2013 VCAA Further Mathematics Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	D	C	B	B	E	E	A	B	A	C	D

SECTION B

Module 1: Number patterns

1	2	3	4	5	6	7	8	9
C	E	E	D	A	C	B	D	B

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
C	D	E	A	C	D	C	B	B

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
B	E	C	D	D	E	A	B	C

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
B	A	B	D	E	C	C	B	A

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
A	E	A	D	E	C	C	D	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
B	D	B	E	C	A	D	B	E

SECTION A Core: Data analysis

Q1 23, 24, 24, 25, 27, 28, 29, 30, 30, 31, 31, 31, 32, 32, 33, 35, 37, 38, 38 There are 19 countries over 22%. C

Q2 $Median = \frac{29+30}{2} = 29.5$ B

Q3 $10+14+7=31$ D

Q4 $\frac{3}{3+11+8} \approx 0.14 = 14\%$ C

Q5 95% $interval = [\mu - 2\sigma\,,\ \mu + 2\sigma] = [6.0\,,\ 8.8]$ B

Q6 $8.1 = \mu + 1\sigma$. The percentage above 8.1 is 16%.
$Number\ of\ students = 16\% \times 1550 = 248$ B

Q7 *Population density* $r^2 = (-0.563)^2 \approx 0.317 = 31.7\%$
House size $r^2 = (0.357)^2 \approx 0.127 = 12.7\%$ E

Q8 By CAS/graphics calculator, $r = 0.9622$ E

Q9 The ages remain in the same order when 69 was incorrectly entered as 96. A

Q10 $(width)^2 = 1.8 + 0.8 \times 120 = 97.8$, $width = \sqrt{97.8} \approx 9.9$ B

Q11 A

Q12 C

Q13

390		
126		
	182.75	
85		191.5
	200.25	
130		
460		

D

SECTION B

Module 1: Number patterns

Q1 $a = 3$, $d = 2$, $t_9 = 3 + (9-1) \times 2 = 19$ C

Q2 $a = 1$, $r = 1$, $t_n = 1 \times 1^{n-1} = 1$ E

Q3 $a = 18$, $d = 12$, $S_{15} = \frac{15}{2}(2 \times 18 + (15-1) \times 12) = 1530$ E

Q4 $a = 64.0$, $r = \frac{60.8}{64.0} = 0.95$, $S_{10} = \frac{64.0(1 - 0.95^{10})}{1 - 0.95} \approx 514$ D

Q5 $a = 1$, $r = \frac{t_{n+1}}{t_n} = 2$, $\therefore\ t_n = ar^{n-1} = 1 \times 2^{n-1}$ A

Q6 200, 250, 300, … form an arithmetic sequence.
$a = 200$, $d = 50$, $S_n = 3000$
$\therefore\ \frac{n}{2}(2 \times 200 + (n-1) \times 50) = 3000$, solve for positive n, $n = 8$ C

Q7 -1, -3, -5 form an arithmetic sequence with $d = -2$.
The term before -1 is $-1 - (-2) = 1$; the term before 1 is $1 - (-2) = 3$, and 2 is skipped. B

Q8 Let $S_{max} = 0.2 \times S_{max} + 15$, $S_{max} - 0.2 \times S_{max} = 15$,
$0.8 \times S_{max} = 15$, $\therefore\ S_{max} = \frac{15}{0.8} = 18.75$ D

Q9 Ten distances are marked by the eleven speed humps.
$S_n = \frac{n}{2}(a + l)$, $S_{10} = \frac{10}{2}(a + 2) = 100$, $a = 18$ B

Module 2: Geometry and trigonometry

Q1 $\frac{100}{5} = 20$ C

Q2 Use the cosine rule D

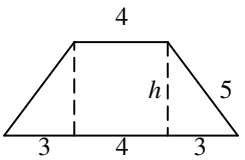
Q3 From *M* to *N* the contour has a negative slope for about a quarter of the way and then a positive slope for the rest. E

Q4 $\frac{P_{small}}{P_{large}} = \frac{V_{small}}{V_{large}} = \left(\frac{h_{small}}{h_{large}}\right)^3$, $\frac{P_{small}}{5.40} = \left(\frac{4}{6}\right)^3$, $P_{small} = \$1.60$ A

Q5 $\frac{1}{50000} \times 4\text{ km} = \frac{1}{50000} \times 400000\text{ cm} = 8.0\text{ cm}$ C

Q6 $\overline{RN} = 4 + 6 = 10$, ΔRMN and ΔRPQ are similar.
$\therefore \frac{\overline{MN}}{\overline{PQ}} = \frac{\overline{RN}}{\overline{RQ}}$, $\overline{MN} = \frac{\overline{RN}}{\overline{RQ}} \times \overline{PQ}$, $\overline{MN} = \frac{10}{4} \times 5 = 12.5$ D

Q7



$h = \sqrt{5^2 - 3^2} = 4$

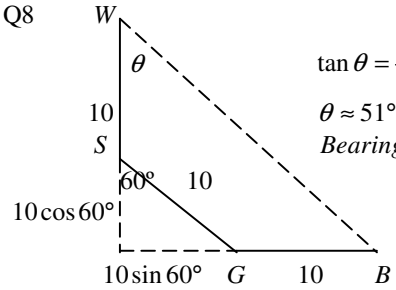
$2 \times$  $= 2 \times \left(\frac{1}{2} \times (4+10) \times 4\right) = 56$

$2 \times$  $= 2 \times (12 \times 5) = 120$

 $= 12 \times 4 = 48$

$Total = 56 + 120 + 48 = 224$ C

Q8



$\tan\theta = \frac{10\sin 60° + 10}{10\cos 60° + 10}$
$\theta \approx 51°$
$Bearing = 180° - 51° = 129°$ B

Q9 Let *x* cm be the side length of the base square.
$x^2 + x^2 + 4^2 = 8^2$, $\therefore x^2 = 24$
$V = x^2 h = 24 \times 4 = 96\text{ cm}^3$ B

Module 3: Graphs and relations

Q1 $m = 1$, $c = 6$ B

Q2 $3 \times (-1) - 2 \times (1) = -5$ E

Q3 $Gradient = \frac{-2000}{160} = -12.5$, *pumping out rate* $= 12.5$ C

Q4 $2j + 1s = 225$, $1s = 3j$, $\therefore 2j + 3j = 225$, $5j = 225$
$\therefore j = 45$, $s = 3 \times 45 = 135$ D

Q5 Read from graph, 25 km/h over, the fine is \$400. D

Q6 Graph E: $Gradient = \frac{\$130 - \$30}{5000MJ} = \$0.02\text{ /MJ} = 2\text{ }^{c}\text{/MJ}$ E

Q7 $Gradient = \frac{25.7}{0.44} \approx 58.4$, $\therefore$ the rule is $H = \frac{58.4}{d^2}$ A

Q8 *Gradient of line MN* $= \frac{-70}{70} = -1$
Gradient of objective function $= -\frac{a}{b}$, $\therefore a = b$ B

Q9 $500 + 1.30n = 4.50n - 300$, $n = 250$ C

Module 4: Business-related mathematics

Q1 $\frac{30}{200} \times 100\% = 15\%$ B

Q2 A

Q3 B

Q4 *Amount owing* $= \$15000 - \$3000 = \$12000$
Interest $= 36 \times \$400 - \$12000 = \$2400$ for 3 years
$I = \frac{\Pr T}{100}$, $2400 = \frac{12000 \times r \times 3}{100}$, $r \approx 6.7$ D

Q5 The amount invested in a perpetuity does not change. E

Q6 Let \$*x* be her salary two years ago.
$1.03^2 x = 46500$, $x \approx 43830$ C

Q7 *Flat rate* $= \frac{50}{1000} \times 100\% = 5\%$ p.a.
$I = \frac{\Pr T}{100}$, $600 = \frac{P \times 5 \times 8}{100}$, $P = 1500$ C

Q8 *Depreciated value after 3 years* $= 25000 \times 0.8^3 = 12800$
Depreciation in the fourth year $= 12800 \times 0.2 = 2560$ B

Q9 Use TVM solver to find $I\% = 6$. When $N = 2$,
$FV = -297995$, i.e. \$297995 owing after 2 months A

Module 5: Networks and decision mathematics

Q1 A

Q2 $Number\ of\ edges = \frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6$ E

Q3 A

Q4

	Kate	***Lexie***	***Mei***	***Nasim***
W	6	3	4	6
X	4	3	5	5
Y	5	7	9	6
Z	3	2	3	2

$5+3+4+2=14$ D

Q5

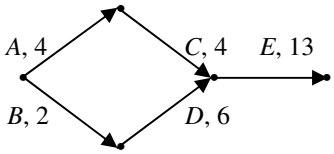
	One-step dominance	Two-step dominance	Sum	Rank
Ash	$A \to B$ $A \to D$	$A \to B \to C$ $A \to D \to C$ $A \to D \to B$	5	1
Binh	$B \to C$	$B \to C \to A$	2	4
Con	$C \to A$	$C \to A \to B$ $C \to A \to D$	3	3
Dan	$D \to B$ $D \to C$	$D \to B \to C$ $D \to C \to A$	4	2

E

Q6 C

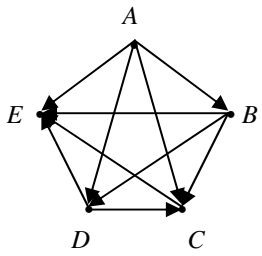
Q7 The graph always has more than one face. False
All vertices are always of even degree. False C

Q8



Reduce the time for each of *B*, *C*, *D* and *E* by one hour at a cost of $4\times\$100 = \$400$. D

Q9



B

Module 6: Matrices

Q1

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}2\\0\\0\\2\end{bmatrix} - 2\times\begin{bmatrix}0\\-1\\-1\\0\end{bmatrix} = \begin{bmatrix}2\\0\\0\\2\end{bmatrix} - \begin{bmatrix}0\\-2\\-2\\0\end{bmatrix} = \begin{bmatrix}2\\2\\2\\2\end{bmatrix}$$ B

Q2 $B\times A = C$
$4\times 3 \quad 3\times 2 \quad 4\times 2$ D

Q3

$$\begin{bmatrix}B\\I\\K\end{bmatrix} = \begin{bmatrix}0.8&0.1&0.1\\0&0.8&0.1\\0.2&0.1&0.8\end{bmatrix}^n\begin{bmatrix}84\\96\\81\end{bmatrix} \to \begin{bmatrix}87\\58\\116\end{bmatrix}$$ for large *n*, i.e. in the long term. B

Q4 $\Delta = 0,\ 2.8k - 0.7\times 1.4 = 0,\ k = 0.35$ E

Q5 C

Q6

$$\begin{bmatrix}10&4\\8&3\end{bmatrix}\begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}360\\280\end{bmatrix},\ \begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}10&4\\8&3\end{bmatrix}^{-1}\begin{bmatrix}360\\280\end{bmatrix}$$

$$\begin{bmatrix}b\\d\end{bmatrix} = \frac{1}{10\times 3 - 4\times 8}\begin{bmatrix}3&-4\\-8&10\end{bmatrix}\begin{bmatrix}360\\280\end{bmatrix} = \begin{bmatrix}-1.5&2\\4&-5\end{bmatrix}\begin{bmatrix}360\\280\end{bmatrix}$$ A

Q7 M is 3×15 and S is 15×1
$M\times S$ is 3×1. Each entry is the sum of the marks in a class.
When divided by 45, each entry is the mean mark for a class. D

Q8

$$S_1 = TS_0 - C = \begin{bmatrix}0.5&0.6\\0.5&0.4\end{bmatrix}\begin{bmatrix}100\\250\end{bmatrix} - \begin{bmatrix}20\\20\end{bmatrix} = \begin{bmatrix}180\\130\end{bmatrix}$$

$$S_2 = TS_1 - C = \begin{bmatrix}0.5&0.6\\0.5&0.4\end{bmatrix}\begin{bmatrix}180\\130\end{bmatrix} - \begin{bmatrix}20\\20\end{bmatrix} = \begin{bmatrix}148\\122\end{bmatrix}$$ B

Q9 Q and S are square matrices and $Q+S$ is not defined, .: they do not have the same order.
Let Q be 3×3 and S be 2×2.
$P = Q\times R\times S$ is defined, .: R is 3×2 and P is 3×2,
.: only $P\times S$ is defined. E

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

