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2013 NSW BOS Mathematics Exam Solutions

Section I

1	2	3	4	5	6	7	8	9	10
D	B	A	A	C	D	B	B	C	D

Q1 Use the quadratic formula: $a=2$, $b=-5$, $c=-1$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(-1)}}{2(2)}=\frac{5\pm\sqrt{33}}{4}$$ D

Q2 $Slope=\tan(180°-60°)=-\sqrt{3}$ B

Q3 $x+3>0$, $x>-3$ A

Q4 $\frac{d}{dx}\left(\frac{x}{\cos x}\right)=\frac{\cos x-x(-\sin x)}{\cos^2 x}=\frac{\cos x+x\sin x}{\cos^2 x}$ A

Q5 $\Pr(X\geq 1)=1-\Pr(X=0)=1-\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}=\frac{5}{6}$ C

Q6 $y=\sin\left(2x+\frac{\pi}{3}\right)=\sin 2\left(x+\frac{\pi}{6}\right)$ is a sine graph with a period of π translated by $\frac{\pi}{6}$ to the left. D

Q7 $\overline{PM}^2=\overline{PF}^2$, $(x-1)^2=(x-5)^2+(y-0)^2$, $y^2=8(x-3)$ B

Q8 The curve has a positive slope at B, .: $f'(x)>0$, and B is an inflection point, .: $f''(x)=0$. B

Q9 $5^x=4$, $\log_e 5^x=\log_e 4$, $x\log_e 5=\log_e 4$, $x=\frac{\log_e 4}{\log_e 5}$ C

Q10 Velocity $\dot{x}$ is negative, .: the particle is moving to the left. Acceleration $\ddot{x}$ is positive, i.e. opposite to $\dot{x}$, .: the particle is slowing down. D

Section II

Q11a $\ln 3=1.10$

Q11b $\lim_{x\to 2}\frac{x^3-8}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x^2+2x+4}{x+2}=3$

Q11c $\frac{d}{dx}(\sin x-1)^8=8\cos x(\sin x-1)^7$

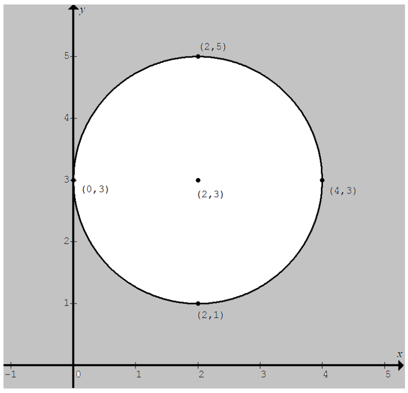
Q11d $\frac{d}{dx}x^2e^x=2xe^x+x^2e^x=x(2+x)e^x$

Q11e $\int e^{4x+1}dx=\frac{1}{4}e^{4x+1}+c$

Q11f Let $u=x^3+1$, $\frac{1}{3}du=x^2dx$. When $x=0$, $u=1$; when $x=1$, $u=2$.

$$\int_0^1\frac{x^2}{x^3+1}dx=\frac{1}{3}\int_1^2\frac{1}{u}du=\frac{1}{3}[\ln u]_1^2=\frac{1}{3}\ln 2$$

Q11g



Q12a $y=ax^3+bx^2+cx+d$, $y'=3ax^2+2bx+c$, $y''=6ax+2b$

At $x=p$, inflection point, $y''=0$, .: $0=6ap+2b$, .: $p=-\frac{b}{3a}$

Q12bi $A(-2,-1)$, $D(22,17)$, $\frac{y-(-1)}{x-(-2)}=\frac{17-(-1)}{22-(-2)}$

.: $\frac{y+1}{x+2}=\frac{18}{24}$, .: $3x-4y+2=0$

Q12bii Let h be the perpendicular distance from B to AD.

Area of parallelogram $=\overline{AD}\times h$

.: $(22-(-2))(42-17)=\sqrt{(22-(-2))^2+(17-(-1))^2}\times h$

$600=30h$, .: $h=20$ units

Q12biii $\overline{EC}=\sqrt{(22-18)^2+(42-39)^2}=5$ units

Q12biv $\overline{AD}=30$, $\overline{FD}=\frac{1}{2}\times 30=15$

Area of trapezium EFDC $=\frac{1}{2}\times(5+15)\times 20=200$ square units

Q12ci Kim: $t_1=30000$, $r=1+0.05=1.05$

$t_{10}=30000\times 1.05^9\approx 46539.85$ dollars

Alex: $t_1=33000$, $d=1500$

$t_{10}=33000+9\times 1500=46500.00$ dollars

Q12cii Kim: $S_{10}=\frac{30000(1.05^{10}-1)}{1.05-1}\approx 377336.77$ dollars

Q12ciii Alex's n year savings:

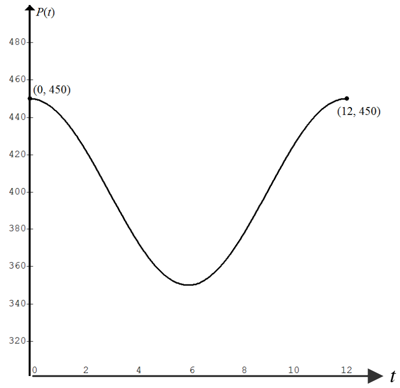
$\frac{1}{3}S_n=87500$, $\frac{1}{3}\times\frac{n}{2}(2\times 33000+(n-1)1500)=87500$ and $n>0$

$n^2+43n-350=0$, $(n-7)(n+50)=0$, .: $n=7$ years.

Q13ai $400+50\cos\left(\frac{\pi t}{6}\right)=375$ and $0\le t\le 12$, i.e. $0\le \frac{\pi t}{6}\le 2\pi$

.: $\cos\left(\frac{\pi t}{6}\right)=-\frac{1}{2}$, .: $\frac{\pi t}{6}=\frac{2\pi}{3}$, $\frac{4\pi}{3}$, .: $t=4$, 8

Q13aii



Q13bi $f(x)=g(x)$, $4x^3-4x^2+3x=2x$, $4x^3-4x^2+x=0$

$x(4x^2-4x+1)=0$, $x(2x-1)^2=0$, $x=0$, $\frac{1}{2}$

.: x-coordinate of T is $\frac{1}{2}$.

Q13bii $Area=\int_0^{\frac{1}{2}}\left(\left(4x^3-4x^2+3x\right)-(2x)\right)dx=\int_0^{\frac{1}{2}}\left(4x^3-4x^2+x\right)dx$

$=\left[x^4-\frac{4x^3}{3}+\frac{x^2}{2}\right]_0^{\frac{1}{2}}=\frac{1}{48}$

Q13c Let $\overline{CD}=x$ cm

Two similar sectors: $\left(\frac{x}{30}\right)^2=\frac{1}{2}$, $x^2=450$, $x=15\sqrt{2}$

Q13di $n=360$, $A_{360}=0$, $P=500000$, $r=1.005$

In the formula, $1+r+r^2+...+r^{n-1}=\frac{r^n-1}{r-1}$.

.: $A_n=P.r^n-M\left(\frac{r^n-1}{r-1}\right)$, $0=500000\times 1.005^{360}-M\left(\frac{1.005^{360}-1}{0.005}\right)$

.: $M\approx 2997.75\approx 2998$ dollars

Q13dii $n=240$, $P=500000$, $r=1.005$, $M=2998$

$A_{240}=500000\times 1.005^{240}-2998\left(\frac{1.005^{240}-1}{0.005}\right)$

$\approx 269903\approx 270000$ dollars

Q13diii $P=370000$, $r=1.005$, $M=2998$, $A_n=0$

$0=370000\times 1.005^n-2998\left(\frac{1.005^n-1}{0.005}\right)$.

Let $x=1.005^n$, $370000x-2998\left(\frac{x-1}{0.005}\right)=0$, .: $x\approx 2.6115$

$1.005^n\approx 2.6115$, $n\approx 192.5$ months, .: just over 16 years

Q14ai $\dot{x}=10-2t$, $a=\ddot{x}=-2$, a constant

Q14aii At rest: $\dot{x}=10-2t=0$, $t=5$

Q14aiii $\dot{x}=10-2t$, $x=\int(10-2t)dt=10t-t^2+c$

$t=0$, $x=+5$, .: $c=+5$, .: $x=10t-t^2+5$

When $t=7$, $x=+26$, i.e. 26 m to the right of the origin

Q14aiv $t=0\to 5$, $\Delta x=\int_0^5(10-2t)dt=+25$

$t=5\to 7$, $\Delta x=\int_5^7(10-2t)dt=-4$

Distance travelled $=25+4=29$ m

Q14bi $\overline{RB}=100-50t$, $\overline{RA}=80t$

$r^2=(80t)^2+(100-50t)^2-2(80t)(100-50t)\cos 60°$

$=12900t^2-18000t+10000$

Let $\frac{d(r^2)}{dt}=25800t-18000=0$, $t=\frac{30}{43}$

.: $r_{\min}=\sqrt{12900\left(\frac{30}{43}\right)^2-18000\left(\frac{30}{43}\right)+10000}\approx 61$ km

Q14c $\overline{BC}=\frac{4}{\tan\frac{\pi}{6}}=4\sqrt{3}$, $\overline{AC}=\sqrt{13^2-\left(4\sqrt{3}\right)^2}=11$

.: $\overline{AD}=11-4=7$, $\overline{BD}=\frac{4}{\sin\frac{\pi}{6}}=8$, $\angle ADB=\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}$

The sine rule: $\frac{\sin x}{7}=\frac{\sin\frac{2\pi}{3}}{13}$, .: $\sin x=\frac{7\sqrt{3}}{26}$

Q14d $2\times(a-3)\times 1=3\times 1$, .: $a=\frac{9}{2}$

Q15ai Trapezoidal rule: $A\approx 2\times\frac{1}{2}(1.5+1.8)(1.2)=3.96\text{ m}^2$

Simpson's rule: $A\approx\frac{2.4}{6}(1.5+4\times 1.8+1.5)=4.08\text{ m}^2$

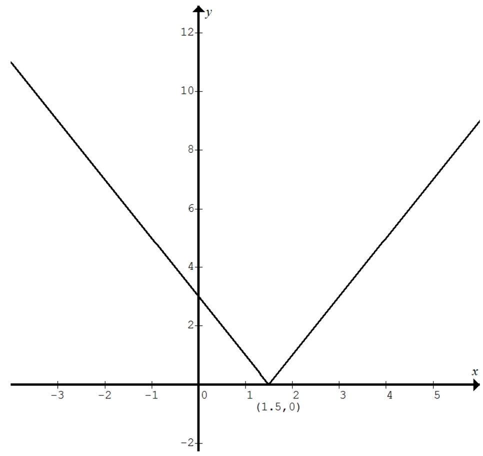
Q15aiii Simpson's rule uses an inverted parabola through the top points of the poles whilst the trapezoidal rule uses straight lines joining the points to estimate the area. In this case, Simpson's rule gives a greater estimation than the trapezoidal rule.

Q15b y-intercept: $x=0$, $y=4$

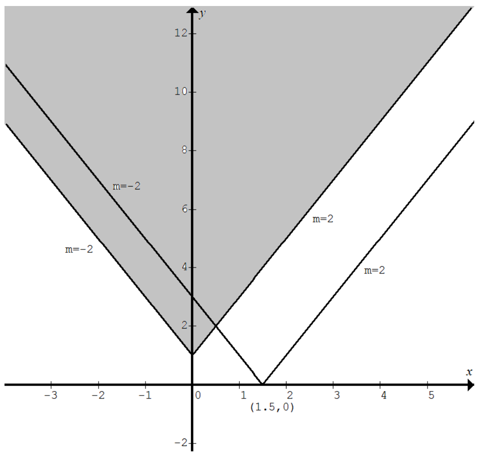
$V=\int_0^4\pi x^2dy=\pi\int_0^4\left(4-4y^{\frac{1}{2}}+y\right)dy$

$=\pi\left[4y-\frac{4y^{\frac{3}{2}}}{\frac{3}{2}}+\frac{y^2}{2}\right]_0^4=\frac{8\pi}{3}$ cubic units

Q15ci



Q15cii



To have exactly one solution, $y = mx + 1$ **either** passes through the shaded region (not including the boundary line $y = -2x + 1$),

.: $m < -2$ or $m \geq 2$,

or through the point $\left(\frac{3}{2}, 0\right)$ and .: $m = -\frac{2}{3}$.

Q15di Pat wins on the first throw: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Q15dii Pat wins on the second throw: $\left(1 - \frac{1}{36}\right)^2 \times \frac{1}{36}$

Pat wins on the first or second throw:

$$\frac{1}{36} + \left(1 - \frac{1}{36}\right)^2 \times \frac{1}{36} = \frac{1}{36} + \left(\frac{35}{36}\right)^2 \times \frac{1}{36}$$

Q15diii Pat eventually wins: $\frac{1}{36} + \left(\frac{35}{36}\right)^2 \times \frac{1}{36} + \left(\frac{35}{36}\right)^4 \times \frac{1}{36} + ...$

is an infinite geometric series, $S_\infty = \frac{\frac{1}{36}}{1 - \left(\frac{35}{36}\right)^2} = \frac{36}{71}$.

Q16a $f'(x) = 4x - 3 = 5$, .: $x = 2$ and $y = 5x - 7 = 3$

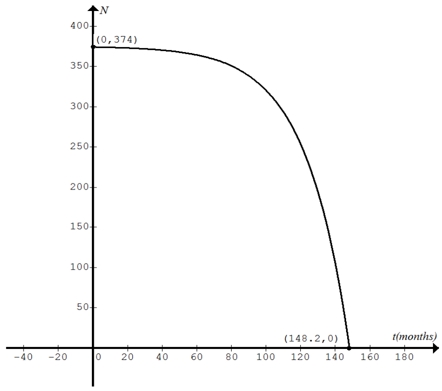
$y = f(x) = \int (4x - 3)dx = 2x^2 - 3x + c$

.: $3 = 2(2^2) - 3(2) + c$, .: $c = 1$, .: $f(x) = 2x^2 - 3x + 1$

Q16bi Trout: $N = 375 - e^{0.04t}$. When $t = 0$, $N = 374$

Q16bii $N = 0$ when $e^{0.04t} = 375$, i.e. $t = \frac{\ln 375}{0.04} \approx 148.2$ months

Q16biii



Q16biv Trout: $\frac{dN}{dt} = -0.04e^{0.04t}$

Carp: $\frac{dP}{dt} = 0.02P$, $\frac{dt}{dP} = \frac{1}{0.02}\frac{1}{P}$, $t = \frac{1}{0.02}\int \frac{1}{P}dP$

$0.02t = \ln P + c$ and $P = 10$ when $t = 0$, .: $c = -\ln 10$

.: $0.02t = \ln P - \ln 10$, $P = 10e^{0.02t}$, .: $\frac{dP}{dt} = 0.2e^{0.02t}$

Let $0.04e^{0.04t} = 0.2e^{0.02t}$

.: $\frac{e^{0.04t}}{e^{0.02t}} = 5$, $e^{0.02t} = 5$, $t \approx 80.5$ months

Q16bv Let $10e^{0.02t} = 375 - e^{0.04t}$.

$e^{0.04t} + 10e^{0.02t} - 375 = 0$, .: $\left(e^{0.02t}\right)^2 + 10e^{0.02t} - 375 = 0$

.: $\left(e^{0.02t} - 15\right)\left(e^{0.02t} + 25\right) = 0$

Since $e^{0.02t} + 25 \neq 0$, .: $e^{0.02t} - 15 = 0$, $t \approx 135.4$ months

Q16ci $XY \parallel BC$, AB, AC are transversals .: there are two pairs of corresponding angles ($\angle AYX = \angle ACB$ and $\angle AXY = \angle ABC$) and a common angle. .: ΔAXY and ΔABC are similar.

Q16cii Similarly, ΔBXY and ΔBAD are similar.

.: $\frac{\overline{XB}}{\overline{AB}} = \frac{\overline{XY}}{\overline{AD}}$ and $\frac{\overline{AX}}{\overline{AB}} = \frac{\overline{XY}}{\overline{BC}}$

.: $\frac{\overline{AX}}{\overline{AB}} + \frac{\overline{XB}}{\overline{AB}} = \frac{\overline{XY}}{\overline{AD}} + \frac{\overline{XY}}{\overline{BC}}$

.: $1 = \overline{XY}\left(\frac{1}{\overline{AD}} + \frac{1}{\overline{BC}}\right)$

.: $\frac{1}{\overline{XY}} = \frac{1}{\overline{AD}} + \frac{1}{\overline{BC}}$

Please inform mathline@itute.com re conceptual and/or mathematical errors.