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Specialist Mathematics

2013

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 Given $|z| = 5$, $|z - 4 + 4i| = 1$ and $Arg(z) = \theta$, $\tan\theta =$

A. -1

B. $-\frac{4}{3}$

C. $\frac{3}{4}$

D. 1

E. $\frac{4}{3}$

Question 2 Given $i^7 z = \frac{\pi}{2} cis\left(-\frac{2\pi}{3}\right)$, $Arg(z) =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{5\pi}{6}$

D. $-\frac{\pi}{3}$

E. $-\frac{\pi}{6}$

Question 3 Given a, b, $c \in R^+$, and $z \in C$ such that $2\text{Re}(z) = \text{Im}(z)$, the maximum number of z satisfying the equation $a|z|^2 + b|z| - c = 0$ is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 4 Given $z = rcis\theta$ and $w = z - r$, where $r \in R^{+}$, then $|w| =$

A. $r\cos\theta$

B. $r\sin\theta$

C. $2r\cos\dfrac{\theta}{2}$

D. $2r\sin\dfrac{\theta}{2}$

E. $2r$

Question 5 The asymptotes of a hyperbola make a $60°$ angle. A possible equation of the hyperbola is

A. $\dfrac{(x+2)^2}{4} - \dfrac{(y+6)^2}{2} = 1$

B. $\dfrac{x^2}{3} - (y-2)^2 = 1$

C. $\dfrac{(x-1)^2}{6} - \dfrac{(y-2)^2}{2} = 1$

D. $\dfrac{(x+2)^2}{4} - \dfrac{(y-2)^2}{9} = 1$

E. $(x-2)^2 - \dfrac{(y+2)^2}{9} = 1$

Question 6 The graph of $y = \dfrac{1}{x^2 - px + q}$, where $p, q \in R \setminus \{0\}$, has a turning point when

A. $p^2 > 4q$ only

B. $p^2 < 4q$ only

C. $p^2 = 4q$

D. $p^2 \neq 4q$

E. $p^2 \neq 2q$

Question 7 A sequence of transformations changing the ellipse $\frac{(x-2)^2}{4}+4y^2=1$ to the circle of radius 1 unit centred at the origin O is

A. Dilate from the y-axis by a factor of 2 and from the x-axis by a factor of 0.5, and then translate in the negative x direction by 1 unit

B. Dilate from the x-axis by a factor of 2 and from the y-axis by a factor of 0.5, and then translate in the positive x direction by 1 unit

C. Translate in the negative x direction by 2 units, and then dilate from the y-axis by a factor of 0.5 and from the x-axis by a factor of 2

D. Translate in the positive x direction by 2 units, and then dilate from the y-axis by a factor of 2 and from the x-axis by a factor of 0.5

E. Translate in the negative x direction by 2 units, and then dilate from the y-axis by a factor of 2 and from the x-axis by a factor of 0.5

Question 8 Given $\sec(a+b)+\operatorname{cosec}(a-b)=0$, a possible value for b is

A. $\frac{a}{2}$

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{6}$

E. $\frac{\pi}{4}$

Question 9 Given $f:\left(0,\frac{1}{a}\right]\to R$, $f(x)=\frac{2}{\pi}\cos^{-1}\left(ax-\frac{1}{2}\right)$ and $a\in R^+$, the domain of f^{-1} is

A. $\left(-\frac{2}{3},\frac{2}{3}\right]$

B. $\left[\frac{2}{3},\frac{4}{3}\right)$

C. $\left(-\frac{4}{3},\frac{4}{3}\right]$

D. $\left[a-\frac{2}{3},a+\frac{2}{3}\right)$

E. $\left(a+\frac{2}{3},a+\frac{4}{3}\right]$

Question 10 The equation $b\tan^{-1}\left(\frac{x-c}{a}\right)-a\tan\left(\frac{x-c}{b}\right)=0$, where a, b and $c \in R^{+}$, and $-\frac{\pi}{2}<\frac{x-c}{b}<\frac{\pi}{2}$, has ***more than one*** solution when

A. $a=b$

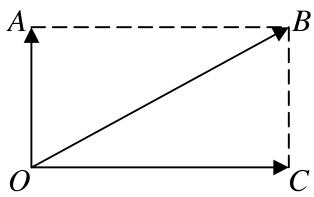
B. $ab=1$

C. $a>b$

D. $a<b$

E. $a+b>2$

Question 11 $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are ***position vectors*** of points *A*, *B* and *C* respectively, and *OABC* is a ***rectangle***. Which one of the following statements is **un**defined in kinematics?



A. $\overrightarrow{OA}+\overrightarrow{OC}=\overrightarrow{OB}$

B. $\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$

C. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

D. $\overrightarrow{OA}\bullet\overrightarrow{OC}=0$

E. $\left|\overrightarrow{OA}\right|^2+\left|\overrightarrow{OC}\right|^2=\left|\overrightarrow{OB}\right|^2$

Question 12 Vectors $3\tilde{k}-a\tilde{i}$, $\tilde{i}-b\tilde{j}$ and $2\tilde{j}-c\tilde{k}$ are linearly **in**dependent if the product $abc \in$

A. $R\backslash\{3\}$

B. $R\backslash\{-4,2\}$

C. $R\backslash\{6\}$

D. $R\backslash\{-3\}$

E. $R\backslash\{2\}$

Question 13 $\tilde{a}$ and $\tilde{b}$ are any two non-parallel vectors of the same magnitude. The scalar resolute of $\tilde{a}$ in the direction of $\tilde{a}+\tilde{b}$ is

A. $|\tilde{a}|+\dfrac{\tilde{a}.\tilde{b}}{|\tilde{a}|}$

B. $\dfrac{1}{2}$

C. $|\tilde{b}|+\dfrac{\tilde{a}.\tilde{b}}{|\tilde{b}|}$

D. $\dfrac{1}{\sqrt{2}}$

E. $\dfrac{1}{2}|\tilde{a}+\tilde{b}|$

Question 14 In terms of a the exact value of the definite integral $\int_{-\frac{a}{2}}^{\frac{a}{2}}\sqrt{a^2-t^2}\,dt$ is

A. $\left(\dfrac{\pi}{6}-\dfrac{\sqrt{3}}{4}\right)a$

B. $\left(\dfrac{\pi}{6}+\dfrac{\sqrt{3}}{4}\right)a$

C. $\left(\dfrac{\pi}{6}-\dfrac{\sqrt{3}}{4}\right)a^2$

D. $\left(\dfrac{\pi}{6}+\dfrac{\sqrt{3}}{4}\right)a^2$

E. $\dfrac{24a^2}{25}$

Question 15 Consider $y=\left(\tan^{-1}x\right)^2,\ x\in R$. The area of the region enclosed by the line $y=\dfrac{\pi^2}{16}$ and the curve $y=\left(\tan^{-1}x\right)^2$ is closest to

A. 0.25

B. 0.35

C. 0.5

D. 0.75

E. 1

Question 16 The differential equation $\left(\tan^{-1} x\right)^2 \frac{dy}{dx} - \frac{1}{1+x^2} = 0$ is equivalent to

A. $t^2 \frac{dy}{dt} - 1 = 0$ where $t = \tan^{-1} x$

B. $t^2 \frac{dy}{dt} - \frac{1}{1+t^2} = 0$ where $t = \tan^{-1} x$

C. $\frac{dy}{dt} - \frac{1}{1+t^2} = 0$ where $t = \tan^{-1} x$

D. $\frac{dy}{dt} - t^2 = 0$ where $t = \tan^{-1} x$

E. $\frac{dy}{dt} - \frac{t}{1+t^2} = 0$ where $t = \tan^{-1} x$

Question 17 The graph of $y = \left(x^2 - 2.5x + 3.1\right)\left(x^2 + 2.5x + 3.1\right)$ has

A. 3 stationary points and 2 inflection points

B. 2 stationary points and 2 inflection points

C. 1 stationary point and 2 inflection points

D. 1 stationary point and 1 inflection point

E. 1 stationary point and 0 inflection point

Question 18 The velocity of a particle at position $x \geq 0$ is given by $v = 2e^{-x} - 1$, and $x = 0$ initially. The velocity of the particle at time *t* is given by

A. $v = \frac{e^t}{2e^{-t} - 1}$

B. $v = \frac{e^t}{2e^t - 1}$

C. $v = \frac{1}{2e^t - 1}$

D. $v = \frac{e^t}{2 - e^t}$

E. $v = \frac{e^t}{2 - e^{-t}}$

Question 19 The momentum (in kg m s^{-1}) of a particle changes uniformly from $6\underset{\sim}{i} - 6\underset{\sim}{j} + 3.0\underset{\sim}{k}$ to $3\underset{\sim}{i} - 3\underset{\sim}{j} + 1.5\underset{\sim}{k}$ in 5.0 seconds. The magnitude of the resultant force on the particle is closest to

A. 0.5 N

B. 1.0 N

C. 1.5 N

D. 3.7 N

E. 3.8 N

Question 20 The velocity-time graph of a particle in rectilinear motion is shown below. The average ***speed*** (m s^{-1}) of the particle in the first 40 seconds is closest to

A. 3.9

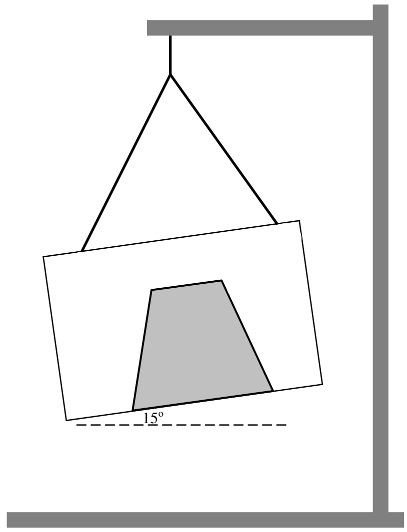
B. 3.85

C. 3.8

D. 3.4

E. 2.5

Question 21 A 1500 kg machine (shaded) is placed inside a 100 kg crate. It is ***lowered*** by a crane with a speed increasing at a rate of 0.8 m s^{-2}. The machine does not slide when the crate is tilted at an angle of 15° while it is lowered. The reaction force of the crate on the machine is



A. 12500 N upward and perpendicular to the floor of the crate

B. 13500 N upward and perpendicular to the floor of the crate

C. 13500 N vertically upward

D. 14400 N upward and perpendicular to the floor of the crate

E. 14400 N vertically upward

Question 22 A *M* kg crate is pushed along a ***rough*** horizontal floor by a horizontal force of 99 N. The acceleration of the crate is 1.00 m s^{-2}. Another *M* kg crate is now stacked securely on top of the first crate. The acceleration is only 0.010 m s^{-2} when the same force of 99 N is used to push the two crates.

The value of *M* is closest to

A. 10

B. 25

C. 50

D. 99

E. 9900

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$.

Question 1 Consider the equation $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$ where $a \in R$ and $z \in C$.

a. $\frac{z}{2} = \sqrt{a - \sqrt{a + \frac{z}{2}}}$ can be expressed in the form $z^4 + lz^2 + mz + n = 0$.
Find the values of l, m and n in terms of a if necessary. 2 marks

b i. $z^4 + lz^2 + mz + n$ can be expressed in factorised form, $(z^2 + 2z + p)(z^2 + rz + q)$.
Find the values of p, q and r in terms of a if necessary. 3 marks

b ii. Hence solve $z^4 + lz^2 + mz + n = 0$ for z in terms of a. 2 marks

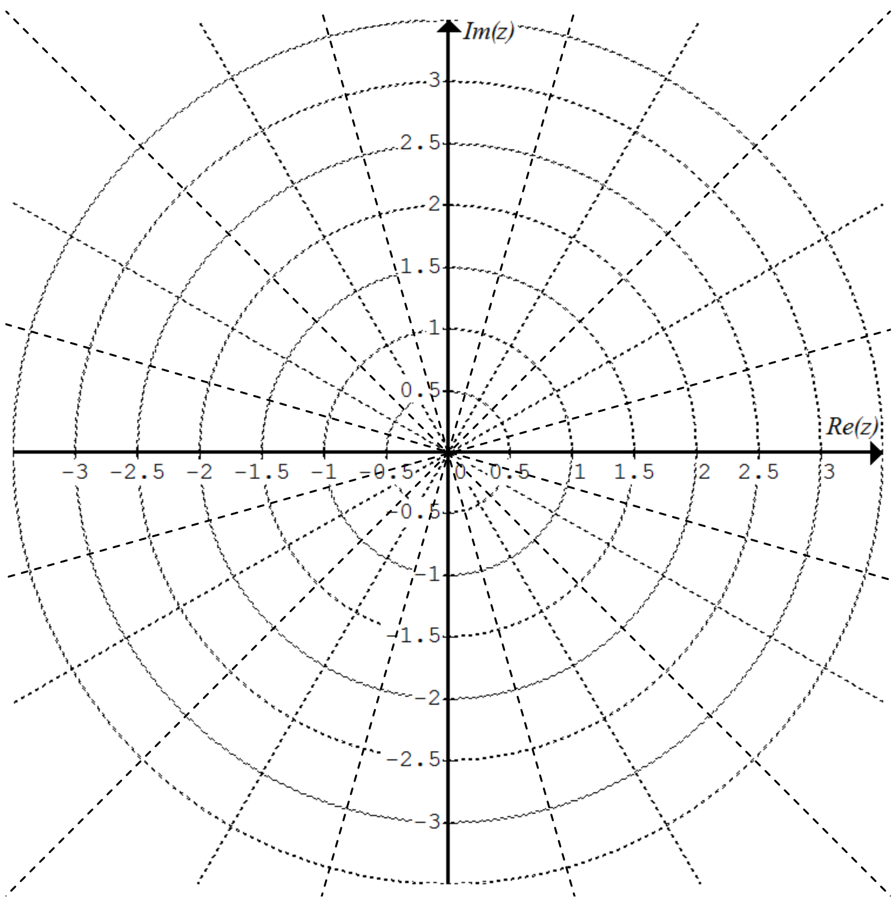
c i. Find the values of a such that all the solutions of $z^4 + lz^2 + mz + n = 0$ are real. 1 mark

c ii. Find the values of a such that all the solutions of $z^4 + lz^2 + mz + n = 0$ have imaginary part. 1 mark

c iii. Find the values of a such that $z^4 + lz^2 + mz + n = 0$ has real solutions and solutions with imaginary part. 1 mark

d i. Express in polar form the solutions to $\frac{z}{2}=\sqrt{a-\sqrt{a+\frac{z}{2}}}$ for $a=\frac{1}{2}$. 2 marks

d ii. Hence plot accurately the solutions on the grid below. Label each one in polar form. 2 marks



Question 2 The position of a particle moving in the *x*-*y* plane is given by $\tilde{r}(t) = \log_e(t+1)\left[\cos(t)\tilde{i} + \sin(t)\tilde{j}\right]$ for $t \geq 0$, where $\tilde{i}$ and $\tilde{j}$ are unit vectors pointing to the east (*x*-direction) and north (*y*-direction) respectively. Distance is measured in metres, time in seconds and speed in m s^{-1}.

a i. Find the initial position of the particle. 1 mark

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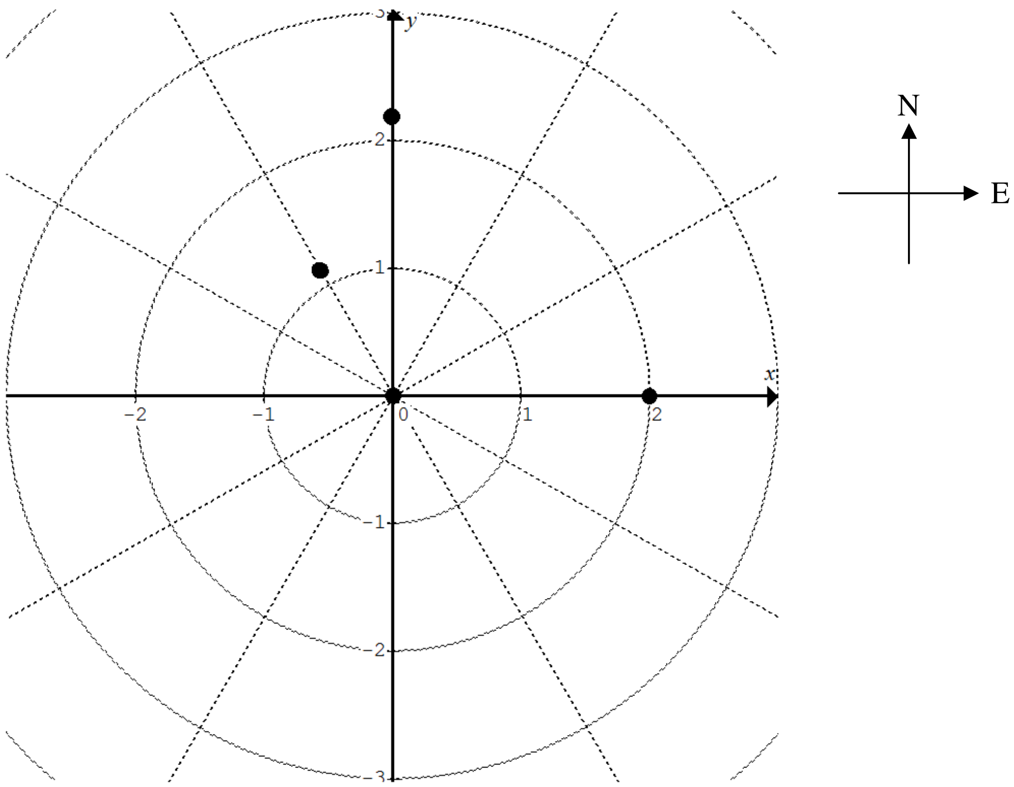
a ii. Complete the following table (correct to two decimal places). 1 mark

t	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{3}$	2π	$\frac{5\pi}{2}$
$\lvert\tilde{r}\rvert$	0.00		0.72	0.94	1.13		1.99	2.18

a iii. The positions of the particle at $t = 0$, $\frac{2\pi}{3}$, 2π and $\frac{5\pi}{2}$ are plotted on the following diagram. Label each point with $t = 0$, $\frac{2\pi}{3}$, 2π or $\frac{5\pi}{2}$. 1 mark



a iv. Plot on the above diagram the positions of the particle at $t = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and $\frac{4\pi}{3}$. 2 marks

b i. Find the velocity $\tilde{v}(t)$ of the particle at time t. 2 marks

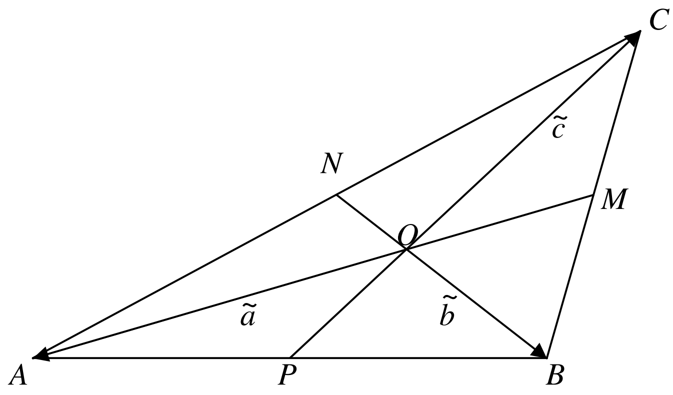
b ii. Show that the initial velocity of the particle is $\tilde{i}$. 1 mark

b iii. On the diagram in part **a iii** draw the initial velocity vector of the particle at its initial position. 1 mark

b iv. Find t (correct to two decimal places) when the particle is first heading ***south***. 2 marks

b v. What is the speed (correct to two decimal places) of the particle when it is first heading south? 1 mark

Question 3 ABC is any triangle. $\overline{AM}$ and $\overline{BN}$ are ***medians*** of the triangle, that is M and N are midpoints of line segments $\overline{BC}$ and $\overline{CA}$ respectively. $\overline{CP}$ is a line segment passing through the intersection O of $\overline{AM}$ and $\overline{BN}$. Let $\overrightarrow{OA} = \tilde{a}$, $\overrightarrow{OB} = \tilde{b}$ and $\overrightarrow{OC} = \tilde{c}$.



a. Express $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$. 2 marks

Let m, n and p be some positive real numbers such that $\overrightarrow{OM} = -m\tilde{a}$, $\overrightarrow{ON} = -n\tilde{b}$ and $\overrightarrow{OP} = -p\tilde{c}$.

b i. Show that $2m\tilde{a} + \tilde{b} + \tilde{c} = \tilde{0}$ and $\tilde{a} + 2n\tilde{b} + \tilde{c} = \tilde{0}$. 2 marks

Hence show/explain parts **b ii** to **b v**.

b ii. $(1-2m)\tilde{a} - (1-2n)\tilde{b} = \tilde{0}$ 1 mark

b iii. $m = n = \frac{1}{2}$. 1 mark

b iv. $\tilde{b} = -\tilde{a} - \tilde{c}$. 1 mark

b v. $\overrightarrow{AB} = -2\tilde{a} - \tilde{c}$. 1 mark

c i. Given $\overrightarrow{AP} = -\tilde{a} - p\tilde{c}$ and $\overrightarrow{AP} = k\overrightarrow{AB}$ where $k \in R$, show that $(2k-1)\tilde{a} + (k-p)\tilde{c} = \tilde{0}$. 1 mark

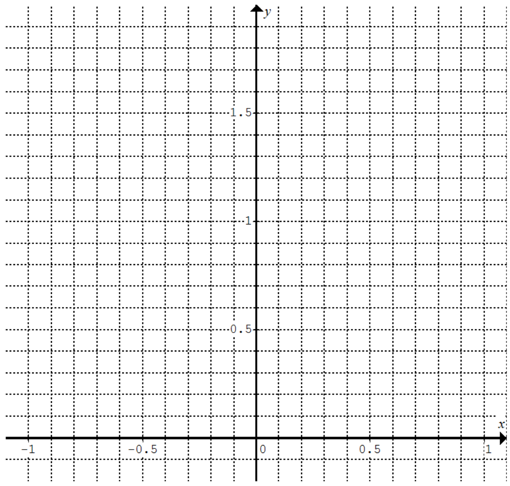
c ii. Hence show that $p = k = \frac{1}{2}$. 1 mark

c iii. Explain why line segment $\overline{CP}$ is a median of triangle *ABC*. 1 mark

Question 4 A top is formed by revolving $y = \sin^{-1} 2x$, $0 \le x \le \frac{1}{2}$, about the y-axis.

a. Draw an accurate graph of $y = \sin^{-1} 2x$, $0 \le x \le \frac{1}{2}$, showing the exact coordinates of the end points.

2 marks



b. Find the exact coordinates of the point where the gradient of the curve $y = \sin^{-1} 2x$ is 4. 2 marks

c. Find the exact area under the curve $y = \sin^{-1} 2x$, $0 \le x \le \frac{1}{2}$. 3 marks

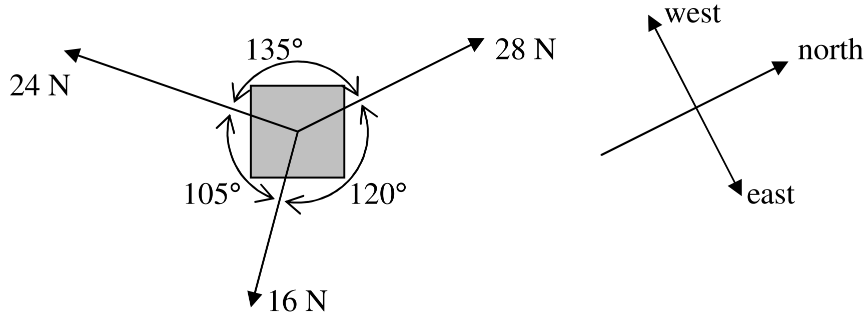
d. The top is cut from a rectangular block of wood of dimensions 1 unit $\times 1$ unit $\times$ 2 units. Find the exact volume of wood to be removed from the block to make the top.

3 marks

e. The shape of a dowel is formed by revolving $y = \dfrac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - x^2}}$, $x \geq 0$ and $y \leq 10$, about the *y*-axis. Find the exact volume of the dowel.

3 marks

Question 5 A box on a rough horizontal floor is pulled by three forces as shown in the following diagram (not drawn to scale). The box is in *limiting* equilibrium. The coefficient of friction is 0.25 between the box and the floor. The 28-N force is directed towards the north.



a. Determine the magnitude (N) and direction (degrees) of the resultant of the three pulling forces, correct to one decimal place. 3 marks

b. Write down the magnitude and direction (correct to one decimal place) of the force of friction on the ***floor***. 1 mark

c. Calculate the mass (kg) of the box, correct to one decimal place. 1 mark

d. Determine the magnitude (in m s^{-2}, correct to one decimal place) and direction of the acceleration of the box when the 24 N force is increased to $20\sqrt{2}$ N. 3 marks

End of Exam 2