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2013 NSW BOS Mathematics Extension 2 Solutions

Section I

1	2	3	4	5	6	7	8	9	10
B	A	D	A	B	D	C	C	B	A

Q1 $\int \tan x\ dx = \int \frac{\sin x}{\cos x}\ dx = \int \frac{-du}{u} = -\ln u + c = -\ln(\cos x) + c$

B

Q2 $4x^2 - 25y^2 = 100$, $\frac{x^2}{25} - \frac{y^2}{4} = 1$, $a^2 = 25$, $b^2 = 4$

$c = \sqrt{a^2 + b^2} = \sqrt{25+4} = \sqrt{29}$, $e = \frac{c}{a}$

Directrices are $x = \pm\frac{a}{e}$, .: $x = \pm\frac{a^2}{c} = \pm\frac{25}{\sqrt{29}}$

A

Q3 Let $z = rcis\theta$, .: $z^2 = r^2 cis 2\theta$.

From the given Argand diagram, $0 < r < 1$ and $0 < \theta < \frac{\pi}{4}$.

.: $0 < r^2 < r < 1$ and $0 < 2\theta < \frac{\pi}{2}$

D

Q4 $P(x)$ has roots α, β and γ. If $P(x)$ is translated 1 unit to the right, the roots become $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.
The transformed polynomial is
$P(x-1) = 4(x-1)^3 + (x-1)^2 - 3(x-1) + 5 = 4x^3 - 11x^2 + 7x + 5$

A

Q5 $|z-1| = \frac{\pi}{4}$ and $|z-1| = \frac{\pi}{3}$ are concentric circles centred at $(1,0)$. Their radii are $\frac{\pi}{4}$ and $\frac{\pi}{3}$ respectively.

.: $\frac{\pi}{4} \le |z-1| \le \frac{\pi}{3}$ consists of the two circles and the region enclosed by them.

B

Q6 $\int \frac{1}{\sqrt{x^2 - 6x + 5}} dx = \int \frac{1}{\sqrt{(x-3)^2 - 4}} dx$

$= \ln\left(x - 3 + \sqrt{(x-3)^2 - 4} \right) + c$

D

Q7 $\omega = 10$ revolutions per minute $= 10 \times 2\pi$ per minute
$v = r\omega = 5 \times 10 \times 2\pi = 100\pi$ cm per minute

C

Q8 The length of the square at x is $2y = 2\sqrt{16 - x^2}$,

.: area of the square $= (2y)^2 = 4(16 - x^2)$

.: volume of the solid $= \int_{-4}^{4} 4(16 - x^2) dx$

C

Q9 As $x \to 0^-$, $y \to 1$; as $x \to 0^+$, $y \to 1$

B

Q10 6 people in 4 rooms with no restrictions: 4^6 ways

6 people in 4 rooms with a room of 6: ${}^4C_1$ ways

6 people in 4 rooms with a room of 5: ${}^4C_1 \times {}^3C_1 \times {}^6C_1$ ways

.: 6 people in 4 rooms with a maximum of 4 in a room:
$4^6 - {}^4C_1 - {}^4C_1 \times {}^3C_1 \times {}^6C_1 = 4020$

A

Section II

Q11ai $z + \overline{w} = (2 - i\sqrt{3}) + (1 - i\sqrt{3}) = 3 - i.2\sqrt{3}$

Q11aii $|w| = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ and $0 < \theta < \frac{\pi}{2}$

.: $w = 2cis\frac{\pi}{3}$

Q11aiii $w^{24} = 2^{24} cis\left(24 \times \frac{\pi}{3}\right) = 2^{24} cis 8\pi = 2^{24}$

Q11b $x^2 + 8x + 11 = A(x^2 + 2) + (Bx + C)(x - 3)$
.: $A + B = 1$, $C - 3B = 8$ and $2A - 3C = 11$
.: $A = 4$, $B = -3$ and $C = -1$

Q11c $z^2 + 4iz + 5 = z^2 + 4iz - 5i^2 = (z + 5i)(z - i)$

Q11d $\int_0^1 x^3\sqrt{1-x^2}dx = \int_0^1 x^2\sqrt{1-x^2}\,xdx$

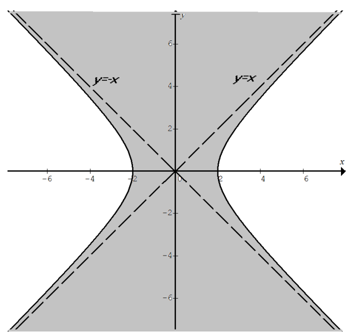
$= -\int_1^0 \frac{1}{2}(1-u)\sqrt{u}du = \int_0^1 \frac{1}{2}(1-u)u^{\frac{1}{2}}du$

$= \int_0^1 \frac{1}{2}\left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du = \left[\frac{u^{\frac{3}{2}}}{3} - \frac{u^{\frac{5}{2}}}{5}\right]_0^1 = \frac{2}{15}$

Let $u = 1 - x^2$
$x = 0$, $u = 1$
$x = 1$, $u = 0$
$\frac{du}{dx} = -2x$
$xdx = -\frac{1}{2}du$

Q11e $z^2 + \overline{z}^2 \le 8$, $(z + \overline{z})^2 - 2z\overline{z} \le 8$, $(2x)^2 - 2(x^2 + y^2) \le 8$

.: $x^2 - y^2 \le 4$, $\frac{x^2}{4} - \frac{y^2}{4} \le 1$



Q12a Let $t=\tan\frac{x}{2}$

$dt=\frac{1}{2}\sec^2\frac{x}{2}dx=\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx=\frac{1}{2}\left(1+t^2\right)dx$

$\cos x=2\cos^2\frac{x}{2}-1=\frac{2}{\sec^2\frac{x}{2}}-1=\frac{2}{1+t^2}-1=\frac{1-t^2}{1+t^2}$

.: $4+5\cos x=\frac{9-t^2}{1+t^2}$, $\frac{1}{4+5\cos x}=\frac{1+t^2}{(3-t)(3+t)}$

$x=0$, $t=0$; $x=\frac{\pi}{2}$, $t=1$

.: $\int_0^{\frac{\pi}{2}}\frac{1}{4+5\cos x}dx=\int_0^{\frac{\pi}{2}}\frac{\left(1+t^2\right)dx}{(3-t)(3+t)}=\int_0^1\frac{2}{(3-t)(3+t)}dt$

$=\frac{1}{3}\int_0^1\left(\frac{1}{3-t}+\frac{1}{3+t}\right)dt=\frac{1}{3}\left[\ln\frac{3+t}{3-t}\right]_0^1=\frac{1}{3}\ln 2$

Q12b $\frac{x}{50}-\log_e 3=\log_e y-\log_e(1000-y)$

$\frac{d}{dy}\left(\frac{x}{50}-\log_e 3\right)=\frac{d}{dy}\left(\log_e y-\log_e(1000-y)\right)$

$\frac{1}{50}\frac{dx}{dy}=\frac{1}{y}+\frac{1}{1000-y}=\frac{1000}{y(1000-y)}=\frac{1}{y\left(1-\frac{y}{1000}\right)}$

.: $\frac{dy}{dx}=\frac{y}{50}\left(1-\frac{y}{1000}\right)$

Q12c Translate 4 units left and reflect in the *y*-axis, $y=e^x\rightarrow y=e^{4-x}$, $x\in[1,3]$. Rotate the region about the *y*-axis to form the same solid.

The shell at *x* has volume $dV=2\pi xe^{4-x}dx$.

Volume of the solid $V=2\pi\int_1^3 xe^{4-x}dx$

Integration by parts:

$V=2\pi\left[-xe^{4-x}\right]_1^3+2\pi\int_1^3 e^{4-x}dx=2\pi\left[-(x+1)e^{4-x}\right]_1^3=4\pi\left(e^3-2e\right)$

Q12di $xy=c^2$, $y=\frac{c^2}{x}$, $\frac{dy}{dx}=-\frac{c^2}{x^2}$

At $P\left(cp,\frac{c}{p}\right)$, $\frac{dy}{dx}=-\frac{1}{p^2}$

Equation of the tangent at *P*: $y-\frac{c}{p}=-\frac{1}{p^2}(x-cp)$

.: $p^2y-cp=-x+cp$, .: $x+p^2y=2cp$

Q12dii $x+p^2y=2cp$

.: *x*-intercept $A(2cp,0)$ and *y*-intercept $B\left(0,\frac{2c}{p}\right)$

.: midpoint of $\overline{AB}$ is $\left(cp,\frac{c}{p}\right)$ which is point *P*

Since $\angle AOB=90°$, .: *A*, *O* and *B* are on the circle with $\overline{AB}$ as a diameter and *P* the centre of the circle.

Q12diii By symmetry, $C(2cq,0)$

Gradient of $\overline{PQ}=\frac{\frac{c}{q}-\frac{c}{p}}{cq-cp}=-\frac{1}{pq}$

Gradient of $\overline{BC}=\frac{\frac{2c}{p}}{-2cq}=-\frac{1}{pq}$.: $\overline{BC}$ is parallel to $\overline{PQ}$.

Q13ai $I_n=\int_0^1\left(1-x^2\right)^{\frac{n}{2}}dx$, $n\ge 0$ is an integer

.: $I_{n-2}=\int_0^1\left(1-x^2\right)^{\frac{n-2}{2}}dx$ for $n\ge 2$

Integration by parts:

$I_n=\left[x\left(1-x^2\right)^{\frac{n}{2}}\right]_0^1+n\int_0^1 x^2\left(1-x^2\right)^{\frac{n-2}{2}}dx=0+n\int_0^1 x^2\left(1-x^2\right)^{\frac{n-2}{2}}dx$

$=n\int_0^1\left(\left(1-x^2\right)^{\frac{n-2}{2}}-\left(1-x^2\right)\left(1-x^2\right)^{\frac{n-2}{2}}\right)dx$

$=n\int_0^1\left(1-x^2\right)^{\frac{n-2}{2}}dx-n\int_0^1\left(1-x^2\right)\left(1-x^2\right)^{\frac{n-2}{2}}dx$

$=n\int_0^1\left(1-x^2\right)^{\frac{n-2}{2}}dx-n\int_0^1\left(1-x^2\right)^{\frac{n}{2}}dx$

.: $I_n=nI_{n-2}-nI_n$

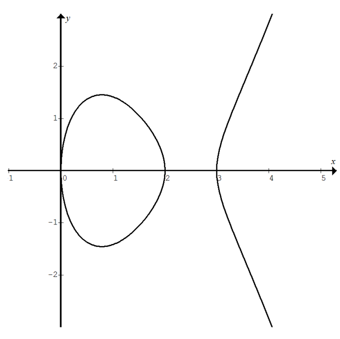
.: $I_n=\frac{n}{n+1}I_{n-2}$ for $n\ge 2$

Q13aii $I_1=\int_0^1\left(1-x^2\right)^{\frac{1}{2}}dx$, $y=\left(1-x^2\right)^{\frac{1}{2}}$ and $0\le x\le 1$ is a quarter of the unit circle centred at *O*.

.: $I_1=\frac{1}{4}\times\pi 1^2=\frac{\pi}{4}$.: $I_5=\frac{5}{6}I_3=\frac{5}{6}\times\frac{3}{4}I_1=\frac{5}{8}I_1=\frac{5\pi}{32}$

Q13bi $y^2=f(x)$, $y=\pm\sqrt{f(x)}$ is defined for $f(x)\ge 0$, i.e. $0\le x\le 2$ or $x\ge 3$. Also, $2y\frac{dy}{dx}=f'(x)$, $\frac{dy}{dx}=\frac{1}{2y}f'(x)$

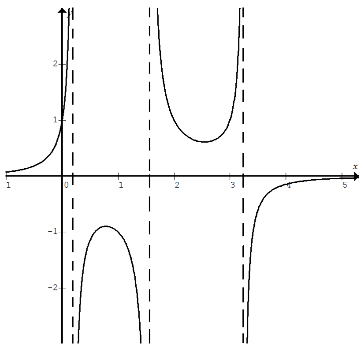
.: as $x\rightarrow x$-intercepts, $y\rightarrow 0$, $\frac{dy}{dx}\rightarrow\infty$



Q13bii $1-f(x)=0$ at $0<x<1$, $1<x<2$ and $x>3$

$\therefore$ $y=\dfrac{1}{1-f(x)}$ has vertical asymptotes at $0<x<1$, $1<x<2$ and $x>3$

Also, at $x=0$, $y=\dfrac{1}{1-f(0)}=1$



Q13ci $ABCD$ is a cyclic quadrilateral,
$\therefore$ $\angle ABC=\pi-\angle ADC=\alpha+\beta$
Consider ΔABC. Since AB is a diameter, $\therefore$ $\angle ACB=90°$
$\therefore$ $\overline{AC}=2r\sin\angle ABC=2r\sin(\alpha+\beta)$

Q13cii Consider ΔABD, $\angle ADB=90°$, $\therefore$ $\angle ABD=\angle ACD=\beta$
$\therefore$ $\overline{AD}=2r\sin\angle ABD=2r\sin\beta$
Consider ΔAED, $\overline{AE}=\overline{AD}\cos\alpha=2r\sin\beta\cos\alpha$

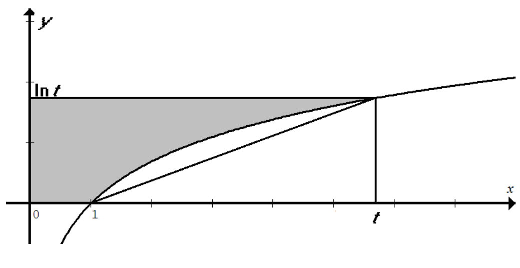
Q13ciii $\overline{ED}=\overline{AD}\sin\alpha=2r\sin\beta\sin\alpha$

Consider ΔCDE, $\dfrac{\overline{ED}}{\overline{EC}}=\tan\beta=\dfrac{\sin\beta}{\cos\beta}$

$\therefore$ $\overline{EC}=\dfrac{\overline{ED}\cos\beta}{\sin\beta}=2r\sin\alpha\cos\beta$

$\overline{AC}=\overline{AE}+\overline{EC}$, $\therefore$ $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$

Q14a



Area of the trapezium $>$ area of the shaded region on the left of $y=\ln x$

$\therefore$ $\dfrac{1}{2}(1+t)\ln t>\displaystyle\int_0^{\ln t}x\,dy$, $\dfrac{1}{2}(1+t)\ln t>\displaystyle\int_0^{\ln t}e^y\,dy$, $\dfrac{1}{2}(1+t)\ln t>\left[e^y\right]_0^{\ln t}$

$\therefore$ $\dfrac{1}{2}(1+t)\ln t>t-1$

$\therefore$ $\ln t>2\left(\dfrac{t-1}{t+1}\right)$ for $t>1$

Q14b $z_2=1+i$ and $z_n=z_{n-1}\left(1+\dfrac{i}{|z_{n-1}|}\right)$ for $n>2$

$|z_2|=\sqrt{2}$, $\therefore$ $|z_n|=\sqrt{n}$ is true for $n=2$

Assume that it is true for $n=k$, i.e. $|z_k|=\sqrt{k}$

$z_{k+1}=z_k\left(1+\dfrac{i}{|z_k|}\right)$, $\therefore$ $z_{k+1}=z_k\left(1+\dfrac{i}{\sqrt{k}}\right)$

$\therefore$ $|z_{k+1}|=|z_k|\left|1+\dfrac{i}{\sqrt{k}}\right|=\sqrt{k}\sqrt{1+\dfrac{1}{k}}=\sqrt{k+1}$

$\therefore$ $|z_n|=\sqrt{n}$ is also true for $n=k+1$

$\therefore$ $|z_n|=\sqrt{n}$ is true for all $n\ge 2$ by mathematical induction.

Q14ci $\sec^{2n}\theta=(\sec^2\theta)^n=(1+\tan^2\theta)^n$

$=\binom{n}{0}(1)^n(\tan^2\theta)^0+\binom{n}{1}(1)^{n-1}(\tan^2\theta)^1+\ldots+\binom{n}{n}(1)^0(\tan^2\theta)^n$

$=\binom{n}{0}\tan^0\theta+\binom{n}{1}\tan^2\theta+\ldots+\binom{n}{n}\tan^{2n}\theta$

$=\displaystyle\sum_{k=0}^{n}\binom{n}{k}\tan^{2k}\theta$

Q14cii $\displaystyle\int\sec^8\theta d\theta=\int\sec^6\theta\sec^2\theta d\theta$

$=\displaystyle\int\left(\binom{3}{0}+\binom{3}{1}\tan^2\theta+\binom{3}{2}\tan^4\theta+\binom{3}{3}\tan^6\theta\right)\sec^2\theta d\theta$

$=\displaystyle\int\left(1+3\tan^2\theta+3\tan^4\theta+\tan^6\theta\right)\sec^2\theta d\theta$

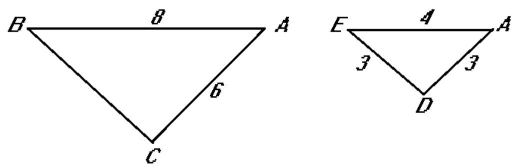
$=\displaystyle\int\left(1+3u^2+3u^4+u^6\right)du$

Let $u=\tan\theta$
$du=\sec^2\theta d\theta$

$=u+u^3+\dfrac{3u^5}{5}+\dfrac{u^7}{7}+c$

$=\tan\theta+\tan^3\theta+\dfrac{3\tan^5\theta}{5}+\dfrac{\tan^7\theta}{7}+c$

Q14di



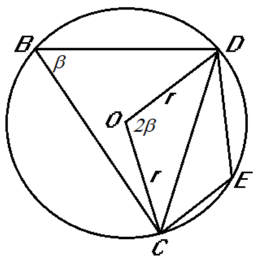
The two triangles have 2 pairs of corresponding sides with the same ratio 2:1 and a common angle *A*.
$\therefore$ ΔABC and ΔAED are similar.

Q14dii Corresponding angles are equal in similar triangles.
$\therefore$ $\angle ABC=\angle AED$
$\therefore$ $\angle ABC+\angle DEC=\angle AED+\angle DEC=\pi$
$\therefore$ $BCED$ is a cyclic quadrilateral.

Q14diii $\overline{BC}=6$ (similar Δs), $\cos\angle ABC=\dfrac{6^2+8^2-6^2}{2\times6\times8}=\dfrac{2}{3}$

$\overline{CD}=\sqrt{5^2+6^2-2\times5\times6\times\dfrac{2}{3}}=\sqrt{21}$

Q14div Let $\beta = \angle DBC = \angle ABC$, .: $\cos\beta = \frac{2}{3}$



Since $BCED$ is a cyclic quadrilateral, the angle at the centre O of the circle is double the angle on the circumference, i.e. 2β .

.: $\cos 2\beta = 2\cos^2\beta - 1 = -\frac{1}{9}$

Consider ΔCOD , $\overline{CD}^2 = r^2 + r^2 - 2r^2\cos 2\beta$

.: $21 = r^2 + r^2 - 2r^2\left(-\frac{1}{9}\right)$, .: $r = \frac{3\sqrt{105}}{10}$

Q15a $A = \frac{1}{2}|z||w|\sin(\theta - \phi)$ where $\phi < \theta$

$z = |z| cis\theta$, $\bar{z} = |z| cis(-\theta)$; $w = |w| cis\phi$, $\bar{w} = |w| cis(-\phi)$

.: $z\bar{w} - w\bar{z} = |z||w| cis(\theta - \phi) - |z||w| cis(\phi - \theta)$

$= |z||w|(cis(\theta - \phi) - cis(-(\theta - \phi))) = i.2|z||w|\sin(\theta - \phi)$

$= 4i\frac{1}{2}|z||w|\sin(\theta - \phi) = 4iA$

Q15bi $P(x) = ax^4 + bx^3 + cx^2 + e$, $P'(x) = 4ax^3 + 3bx^2 + 2cx$

$P(1) = a + b + c + e = -3$ …… (1)

$P(-1) = a - b + c + e = 0$ …… (2)

$P'(-1) = -4a + 3b - 2c = 0$ … (3)

(1) – (2): $b = -\frac{3}{2}$

Substitute in (3): $4a + 2c = -\frac{9}{2}$

Q15bii When $x = 1$, $P'(1) = 4a + 2c + 3b = -\frac{9}{2} + 3\left(-\frac{3}{2}\right) = -9$

Q15ci $\Pr(4 days) = 0.7^4 = 0.2401$

Q15cii Binomial: $n = 8$, $p \approx 0.24$, $q \approx 0.76$

$\Pr(X \ge 3) = 1 - \Pr(0) - \Pr(1) - \Pr(2)$

$= 1 - \binom{8}{0}(0.24^0)(0.76^8) - \binom{8}{1}(0.24^1)(0.76^7) - \binom{8}{2}(0.24^2)(0.76^6)$

Q15di Falling: Taking the direction of motion as the positive direction, $m\dot{v} = mg - kv^2$, v_T is reached when $\dot{v} = 0$

.: $mg - kv_T{}^2 = 0$, .: terminal velocity $v_T = \sqrt{\frac{mg}{k}}$

Q15dii Going up: Taking the direction of motion as the positive direction, the equation of motion is $m\dot{v} = -(mg + kv^2)$.

.: $\frac{m}{k}\dot{v} = -\left(\frac{mg}{k} + v^2\right)$, $\frac{v_T{}^2}{2g}\frac{d(v^2)}{dx} = -(v_T{}^2 + v^2)$

.: $\int_0^H dx = -\frac{v_T{}^2}{2g}\int_{u^2}^0 \frac{1}{v_T{}^2 + v^2} d(v^2)$.: $[x]_0^H = -\frac{v_T{}^2}{2g}\left[\ln(v_T{}^2 + v^2)\right]_{u^2}^0$

.: $H = \frac{v_T{}^2}{2g}\ln\left(\frac{v_T{}^2 + u^2}{v_T{}^2}\right)$.: $H = \frac{v_T{}^2}{2g}\ln\left(1 + \frac{u^2}{v_T{}^2}\right)$

Q15diii Falling: Taking the direction of motion as the positive direction, $m\dot{v} = mg - kv^2$

.: $\frac{v_T{}^2}{2g}\frac{d(v^2)}{dx} = v_T{}^2 - v^2$, $\int_0^H dx = \frac{v_T{}^2}{2g}\int_0^{w^2}\frac{1}{v_T{}^2 - v^2} d(v^2)$

.: $[x]_0^H = \frac{v_T{}^2}{2g}\left[-\ln(v_T{}^2 - v^2)\right]_0^{w^2}$.: $H = \frac{v_T{}^2}{2g}\ln\left(\frac{v_T{}^2}{v_T{}^2 - w^2}\right)$

.: $\frac{v_T{}^2}{v_T{}^2 - w^2} = \frac{v_T{}^2 + u^2}{v_T{}^2}$

Expand and simplify to $\frac{1}{w^2} = \frac{1}{u^2} + \frac{1}{v_T{}^2}$

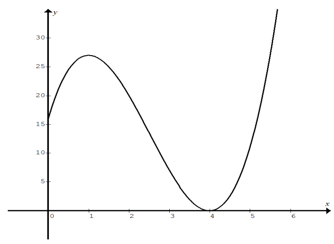
Q16ai $P(x) = 2x^3 - 15x^2 + 24x + 16$

$P'(x) = 6x^2 - 30x + 24 = 6(x-1)(x-4)$

.: Stationary points at $x = 1, 4$

For $x \ge 0$, the minimum occurs at $x = 4$

.: minimum value of $P(x) = P(4) = 0$



Q16aii Since the minimum value of $P(x) = 0$ for $x \ge 0$,

.: $P(x) \ge 0$ for $x \ge 0$

.: $2x^3 - 15x^2 + 24x + 16 \ge 0$, $2x^3 + 10x^2 + 24x + 16 \ge 25x^2$

$2x^3 + 10x^2 + 24x + 16$ is the expansion of $(x+1)(x^2 + (x+4)^2)$.

.: $(x+1)(x^2 + (x+4)^2) \ge 25x^2$

Q16aiii $x^2 + (x+4)^2 \ge \frac{25x^2}{x+1}$

Let $x = m + n$, where $m \ge 0$ and $n \ge 0$.

.: $(m+n)^2 + (m+n+4)^2 \ge \frac{25(m+n)^2}{m+n+1} = \frac{25(m^2 + 2mn + n^2)}{m+n+1}$

Since $(m-n)^2 \ge 0$, .: $m^2 - 2mn + n^2 \ge 0$, .: $m^2 + n^2 \ge 2mn$

.: $(m+n)^2 + (m+n+4)^2 \ge \frac{25(m^2 + 2mn + n^2)}{m+n+1} \ge \frac{25(2mn + 2mn)}{m+n+1}$

i.e. $(m+n)^2 + (m+n+4)^2 \ge \frac{100mn}{m+n+1}$

Q16bi $\overline{SP}+\overline{PS'}=2a$ is a constant, .: locus of P is an ellipse with foci S and S'.

Q16bii $\dfrac{\overline{SP}}{\overline{PM}}=e$, where M is a point on the directrix

.: $\overline{SP}=e\overline{PM}=e\left(\dfrac{a}{e}-a\cos\theta\right)=a(1-e\cos\theta)$

Q16biii $\overline{PS'}=2a-\overline{SP}=2a-a(1-e\cos\theta)=a(1+e\cos\theta)$

$\overline{QS'}=\overline{OS'}+\overline{OQ}=ae+a\cos\theta=a(e+\cos\theta)$

.: $\sin\beta=\dfrac{\overline{QS'}}{\overline{PS'}}=\dfrac{e+\cos\theta}{1+e\cos\theta}$

Q16biv Let α be $\angle QPS$.

Similarly, $\sin\alpha=\dfrac{\overline{SQ}}{\overline{SP}}=\dfrac{a(e-\cos\theta)}{a(1-e\cos\theta)}=\dfrac{e-\cos\theta}{1-e\cos\theta}$

Vertically: $T\sin\alpha+mg=T\sin\beta$

.: $mg=T(\sin\beta-\sin\alpha)=T\left(\dfrac{e+\cos\theta}{1+e\cos\theta}-\dfrac{e-\cos\theta}{1-e\cos\theta}\right)$, which can be simplified to $mg=\dfrac{2T(1-e^2)\cos\theta}{1-e^2\cos^2\theta}$ …… (A)

Q16bv Horizontally: $mr\omega^2=T(\cos\alpha+\cos\beta)$

.: $mr\omega^2=T\left(\dfrac{\overline{QP}}{\overline{SP}}+\dfrac{\overline{QP}}{\overline{PS'}}\right)$

and $\overline{QP}=\sqrt{\overline{SP}^2-\overline{SQ}^2}=a\sqrt{1-e^2}\sin\theta$

.: $mr\omega^2=T\times\overline{QP}\left(\dfrac{\overline{PS'}+\overline{SP}}{\overline{SP}\times\overline{PS'}}\right)$

.: $mr\omega^2=T\times a\sqrt{1-e^2}\sin\theta\left(\dfrac{2}{a(1-e^2\cos^2\theta)}\right)$

.: $mr\omega^2=\dfrac{2T\sqrt{1-e^2}\sin\theta}{1-e^2\cos^2\theta}$ …… (B)

Q16bvi $\dfrac{(B)}{(A)}$: $\dfrac{2T\sqrt{1-e^2}\sin\theta}{2T(1-e^2)\cos\theta}=\dfrac{mr\omega^2}{mg}$

.: $\dfrac{\tan\theta}{\sqrt{1-e^2}}=\dfrac{r\omega^2}{g}$ or $\tan\theta=\dfrac{r\omega^2}{g}\sqrt{1-e^2}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.

