frac{\$55200}{12}=\$4600.00$ C

Another interpretation:

$Gross\ annual\ salary=(\$45000+\$7500)\times(1+0.06)=\$55650$

$Gross\ monthly\ pay=\frac{\$55650}{12}=\$4637.50$ D

Q12 $Height\ of\ the\ pyramid=19-9=10$

$Volume\ of\ solid=volume\ of\ cube+volume\ of\ pyramid$

$=9^3+\frac{1}{3}\times9^2\times10=999\ \text{cm}^3$ B

Q13 $Interest=\$12000-\$11000=\$1000$

$I=P\times r\times n$, $\$1000=\$11000\times r\times2$, $r\approx0.0455=4.55\%$ B

Q14 C

Q15 $Mean=\frac{3\times3+4\times7+5\times5+6\times1+8\times4}{20}=5$ goals per game C

Q16 $Area\ of\ shaded\ region=area\ of\ quadrant-area\ of\ rectangle$

$=\frac{1}{4}\times\pi\times9^2-6\times2\approx52\ \text{cm}^2$ B

Q17 $\frac{x}{y}=\frac{15}{10}$, $x=y\times\frac{15}{10}$ C

Q18 $\Pr(sum=6)=\Pr((1,5),(5,1),(2,4),(4,2),(3,3))=\frac{5}{36}$ D

Q19

$Area\ of\ logo=area\ of\ large\ semicircle-area\ of\ small\ semicircle$

$=\frac{1}{2}\pi5^2-\frac{1}{2}\pi2^2\approx33\ \text{cm}^2$ B

Q20 $Score\ between\ 1\ and\ 2\ standard\ deviations\ above\ the\ mean$

$=\frac{95\%-68\%}{2}=13.5\%$

$Number\ of\ students=60000\times0.135=8100$ A

Q21

$S=800(1-r)$, $S=800-800r$, $800r=800-S$, $r=\frac{800-S}{800}$ A

Q22 $Length=20-2x$, $A=x(20-2x)=20x-2x^2\ \text{m}^2$ D

Q23 $Interest\ rate=\frac{3\%}{2}=1.5\%=0.015$ per 6 month period

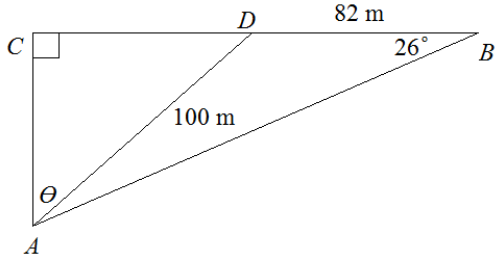
End of first 6 months, $amount=\$1200$

End of second 6 months, $amount=\$1200+\$1200\times1.015=\$2418$

End of third 6 months, $amount=\$1200+\$2418\times1.015=\$3654.27$

End of 4th 6 months, $amount=\$1200+\$3654.27\times1.015=\$4909.08$ A

Q24



$\frac{\sin\angle BAD}{82}=\frac{\sin26°}{100}$, $\sin\angle BAD=\frac{82\times\sin26°}{100}$, $\angle BAD\approx21°$

$\therefore\ \angle ADC\approx26°+21°=47°$, $\therefore\ \theta\approx90°-47°=43°$ C

Q25 The solid is a prism with the trapezium as its cross-section.

Volume $=\frac{1}{2}\times(5+11)\times8\times9=576\ \text{cm}^3$ D

Section II

Q26a $98^2 = 53^2 + 66^2 - 2(53)(66)\cos Q$

$$Q = \cos^{-1}\left(\frac{53^2 + 66^2 - 98^2}{2(53)(66)}\right) \approx 110°$$

Q26b 12, 12, 12, 12, 12, 24

Q26ci The commentator is incorrect because $\frac{9}{40}$ is the probability that Michael will score **less than or equal to** 100 points.

Q26cii $\Pr(SS) = \frac{31}{40} \times \frac{31}{40} = \frac{961}{1600} \approx 0.6$

Q26di $A = 560.9 + 523.5 = 1084.4$

Q26dii *Amount saved* $= \$(0.477700 - 0.096000) \times 154 = \$58.78$

Q26e *Compounded value* $= \$3500 \times 1.172 = \$4102$

Q26fi 52 minutes

Q26fii $Median = \frac{50 + 51}{2} = 50.5$ minutes

Q26fiii
With tolls: The set of data has a greater spread and it is positively skewed.
Without tolls: The set of data has a smaller spread and it is almost symmetric.

Q27ai *Total distance* $= 35 + 35 = 70$ km

Q27aii *Riding time* $= 2 + 1.5 + 2 + 1 = 6.5$ hours

Q27b *Taxable income* $= \$84000 - \$1000 - \$500 = \$82500.00$
Medicare levy $= \$82500 \times 0.015 = \$1237.50$
Tax payable $= \$17547 + \$0.37 \times (82500 - 80000) = \$18472.00$
Already paid $= \$18500.00$
Amount owed $= \$1237.50 + \$18472.00 - \$18500.00 = \$1209.50$

Q27ci $Q_1 = 190$, $Q_3 = 470$, *interquartile range* $= 470 - 190 = 280$

Q27cii Yes, Oscar is correct, generally speaking. $28 - 22 = 6$ weeks for the class centred at 300.
Alternative answer: No, Oscar is incorrect, strictly speaking. In the 6 weeks, the number of televisions sold each week was in the range of 250 to 350.

Q27di *Length* $= 25.0 \pm 0.5$ cm, *%error* $= \frac{0.5}{25.0} = 2\%$

Q27dii *Lower limit* $= 16.5 \times 24.5 = 404.25$ cm^2
Upper limit $= 17.5 \times 25.5 = 446.25$ cm^2
The actual area lies between 404.25 and 446.25 cm^2.

Q27ei Athens is 7 hours ahead of New York.
∴ 10 pm Tuesday Athens time is 3 pm Tuesday New York time.

Q27eii 9 am Wednesday New York time is 4 pm Wednesday Athens time. 11 hours later it will be 3 am Thursday Athens time.

Q28a $\angle AOB = 50° + 21° = 71°$

Area of $\Delta AOB = \frac{1}{2}(75)(60)\sin 71° \approx 2127$ m^2

Q28bi *Gradient* $= \frac{176 - 146}{16 - 11} = \frac{30}{5} = 6$

Q28bii The height of males aged 11-16 years increases by 6 cm on average every year.

Q28biii $h = 6a + constant$, and $a = 11$, $h = 146$
∴ *constant* $= 146 - 6 \times 11 = 80$
∴ $h = 6a + 80$

Q28biv At $a = 17$, $h = 6 \times 17 + 80 = 182$ cm

Q28bv Growth rate of adults is different from that of teenagers.

Q28c $\Delta\theta = 47° - 13° = 34°$, *distance* $= \frac{34}{360} \times 2\pi(6400) \approx 3798$ km

Q28di *Current value* $= \$1.50 \times 2000 = \$3000.00$

Q28dii *Dividend yield* $= \frac{0.30}{1.50} = 0.2 = 20\%$

Q28e Method 1: *Depreciation* $= \$1250 \times 2 = \$2500$
Method 2: *Depreciated value* $= \$5000 \times (1 - 0.35)^2 = \$2112.50$
Depreciation $= \$5000.00 - \$2112.50 = \$2887.50$
∴ Method 2 gives the greatest depreciation over the two years.

Q29a

$$\frac{W + 4}{3} - \frac{2W - 1}{5} = 1 \quad \text{..... Line 1}$$
$$5W + 20 - 6W + 3 = 15 \quad \text{..... Line 2}$$
$$23 - W = 15 \quad \text{..... Line 3}$$
$$W = 8 \quad \text{..... Line 4}$$

Q29bi First test: $mean = 68.5$, $sd \approx 5.5$

Q29bii Second test: $mean = 74.4$, $sd = 12.4$
For the score 62 on the first test, $z \approx \frac{62 - 68.5}{5.5} \approx -1.1818$
Let x be the mark required in the second test.
$z = \frac{x - 74.4}{12.4} \approx -1.1818$, ∴ $x \approx 60$

Q29c $26^2 \times 10^n \geq 3000000$, $n \geq 3.6$, ∴ *minimum* $n = 4$

Q29d Expected win/loss $= \$6\times\frac{1}{4}+\$1\times\frac{1}{2}+\$2\times\frac{1}{4}-\$4=-\$1.50$

Jane will expect a loss of \$1.50.

Q29e

$P=\$300000$, $r=\frac{0.06}{12}=0.005$, $M=\$1798.65+\$250=\$2048.65$

Let $P(1+r)^n-M\left(\frac{(1+r)^n-1}{r}\right)=0$

$\therefore\ 300000(1.005)^n-2048.65\left(\frac{1.005^n-1}{0.005}\right)=0$, $1.005^n\approx 3.7340$

$\therefore\ n>264$, i.e. more than 22 years

∴ Jack will not be able to pay off the loan in 20 years.

Q30ai $T=20000w^3$

When $w=7.3$ m/s, $T=20000(7.3)^3=7780340\approx 7.780\times10^6$ watts

Q30aii $A=40\%T$, $A=0.40\times20000w^3$, $A=0.40\times20000w^3$

$\therefore\ A=8000w^3$

Q30aiii When $w=9$ m/s, *difference*

$=T-A=(20000-8000)w^3=12000\times9^3=8.748\times10^6$ watts

Q30aiv $A\geq 4.4\times10^6$ watts, $\therefore\ 8000w^3\geq 4.4\times10^6$, $w^3\geq 550$

$\therefore\ w\geq 8.2$ m/s approximately

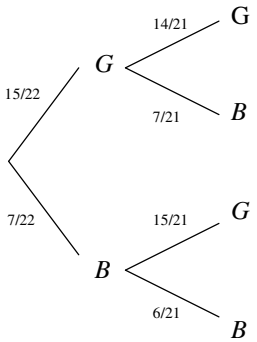
Q30av $P=0.61\times\pi\times r^2\times w^3$ where $r=43$

When $w=8$, $P\approx 1814206$; when $w=8.8$, $P\approx 2414708$

$\therefore\ \Delta P\approx 2414708-1814206=600502$

$\therefore$ *percentage increase* $\approx\frac{600502}{1814206}\approx 0.33=33\%$

Q30bi



Q30bii $\Pr(same.gender)=\Pr(GG)+\Pr(BB)$

$=\frac{15}{22}\times\frac{14}{21}+\frac{7}{22}\times\frac{6}{21}=\frac{6}{11}$

Q30ci For the leaf blower, *petrol* : *oil* = 40 : 1

$\therefore\ \frac{5}{oil}=\frac{40}{1}$, $oil=\frac{1}{8}=0.125$ L, i.e. 125 mL

Q30cii For the leaf blower, 4.1 litres of fuel contain $4.1\times\frac{40}{41}=4$ L of petrol and 0.1 L of oil.

For the lawnmower, *petrol* : *oil* = 25 : 1

$\therefore\ \frac{4}{oil}=\frac{25}{1}$, $oil=\frac{4}{25}=0.16$ L

∴ 0.06 L, i.e. 60 mL of oil must be added.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.