2013 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

Q1ai Frequency for very high development index is 31.

Q1b *Low index + high index* $= 45 + 49 = 94$, $\frac{94}{153}\times 100\% \approx 61\%$

Q2a $Mode = 78$, $range = 79 - 70 = 9$

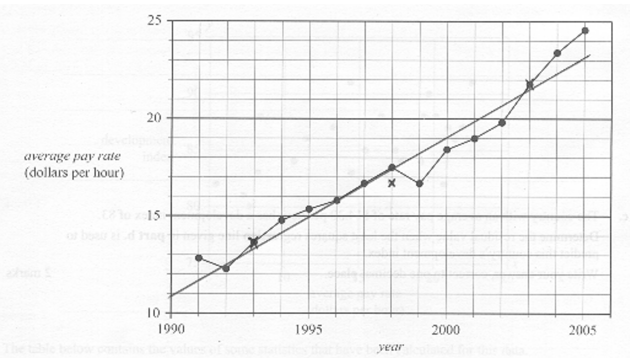
Q2b $Q_1 = 75$, $Q_3 = 78$, $IQR = 78 - 75 = 3$
$Q_1 - 1.5\times IQR = 75 - 1.5\times 3 = 70.5$, $70 < 70.5$, .: 70 is an outlier.

Q3a $z = \frac{91 - 85.6}{2.99} \approx 1.8$

Q3b $b = 0.488\times\frac{2.99}{5.37} \approx 0.272$, $a = 85.6 - 0.272\times 15.7 \approx 81.3$
.: $y = 81.3 + 0.272x$

Q3c $y_{predicted} = 81.3 + 0.272\times 14.30 \approx 85.19$,
$residual = 83 - 85.19 \approx -2.2$

Q4ai and ii
The three medians are approximately:
$(1993,13.7)$, $(1998,16.7)$, $(2003,21.8)$



Q4b $Slope \approx \frac{21.8 - 13.7}{2003 - 1993} \approx 0.8$

Q4c The average pay rate on average increases by \$0.80 every year from 1991 to 2005

Module 1: Number patterns

Q1a $2 + t_2 = 5$, $t_2 = 3$

Q1b Fibonacci sequence: 2, 3, 5, 8, 13, 21, …
$t_5 = 5 + 8 = 13$, $t_6 = 8 + 13 = 21$

Q2a $r = \frac{9.6}{8} = 1.2$, $t_5 = 8\times 1.2^4 \approx 16.6$

Q2b $t_n = 8\times 1.2^{n-1} > 30$, $n = 9$

Q2c $S_{10} = \frac{8\left(1.2^{10} - 1\right)}{1.2 - 1} \approx 208$

Q3ai $P_1 = c$, $P_2 = P_1 + 50 = c + 50$,
$P_3 = P_2 + 50 = c + 50 + 50 = c + 100 = 250$, .: $c = 150$

Q3aii Arithmetic sequence

Q3b $P_n = 150 + (n-1)\times 50 = 100 + 50n$, .: $a = 100$, $b = 50$

Q4a $(1 - 0.8)\times 100\% = 20\%$

Q4b $A_1 = 18000$, $A_2 = 0.8\times 18000 + 100 = 14500$
$A_3 = 0.8\times 14500 + 100 = 11700$

Q4c Let $0.8\times 18000 + k = 18000$, $k = 3600$

Q5a $r = \frac{18}{20} = 0.9$, $L_{n+1} = 0.9L_n$

Q5b 20, 18, … , L_n, … is a geometric sequence, $L_n = 20\times 0.9^{n-1}$
Let I_n be the \$income per tonne during year *n*.
1500, 1300, …, I_n, … is an arithmetic sequence where $d = -200$
and $I_n = 1500 + (n-1)(-200) = 1700 - 200n$
Let $I_n \times L_n < 10000$, i.e. $(1700 - 200n)\times 20\times 0.9^{n-1} < 10000$
.: $n = 5$
$n = 1$ corresponds to year 2012, .: $n = 5$ is 2016

Module 2: Geometry and trigonometry

Q1a $\overline{SX} = \sqrt{40^2 + 13^2} \approx 42$ m

Q1bi $\overline{XY} = \sqrt{40^2 + 52^2 - 2(40)(52)\cos 113°} \approx 77$ m

Q1bii $Area\Delta XGY = \frac{1}{2}(40)(52)\sin 113° \approx 957$ m^2

Q1c $Angle.elevation = \tan^{-1}\left(\frac{13}{52}\right) \approx 14°$

Q2a $Shaded.rect.area = \frac{1.6}{10}\times\frac{3.0}{10} = 0.048$ m^2

Q2b There are 55 shaded rectangles at the side elevation.
$Shaded.area = 0.048\times 55 = 2.64$ m^2
$Volume = 2.64\times 2.5 = 6.6$ m^3

Q3a $Perimeter = \pi\times diameter + 2\times 100 = 400$
.: $\pi\times diameter = 200$, .: $diameter = \frac{200}{\pi} \approx 63.66$ m

Q3b $Area = 63.66\times 100 + \pi\left(\frac{63.66}{2}\right)^2 \approx 9549$ m^2

Q3c *Area of the region enclosed by the outer perimeter of the running track* $= (63.66+16)\times 100 + \pi\left(\frac{63.66+16}{2}\right)^2 \approx 12950 \text{ m}^2$

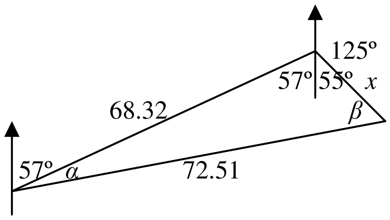
Area of the running track $\approx 12950 - 9549 = 3401 \text{ m}^2$

Volume of the running track $\approx 3401 \times 0.1 \approx 340 \text{ m}^3$

Q4 Let k be the value such that $V_{sen} = kV_{\text{int}}$.

.: $k = \frac{V_{sen}}{V_{\text{int}}} = \left(\sqrt{\frac{720}{500}}\right)^3 = 1.728$

Q5



$57° + 55° = 112°$

$\frac{\sin\beta}{68.32} = \frac{\sin 112°}{72.51}$, $\beta \approx 60.88°$

.: $\alpha = 180° - 112° - 60.88° = 7.12°$

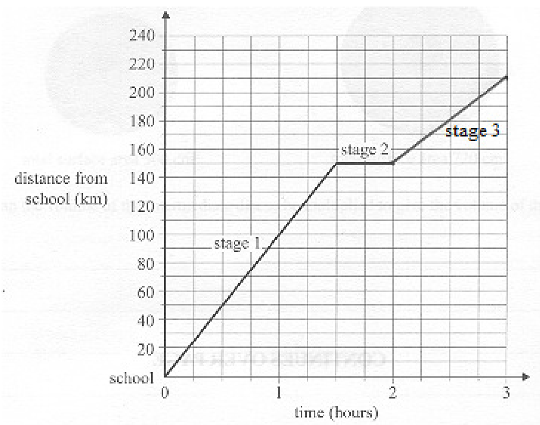
$\frac{x}{\sin 7.12°} = \frac{72.51}{\sin 112°}$, $x \approx 9.7$ m

Module 3: Graphs and relations

Q1a *Speed* $= \frac{150}{1.5} = 100 \text{ km h}^{-1}$

Q1b 30 minutes

Q1c



Q1d *Average speed* $= \frac{210}{3} = 70 \text{ km h}^{-1}$

Q1e $D = 60t + k$, when $t = 2$, $D = 150$

.: $150 = 60\times 2 + k$, $k = 30$ (km)

Q2a $30c + 20s \le 200$, $c = 2$, .: $30\times 2 + 20s \le 200$, .: $s \le 7$

.: maximum 7 hours

Q2b Let $c = s$, $30c + 20c \le 200$, $c \le 4$

.: maximum 4 hours

Q3a $P(n) = (24-6)n - 260$, .: $P(n) = 18n - 260$

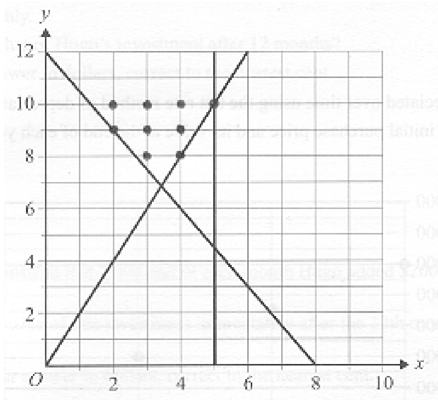
Q3b $18n - 260 \ge 500$ and n is a whole number, .: $n \ge 43$

.: minimum 43 students

Q4a $a = 6$ and $b = 4$

Q4b $y \ge 2$ for $x = 1$, .: $y \ge 2x$

Q4c



Q4d Points $(2,9)$ or $(3,8)$, $2+9 = 3+8 = 11$

.: minimum 11 camp sites

Q4ei Point $(2,9)$, $C = 60\times 2 + 30\times 9 = 390$

.: minimum \$390 per day

Q4eii 24 students in each type of camp sites,

.: $x \ge 4$ and $y \ge 6$

Since $y \ge 2x$, .: $x = 4$ and $y = 8$

$C = 60\times 4 + 30\times 8 = 480$

.: minimum \$480 per day

Module 4: Business-related mathematics

Q1a Read from graph, \$8000

Q1bi $\$8000 - \$6500 = \$1500$

Q1bii $\$8000 - \$1500\times 5 = \$500$

Q1c Let x be the number of kilometres travelled.

$0.25x = 1500\times 2$, $x = 12000$

Q2a $\$5000 \times \left(1 + \frac{4.8}{12\times 100}\right)^{12} \approx \$5245.35$

Q2bi TVM Solver: \$7698.86

Q2bii $I = \$7698.86 - \$5000 - \$200\times 12 = \$298.86$

Q3a $\frac{1}{24}(P+I) = \frac{1}{24}\left(\$7500 + \frac{\$7500\times 8\times 2}{100}\right) \approx \$362.50$

Q3b *Effective rate* $= \frac{2n}{n+1} \times$ *flat rate* $= \frac{2\times 24}{24+1} \times 8\% \approx 15.36\%$

Q3c Flat rate is charged on the initially borrowed amount whilst effective rate is charged on the reduced monthly balance.
.: effective rate is higher than flat rate to yield the same amount of interest.

Q3d $\textit{Fraction of value retained after each year} = \dfrac{6375}{7500}$

$\textit{Value after 5 years} = \$7500 \times \left(\dfrac{6375}{7500}\right)^5 \approx \$3328$

Q4 Use TVM Solver to find *quarterly* payment $1990..2712
Use TVM Solver to find amount still owing after 2 years
$13971.0914
$\textit{Repaid amount} = \$25000 - \$13971.09 \approx \$11029$

Module 5: Networks and decision mathematics

Q1a 3 edges, .: degree 3

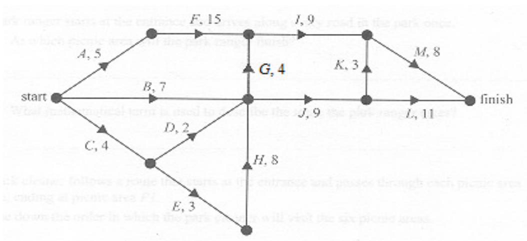
Q1b $600 + 400 = 1000$ metres

Q1ci A possible route:
$entrance - P_5 - P_4 - P_3 - P_2 - P_1 - entrance - P_6 - P_1 - P_3 - P_6 - P_4$
.: will finish at P_4

Q1cii Euler path

Q1d $entrance - P_5 - P_4 - P_6 - P_3 - P_2 - P_1$

Q2a



Q2b $\textit{EST of } H = 4 + 3 = 7$ hours after starting

Q2ci *AFIM*

Q2cii $\textit{LST for } D = 20 - (2+4) = 14$ hours after starting

Q2d The crashed activity is one of the four activities on the critical path.

Q2e There is a new critical path (*CEHGIM*) when *F* is crashed by 2 hours, .: minimum completion time for the project is 36 hours.

Q3a $\textit{Minimum cut} = 24 + 13 = 37$, .: 37 people are permitted to walk to *D* each day.

Q3bi Group 1 (17 students) will take the path *ABECD*.

Q3bii A possibility:

Group	Maximum group size	Path taken from *A* to *D*
1	17	*ABECD*
2	11	*AFED*
3	7	*AGFBCD*
4	2	*ABED*

Module 6: Matrices

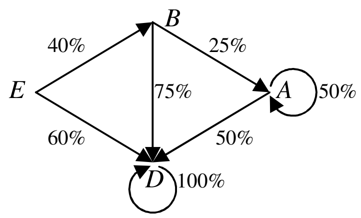
Q1a $1+0+0+1+0=2$

Q1b The sum represents the number of pipes connected to pond *R*.

Q1c $N = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & & & & \\ 0 & & & & \\ 0 & & & & \\ 0 & & & & \end{bmatrix}$

Q2ai $0.6 \times 10000 = 6000$ eggs die in the first year

Q2aii
$E \xrightarrow{40\%} B$, $E \xrightarrow{60\%} D$, $B \xrightarrow{25\%} A$, $B \xrightarrow{75\%} D$,
$A \xrightarrow{50\%} A$, $A \xrightarrow{50\%} D$, $D \xrightarrow{100\%} D$



Q2bi
$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

Q2bii
$$S_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}^4 \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 331 \\ 11469 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$
.: 331 adult trouts

Q2ci 13 years

Q2cii The largest number is predicted to be 1325 after 2 years.

Q2d From S_0 to S_1, $E = -10000$, $B = +3000$ and $A = -150$.
To maintain a constant population, add 10000 eggs, remove 3000 baby trouts and add 150 adult trouts.

Q2ei
$$S_1 = (T + 500M)S_0 = \begin{bmatrix} 0 & 0 & 250 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 200000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

Q2eii
$$S_2 = \begin{bmatrix} 0 & 0 & 250 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}^2 \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 162500 \\ 80000 \\ 1325 \\ 130475 \end{bmatrix} \begin{matrix} E \\ B \\ A \\ D \end{matrix}$$
.: there are 162500 eggs after 2 years.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors