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2014 NSW BOS Mathematics Extension 2 Solutions

Section I

1	2	3	4	5	6	7	8	9	10
D	A	B	C	C	D	B	B	A	D

Q1 $\frac{5x^2-x+1}{x(x^2+1)}=\frac{a}{x}+\frac{bx+c}{x^2+1}=\frac{a(x^2+1)+x(bx+c)}{x(x^2+1)}$

$\therefore\ a=1$, $c=-1$, $a+b=5$, $\therefore\ b=4$ **D**

Q2 $P(z)=(z-2+i)(z-2-i)Q(z)=(z^2-4z+5)Q(z)$ **A**

Q3 $9x^2+16y^2=25$, $\frac{x^2}{\frac{25}{9}}+\frac{y^2}{\frac{25}{16}}=1$

$\frac{25}{16}=\frac{25}{9}(1-e^2)$, $\frac{9}{16}=(1-e^2)$, $e=\frac{\sqrt{7}}{4}$ **B**

Q4 $z=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$, $\bar{z}=2\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$

$(\bar{z})^{-1}=2^{-1}\left(\cos\left(-\frac{\pi}{3}\right)-i\sin\left(-\frac{\pi}{3}\right)\right)=\frac{1}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ **C**

Q5 $y^2=x^2-2x$, $y^2=(x^2-2x+1)-1$, $(x-1)^2-y^2=1$ **C**

Q6 Reflect $y^2=8x$ in the y-axis and translate 2 units to the right to obtain $y^2=-8(x-2)$, $\therefore\ x=2-\frac{y^2}{8}$

$V=2\int_0^4\pi x^2dy=2\pi\int_0^4\left(2-\frac{y^2}{8}\right)^2dy$ **D**

Q7 $\int\frac{1}{1-\sin x}dx=\int\frac{1+\sin x}{(1-\sin x)(1+\sin x)}dx=\int\frac{1+\sin x}{\cos^2 x}dx$

$=\int\left(\sec^2 x+\frac{\sin x}{\cos^2 x}\right)dx=\tan x-\int\frac{1}{u^2}du$ where $u=\cos x$

$\therefore\ \int\frac{1}{1-\sin x}dx=\tan x+\sec x+c$ **B**

Q8 $Arg(u)=Arg(z)+Arg(w)=Arg(zw)$, $\therefore$ either A or B.
$\text{Im}(u)=0$, $\therefore$ B **B**

Q9 When $t=0$, $x=1$, $v=2$, $\therefore$ either A or B.

Consider A: $v=2\sin(x-1)+2$, $\frac{dv}{dx}=2\cos(x-1)$

$a=v\frac{dv}{dx}=(2\sin(x-1)+2)(2\cos(x-1))=4$ at $x=1$ **A**

Q10 It has to be true for all $f(x)$. A and B will be eliminated by considering $f(x)=1$.
C will be eliminated by considering $f(x)=x+1$

D: $\int_0^a f(x-a)+f(a-x)dx=\int_0^a f(x-a)dx+\int_0^a f(a-x)dx$

$=\int_{-a}^0 f(u)du-\int_a^0 f(v)dv=\int_{-a}^0 f(u)du+\int_0^a f(v)dv$

$=\int_{-a}^a f(x)dx$ **D**

Section II

Q11ai $z+w=(-2-2i)+(3+i)=1-i$

$=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$ or $\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

Q11aii $\frac{z}{w}=\frac{-2-2i}{3+i}\times\frac{3-i}{3-i}=\frac{-8-4i}{10}=-\frac{4}{5}-\frac{2}{5}i$

Q11b $\int_a^b v\frac{du}{dx}dx=[uv]_a^b-\int_a^b u\frac{dv}{dx}dx$

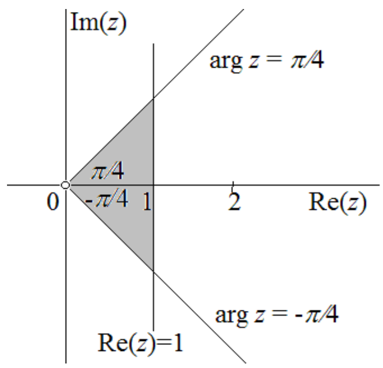
For $\int_0^{\frac{1}{2}}(3x-1)\cos(\pi x)dx$, let $v=3x-1$ and $\frac{du}{dx}=\cos(\pi x)$

$\therefore\ \frac{dv}{dx}=3$ and $u=\frac{\sin(\pi x)}{\pi}$

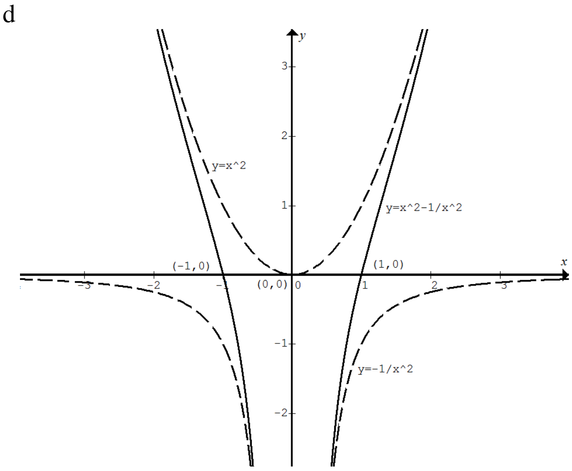
$\therefore\ \int_0^{\frac{1}{2}}(3x-1)\cos(\pi x)dx=\left[\frac{\sin(\pi x)}{\pi}(3x-1)\right]_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}\frac{3\sin(\pi x)}{\pi}dx$

$=\left[\frac{\sin(\pi x)}{\pi}(3x-1)\right]_0^{\frac{1}{2}}-\left[\frac{-3\cos(\pi x)}{\pi^2}\right]_0^{\frac{1}{2}}=\frac{1}{2\pi}-\frac{3}{\pi^2}$

Q11c



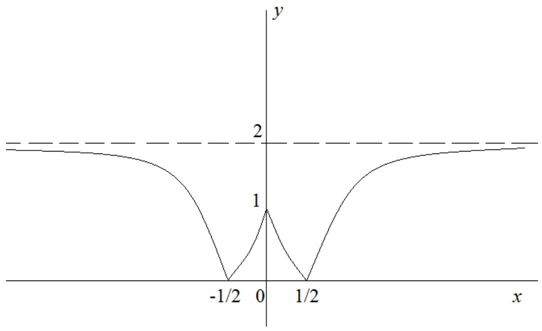
Q11d



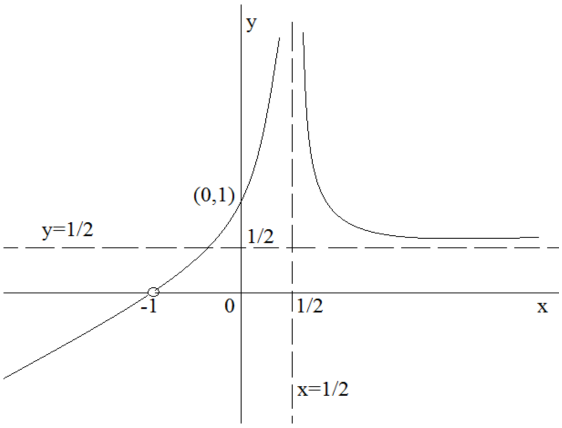
Q11e $V=\int_0^6 2\pi\,yx\,dy=\int_0^6 2\pi\,y(6y-y^2)dy=2\pi\int_0^6(6y^2-y^3)dy$

$=2\pi\left[2y^3-\frac{y^4}{4}\right]_0^6=216\pi$ cubic units

Q12ai $y = f(|x|)$



Q12aii $y = \dfrac{1}{f(x)}$



Q12bi $x = 2\cos\theta$ is a solution of $x^3 - 3x = \sqrt{3}$

$\therefore\ (2\cos\theta)^3 - 3(2\cos\theta) = \sqrt{3}$, $2(4\cos^3\theta - 3\cos\theta) = \sqrt{3}$

$\therefore\ 2\cos 3\theta = \sqrt{3}$, $\cos 3\theta = \dfrac{\sqrt{3}}{2}$

Q12bii $\cos 3\theta = \dfrac{\sqrt{3}}{2}$, $3\theta = \pm\dfrac{\pi}{6} + 2\pi n$ where n is an integer

$\therefore\ \theta = \pm\dfrac{\pi}{18} + \dfrac{2\pi n}{3}$, $x = 2\cos\left(\pm\dfrac{\pi}{18} + \dfrac{2\pi n}{3}\right)$

Let $n = 0$, $x = 2\cos\left(\pm\dfrac{\pi}{18}\right) = 2\cos\dfrac{\pi}{18}$

Let $n = 1$, $x = 2\cos\left(\pm\dfrac{\pi}{18} + \dfrac{2\pi}{3}\right) = 2\cos\dfrac{11\pi}{18}$ or $2\cos\dfrac{13\pi}{18}$

Other values of n give the same solutions.

Q12c $x^2 - y^2 = 5$, $2x - 2y\dfrac{dy}{dx} = 0$

Gradient of the tangent $= m_1 = \dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{x_0}{y_0}$ at $P(x_0, y_0)$

$xy = 6$, $y + x\dfrac{dy}{dx} = 0$

Gradient of the tangent $m_2 = \dfrac{dy}{dx} = -\dfrac{y}{x} = -\dfrac{y_0}{x_0}$ at $P(x_0, y_0)$

Since $m_2 = \dfrac{-1}{m_1}$, $\therefore$ the two tangents are perpendicular.

Q12di $I_n = \int_0^1 \dfrac{x^{2n}}{x^2+1}dx$

$I_0 = \int_0^1 \dfrac{1}{x^2+1}dx = \left[\tan^{-1}x\right]_0^1 = \tan^{-1}1 = \dfrac{\pi}{4}$

Q12dii $I_n + I_{n-1} = \int_0^1 \dfrac{x^{2n}}{x^2+1}dx + \int_0^1 \dfrac{x^{2n-2}}{x^2+1}dx$

$= \int_0^1 \left(\dfrac{x^{2n}}{x^2+1} + \dfrac{x^{2n-2}}{x^2+1}\right)dx = \int_0^1 \dfrac{(x^2+1)x^{2n-2}}{x^2+1}dx$

$= \int_0^1 x^{2n-2}dx = \left[\dfrac{x^{2n-1}}{2n-1}\right]_0^1 = \dfrac{1}{2n-1}$

Q12diii $\int_0^1 \dfrac{x^4}{x^2+1}dx = I_2$

From part ii, $I_n = \dfrac{1}{2n-1} - I_{n-1}$

$\therefore\ I_1 = 1 - I_0 = 1 - \dfrac{\pi}{4}$, $I_2 = \dfrac{1}{3} - I_1 = \dfrac{1}{3} - \left(1 - \dfrac{\pi}{4}\right) = \dfrac{\pi}{4} - \dfrac{2}{3}$

Q13a Let $t = \tan\dfrac{x}{2}$

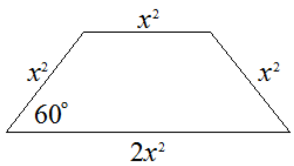
$\therefore\ \tan x = \dfrac{2t}{1-t^2}$, $\sin x = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$, $\dfrac{dt}{dx} = \dfrac{1+t^2}{2}$

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \dfrac{1}{3\sin x - 4\cos x + 5}dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \dfrac{1}{3\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) + 5}dx$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \dfrac{1+t^2}{(1+3t)^2}dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \dfrac{2}{(1+3t)^2}\dfrac{dt}{dx}dx = \int_{\frac{1}{\sqrt{3}}}^{1} \dfrac{2}{(1+3t)^2}dt$

$= \left[-\dfrac{2}{3(1+3t)}\right]_{\frac{1}{\sqrt{3}}}^{1} = \dfrac{2\sqrt{3}-3}{6}$ or $\dfrac{1}{\sqrt{3}} - \dfrac{1}{2}$

Q13b



$V = \int_0^2 \dfrac{1}{2}(x^2 + 2x^2)x^2 \sin 60° dx = \int_0^2 \dfrac{3\sqrt{3}}{4}x^4 dx$

$= \left[\dfrac{3\sqrt{3}}{20}x^5\right]_0^2 = \dfrac{24\sqrt{3}}{5}$ cubic units

Q13ci $M\left(\dfrac{a(t^2+1)}{2t}, \dfrac{b(t^2-1)}{2t}\right)$, $x = \dfrac{a(t^2+1)}{2t}$ and $y = \dfrac{b(t^2-1)}{2t}$

$\dfrac{x^2}{a^2} = \dfrac{(t^2+1)^2}{4t^2}$ and $\dfrac{y^2}{b^2} = \dfrac{(t^2-1)^2}{4t^2}$

$\therefore\ \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \dfrac{(t^2+1)^2}{4t^2} - \dfrac{(t^2-1)^2}{4t^2} = 1$, $\therefore$ M lies on $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Q13cii Gradient of $PQ = \dfrac{bt-\left(-\frac{b}{t}\right)}{at-\frac{a}{t}} = \dfrac{b\left(t^2+1\right)}{a\left(t^2-1\right)} = \dfrac{b^2\,\frac{a\left(t^2+1\right)}{2t}}{a^2\,\frac{b\left(t^2-1\right)}{2t}} = \dfrac{b^2x}{a^2y}$

Gradient of the hyperbola at *M*:

$\dfrac{d}{dx}\left(\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}\right)=0$, $\dfrac{2x}{a^2}-\dfrac{2y}{b^2}\dfrac{dy}{dx}=0$, .: $\dfrac{dy}{dx}=\dfrac{b^2x}{a^2y}$

Hence line *PQ* is a tangent to the hyperbola at *M*.

Q13ciii $OP^2\times OQ^2 = \left((at)^2+(bt)^2\right)\left(\left(\dfrac{a}{t}\right)^2+\left(\dfrac{b}{t}\right)^2\right)$

$= a^4+2a^2b^2+b^4=\left(a^2+b^2\right)^2=\left(a^2e^2\right)^2=\left(OS^2\right)^2$

.: $OP\times OQ = OS^2$

Q13civ *P* and *S* have the same *x*-coordinate, .: $at = ae$, $t=e$.

Gradient of $MS = \dfrac{\frac{b\left(t^2-1\right)}{2t}-0}{\frac{a\left(t^2+1\right)}{2t}-ae} = \dfrac{\frac{b\left(e^2-1\right)}{2e}}{\frac{a\left(e^2+1\right)}{2e}-ae} = -\dfrac{b}{a}$

.: *MS* is parallel to asymptote $y=-\dfrac{b}{a}x$

Q14ai $P(x)=x^5-10x^2+15x-6$, $P(1)=0$

$P'(x)=5x^4-20x+15$, $P'(1)=0$; $P''(x)=20x^3-20$, $P''(1)=0$

.: 1 is a root of $P(x)$ of multiplicity three.

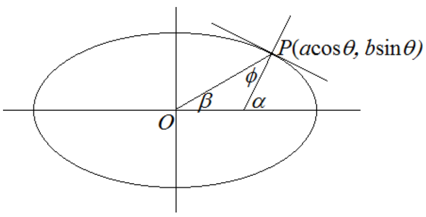
Q14aii $P(x)=x^5-10x^2+15x-6=(x-1)^3\left(x^2+bx+c\right)$

$=\left(x^3-3x^2+3x-1\right)\left(x^2+bx+c\right)$.: $c=6$ and $b=3$

Let $x^2+3x+6=0$, $x=\dfrac{-3\pm\sqrt{3^2-4(1)(6)}}{2(1)}$.: $x=-\dfrac{3}{2}\pm\dfrac{\sqrt{15}}{2}i$

Q14bi $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, gradient of the normal $=-\dfrac{dx}{dy}=\dfrac{a^2y}{b^2x}$

At $P(a\cos\theta, b\sin\theta)$, gradient of the normal $=\dfrac{a\sin\theta}{b\cos\theta}=\tan\alpha$



$\phi=\alpha-\beta$

$\tan\phi=\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\dfrac{\frac{a}{b}\tan\theta-\frac{b}{a}\tan\theta}{1+\frac{a}{b}\tan\theta\times\frac{b}{a}\tan\theta}$

$=\dfrac{\left(\frac{a}{b}-\frac{b}{a}\right)\tan\theta}{1+\tan^2\theta}=\dfrac{\left(\frac{a^2-b^2}{ab}\right)\tan\theta}{\sec^2\theta}=\left(\dfrac{a^2-b^2}{ab}\right)\sin\theta\cos\theta$ for all θ.

Q14bii $\tan\phi=\left(\dfrac{a^2-b^2}{ab}\right)\sin\theta\cos\theta=\dfrac{1}{2}\left(\dfrac{a^2-b^2}{ab}\right)\sin 2\theta$

Let $\dfrac{d}{d\theta}\tan\phi=\left(\dfrac{a^2-b^2}{ab}\right)\cos 2\theta=0$, $\cos 2\theta=0$, $\theta=\dfrac{\pi}{4}$ to maximise ϕ.

Q14ci Sum of forces $=m\ddot{x}$

$F-kv^2=m\ddot{x}$, $F-k\times 300^2=0$ ($t\to\infty$, $\ddot{x}\to 0$, $v=\dot{x}\to 300$)

$k=\dfrac{F}{300^2}$, .: $m\ddot{x}=F-\dfrac{Fv^2}{300^2}=F\left(1-\left(\dfrac{v}{300}\right)^2\right)$

Q14cii $\dfrac{dv}{dt}=\dfrac{F}{m}\left(1-\left(\dfrac{v}{300}\right)^2\right)=\dfrac{F}{m}\times\dfrac{(300-v)(300+v)}{300^2}$

$\dfrac{dt}{dv}=\dfrac{m}{F}\times\dfrac{300^2}{(300-v)(300+v)}=\dfrac{150m}{F}\left(\dfrac{1}{300-v}+\dfrac{1}{300+v}\right)$

.: $t=\dfrac{150m}{F}\displaystyle\int\left(\dfrac{1}{300-v}+\dfrac{1}{300+v}\right)dv$

.: $t=\dfrac{150m}{F}\log_e\dfrac{300+v}{300-v}$ ($t=0$, $x=0$, $v=\dot{x}=0$)

When $v=200$, $t=\dfrac{150m}{F}\log_e 5$ hours

Q15a $a+b+c=1$

$(a+b+c)^2=1$, .: $\left(a^2+2ab+2ca\right)+\left(b^2+2bc\right)+c^2=1$

.: $\left(a^2+2aa+2aa\right)+\left(b^2+2bb\right)+c^2\le 1$ since $0<a\le b\le c$

.: $5a^2+3b^2+c^2\le 1$

Q15bi $(1+i)^n+(1-i)^n$

$=\left(\sqrt{2}\right)^n\left(\cos\dfrac{n\pi}{4}+i\sin\dfrac{n\pi}{4}\right)+\left(\sqrt{2}\right)^n\left(\cos\dfrac{-n\pi}{4}+i\sin\dfrac{-n\pi}{4}\right)$

$=\left(\sqrt{2}\right)^n\left(\cos\dfrac{n\pi}{4}+i\sin\dfrac{n\pi}{4}+\cos\dfrac{n\pi}{4}-i\sin\dfrac{n\pi}{4}\right)$

$=2\left(\sqrt{2}\right)^n\cos\dfrac{n\pi}{4}$

Q15bii $(1+i)^n+(1-i)^n=2\left(\sqrt{2}\right)^n\cos\dfrac{n\pi}{4}$, for $n=4k$ where *k* is a positive integer, .: $i^n=\left(i^4\right)^k=1$, $i^{n-2}=\dfrac{i^n}{i^2}=-1$, etc

LHS

$=\binom{n}{0}i^n+\binom{n}{1}i^{n-1}+\binom{n}{2}i^{n-2}+\binom{n}{3}i^{n-3}+\binom{n}{4}i^{n-4}+\cdots+\binom{n}{n}i^0$

$+\binom{n}{0}i^n-\binom{n}{1}i^{n-1}+\binom{n}{2}i^{n-2}-\binom{n}{3}i^{n-3}+\binom{n}{4}i^{n-4}-\cdots+\binom{n}{n}i^0$

$=2\binom{n}{0}+2\binom{n}{2}i^{n-2}+2\binom{n}{4}i^{n-4}+\cdots+2\binom{n}{n}i^0$

$=2\binom{n}{0}-2\binom{n}{2}+2\binom{n}{4}-2\binom{n}{6}+\cdots+2\binom{n}{n}$

RHS

$=2\left(\sqrt{2}\right)^n\cos\dfrac{n\pi}{4}=2\left(\sqrt{2}\right)^n\cos k\pi=2\left(\sqrt{2}\right)^n(-1)^k=2\left(\sqrt{2}\right)^n(-1)^{\frac{n}{4}}$

.: LHS $=$ RHS

Q15ci Vertical direction: $kv^2 - mg - T\sin\phi = 0$

$\therefore\ T\sin\phi = kv^2 - mg$

Horizontal direction: $T\cos\phi = \dfrac{mv^2}{r}$ where $r = \ell\cos\phi$

$\therefore\ T\cos^2\phi = \dfrac{mv^2}{\ell}$, $\therefore\ \dfrac{\sin\phi}{\cos^2\phi} = \dfrac{kv^2 - mg}{\frac{mv^2}{\ell}}$, $\dfrac{\sin\phi}{\cos^2\phi} = \dfrac{\ell k}{m} - \dfrac{\ell g}{v^2}$

Q15cii $\dfrac{\sin\phi}{\cos^2\phi} < \dfrac{\ell k}{m}$, $\dfrac{\sin\phi}{1-\sin^2\phi} < \dfrac{\ell k}{m}$

$\sin^2\phi + \dfrac{m}{\ell k}\sin\phi < 1$, $\sin^2\phi + \dfrac{m}{\ell k}\sin\phi + \left(\dfrac{m}{2\ell k}\right)^2 < 1 + \dfrac{m^2}{4\ell^2k^2}$

$\left(\sin\phi + \dfrac{m}{2\ell k}\right)^2 < \dfrac{m^2 + 4\ell^2k^2}{4\ell^2k^2}$ where $\sin\phi + \dfrac{m}{2\ell k} > 0$

$\therefore\ \sin\phi + \dfrac{m}{2\ell k} < \dfrac{\sqrt{m^2 + 4\ell^2k^2}}{2\ell k}$, $\therefore\ \sin\phi < \dfrac{\sqrt{m^2 + 4\ell^2k^2} - m}{2\ell k}$

Q15ciii $\dfrac{d}{d\phi}\left(\dfrac{\sin\phi}{\cos^2\phi}\right) = \dfrac{\cos^2\phi\cos\phi - \sin\phi(2\cos\phi)(-\sin\phi)}{\cos^4\phi}$

$= \dfrac{\cos^2\phi + 2\sin^2\phi}{\cos^3\phi} = \dfrac{1+\sin^2\phi}{\cos^3\phi}$

For $-\dfrac{\pi}{2} < \phi < \dfrac{\pi}{2}$, $\cos\phi > 0$

$\therefore\ \dfrac{d}{d\phi}\left(\dfrac{\sin\phi}{\cos^2\phi}\right) > 0$, $\therefore\ \dfrac{\sin\phi}{\cos^2\phi}$ is an increasing function of ϕ.

Q15civ As v increases, $\dfrac{\ell k}{m} - \dfrac{\ell g}{v^2}$ increases.

$\therefore\ \dfrac{\sin\phi}{\cos^2\phi} = \dfrac{\ell k}{m} - \dfrac{\ell g}{v^2}$ increases.

From part iv, ϕ increases.

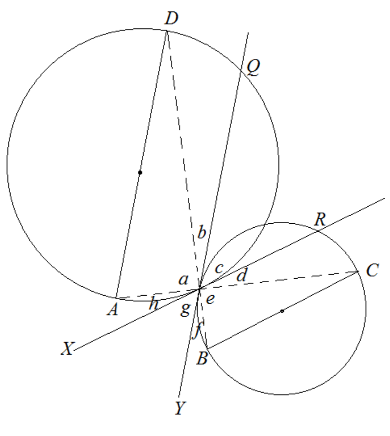
The toy aeroplane goes higher when its speed increases.

Q16ai $\angle APX = \angle ADP$, angles in alternate segments are equal

$\angle ADP = \angle DPQ$, alternate angles are equal, $AD \parallel PQ$

$\therefore\ \angle APX = \angle DPQ$

In the diagram, angles defined by solid/dotted lines are labelled as *a*, *b*, *c*, …, *h*. The lines intersect at point *P* (not shown).



Q16aii $a = e = 90°$, angles subtended by diameters

$c = g$, vertically opposite angles are equal

$b = h$, from part i

Similar to part i, $d = f$

$a+b+c+d+e+f+g+h = 360°$

$\therefore\ 2(a+b+c+d) = 360°$, $\therefore\ a+b+c+d = 180°$

$\therefore$ *A*, *P* and *C* are collinear.

Q16aiii $\angle ADP = h$ angles in alternate segments

$d = \angle PCB$ alternate angles, $d = h$ vertically opposite angles

$\therefore\ \angle ADP = \angle PCB$

Since both angles are subtended by the same arc *AB* $\therefore$ *A, B, C* and *D* define a circle, $\therefore$ *ABCD* is a cyclic quadrilateral.

Q16bi Let $a \geq 0$.

$\therefore\ -1 \leq \dfrac{(-1)^n}{1+a} \leq 1$ where *n* is a positive integer.

$\therefore\ -a^n \leq \dfrac{(-1)^n a^n}{1+a} \leq a^n$, $-a^n \leq \dfrac{1-1+(-1)^n a^n}{1+a} \leq a^n$

$\therefore\ -a^n \leq \dfrac{1}{1+a} - \dfrac{1-(-1)^n a^n}{1+a} \leq a^n$

$\therefore\ -a^n \leq \dfrac{1}{1+a} - \left(1 - a + a^2 - a^3 + \cdots + (-1)^{n-1}a^{n-1}\right) \leq a^n$

Let $a = x^2$ to obtain:

$-x^{2n} \leq \dfrac{1}{1+x^2} - \left(1 - x^2 + x^4 - x^6 + \cdots + (-1)^{n-1}x^{2n-2}\right) \leq x^{2n}$

Q16bii

$\int_0^1 -x^{2n}dx \leq \int_0^1 \frac{1}{1+x^2}dx - \int_0^1\left(1 - x^2 + x^4 - x^6 + \cdots + (-1)^{n-1}x^{2n-2}\right)dx \leq \int_0^1 x^{2n}dx$

$\therefore\ -\dfrac{1}{2n+1} \leq \dfrac{\pi}{4} - \left(1 - \dfrac{1}{3} + \dfrac{1}{5} - \cdots + (-1)^{n-1}\dfrac{1}{2n-1}\right) \leq \dfrac{1}{2n+1}$

Q16biii As $n \to \infty$, $\dfrac{\pi}{4} - \left(1 - \dfrac{1}{3} + \dfrac{1}{5} - \cdots + (-1)^{n-1}\dfrac{1}{2n-1}\right) \to 0$

$\therefore\ \dfrac{\pi}{4} = \lim_{n\to\infty}\left(1 - \dfrac{1}{3} + \dfrac{1}{5} - \cdots + (-1)^{n-1}\dfrac{1}{2n-1}\right) = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$.

Q16c Let $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{\ln x}{(1+\ln x)^2}$, $\therefore\ \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \dfrac{\ln x}{(1+\ln x)^2}$

$\therefore\ \dfrac{1}{v}\dfrac{du}{dx} - \dfrac{u}{v^2}\dfrac{dv}{dx} = \dfrac{1}{1+\ln x} - \dfrac{1}{(1+\ln x)^2}$

Let $\dfrac{du}{dx} = 1$ and $v = 1 + \ln x$, $\therefore\ \dfrac{dv}{dx} = \dfrac{1}{x}$ and $u = x$

Hence $\displaystyle\int \frac{\ln x}{(1+\ln x)^2}dx = \int \frac{d}{dx}\left(\frac{u}{v}\right)dx = \frac{u}{v} + c = \frac{x}{1+\ln x} + c$

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