2014 NSW BOS Mathematics Exam Solutions

Section I

1	2	3	4	5	6	7	8	9	10
A	B	D	C	B	D	B	C	A	D

Q1 $\frac{\pi^2}{6}=1.64$ (3 significant figures) **A**

Q2 The graph of $y=(x-1)^2$ is the graph of $y=x^2$ translated to the right by 1 unit. **B**

Q3 $\log_2(x-1)=8$, $x-1=2^8$, $x=257$ **D**

Q4 $\int e^{2x}dx=\frac{e^{2x}}{2}+c$ **C**

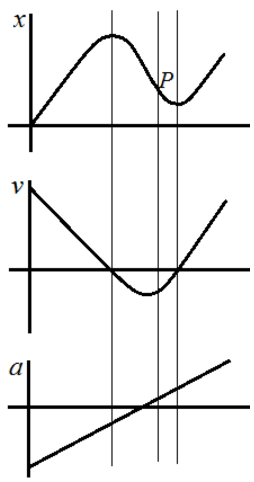
Q5 Gradient of $2x-3y=8$ is $\frac{2}{3}$, .: gradient of perpendicular line is $-\frac{3}{2}$. $3x+2y=6$ has gradient $-\frac{3}{2}$ and passes through the point $(2, 0)$. **B**

Q6 $8x^3+27=(2x)^3+3^3=(2x+3)\left((2x)^2-3(2x)+3^2\right)$
$=(2x+3)(4x^2-6x+9)$ **D**

Q7 $(\sin x-1)(\tan x+2)=0$, .: $\tan x+2=0$ which has 2 solutions between 0 and 2π. $\sin x-1=0$ has 1 solution, i.e. $x=\frac{\pi}{2}$ which is undefined for $\tan x$.
.: Total number of solutions is 2. **B**

Q8 $a=3x$, $r=-2x$, $t_{11}=(3x)(-2x)^{10}=3072x^{11}$ **C**

Q9 **A**



Q10
$\Pr(\text{at least one})=1-\Pr(\text{none})=1-\left(1-\frac{1}{4}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{2}{5}\right)=\frac{5}{8}$ **D**

Section II

Q11a $\frac{1}{\sqrt5-2}\times\frac{\sqrt5+2}{\sqrt5+2}=\frac{\sqrt5+2}{1}=\sqrt5+2$

Q11b $3x^2+x-2=(3x-2)(x+1)$

Q11c $\frac{d}{dx}\left(\frac{x^3}{x+1}\right)=\frac{(x+1)(3x^2)-(x^3)(1)}{(x+1)^2}=\frac{x^2(2x+3)}{(x+1)^2}$

Q11d $\int\frac{1}{(x+3)^2}dx=\int(x+3)^{-2}dx=\frac{(x+3)^{-1}}{-1}+c=-\frac{1}{x+3}+c$

Q11e $\int_0^{\frac{\pi}{2}}\sin\frac{x}{2}dx=\left[-\frac{\cos\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}=-2\cos\frac{\pi}{4}+2\cos 0=-\sqrt2+2$

Q11f $f'(x)=4x-5$, $f(x)=2x^2-5x+c$
The curve passes through $(2, 3)$.
.: $f(2)=2(2)^2-5(2)+c=3$, $c=5$. .: $y=2x^2-5x+5$

Q11g Arc length $=r\theta=8\times\frac{\pi}{7}=\frac{8\pi}{7}$ cm

Perimeter $=8+8+\frac{8\pi}{7}=8\left(2+\frac{\pi}{7}\right)$ cm

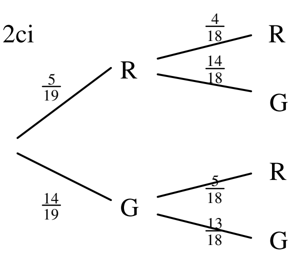
Q12a $a=2$, $d=3$, $t_n=1094$, .: $2+(n-1)3=1094$, $n=365$
$S_{365}=\frac{365}{2}(2+1094)=200020$

Q12bi Gradient $=\frac{1-4}{6-0}=-\frac{1}{2}$, .: $y=-\frac{1}{2}x+4$, .: $x+2y-8=0$

Q12bii Let $P(u,v)$ be a point on AC, and $BP\perp AC$.
.: $u+2v-8=0$ and $\frac{v-0}{u-3}\times\left(-\frac{1}{2}\right)=-1$
.: $\frac{v}{5-2v}=2$, $v=2$, .: $u=4$
$BP=\sqrt{(4-3)^2+(2-0)^2}=\sqrt5$

Q12biii $\overline{AC}=\sqrt{(6-0)^2+(1-4)^2}=\sqrt{45}$
Area of $\Delta ABC=\frac{1}{2}\sqrt{45}\sqrt5=\frac{15}{2}$ square units

Q12ci



RR $\frac{5}{19}\times\frac{4}{18}=\frac{10}{171}$

RG $\frac{5}{19}\times\frac{14}{18}=\frac{35}{171}$

GR $\frac{14}{19}\times\frac{5}{18}=\frac{35}{171}$

GG $\frac{14}{19}\times\frac{13}{18}=\frac{91}{171}$

Q12cii $\Pr(\text{diff. colours}) = \Pr(RG) + \Pr(GR) = \frac{35}{171} + \frac{35}{171} = \frac{70}{171}$

Q12di $y = -2x^2 + 8x$ and $y = 2x$

.: $2x = -2x^2 + 8x$, $2x^2 - 6x = 0$, $2x(x-3) = 0$, $x_A = 3$

Q12dii Area $= \int_0^3 (-2x^2 + 8x - 2x)dx = \int_0^3 (-2x^2 + 6x)dx$

$= \left[\frac{-2x^3}{3} + 3x^2\right]_0^3 = 9$ square units

Q13ai $\frac{d}{dx}(3 + \sin 2x) = \frac{d}{dx}\sin 2x = 2\cos 2x$

Q13aii Let $u = 3 + \sin 2x$, $\frac{1}{2}\frac{du}{dx} = \cos 2x$

$\int \frac{\cos 2x}{3 + \sin 2x}dx = \int \frac{\frac{1}{2}\frac{du}{dx}}{u}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\log_e u + c$

$= \frac{1}{2}\log_e(3 + \sin 2x) + c$

Q13bi $M = Ae^{-kt}$, $\frac{dM}{dt} = -kAe^{-kt}$, .: $\frac{dM}{dt} = -kM$

$M = Ae^{-kt}$ is a solution because it satisfies the equation $\frac{dM}{dt} = -kM$.

Q13bii $t = 0$, $M = 20$ kg, .: $A = 20$, $M = 20e^{-kt}$

At $t = 300$, $M = \frac{1}{2} \times 20$, .: $e^{-300k} = \frac{1}{2}$, $k = \frac{1}{300}\log_e 2$

.: at $t = 1000$, $M = 20e^{-\frac{\log_e 2}{300} \times 1000} \approx 1.98$ kg

Q13ci $x = t - \frac{1}{1+t}$; $v = \frac{dx}{dt} = 1 + \frac{1}{(1+t)^2}$; $a = \frac{dv}{dt} = -\frac{2}{(1+t)^3}$

.: $a < 0$ (i.e. a is negative) for $t \geq 0$

Q13cii As $t \to \infty$, $v = 1 + \frac{1}{(1+t)^2} \to 1$ m s^{-1}

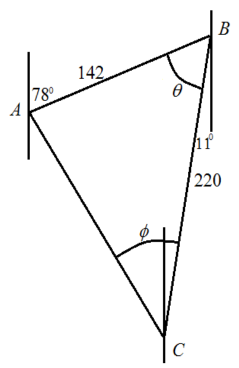
Q13di

$\theta = 78 - 11 = 67°$

$CA = \sqrt{142^2 + 220^2 - 2(142)(220)\cos 67°}$

≈ 210.12

.: CA is approximately 210 km



Q13dii $\frac{\sin\phi}{142} = \frac{\sin 67°}{210.12}$, .: $\phi \approx 38.47°$

.: bearing is $360° - (38.47° - 11°) \approx 333°$

Q14a $y = e^x - ex$, $\frac{dy}{dx} = e^x - e$

$\frac{dy}{dx} = 0$ at a stationary point, .: $e^x - e = 0$, .: $x = 1$ and $y = 0$

x	<1	1	>1
$\frac{dy}{dx}$	<0	0	>0

The stationary point is $(1, 0)$. It is a minimum turning point of $y = e^x - ex$.

Q14bi $2x^2 + 8x + k = 2\left(x^2 + 4x + \frac{k}{2}\right)$

$= 2(x-\alpha)(x-\beta) = 2(x^2 - (\alpha+\beta)x + \alpha\beta)$

.: $\alpha + \beta = -4$

Q14bii From bi, $\alpha\beta = \frac{k}{2}$

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$, .: $6 = \frac{k}{2}(-4)$, .: $k = -3$

Q14c Volume $= \int_0^4 \pi y^2 dx = \pi\int_0^4 (1 + 2\sqrt{x} + x)dx$

$= \pi\left[x + \frac{4x^{\frac{3}{2}}}{3} + \frac{x^2}{2}\right]_0^4 = \frac{68\pi}{3}$ cubic units

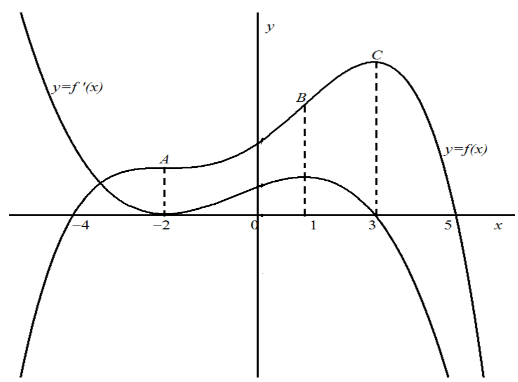
Q14di $\frac{1}{3} \times 10 + 10 = \frac{40}{3}$ mL

Q14dii Immediately after the nth dose, the total amount

$= 10 + \frac{1}{3} \times 10 + \left(\frac{1}{3}\right)^2 \times 10 + ... + \left(\frac{1}{3}\right)^{n-1} \times 10$ is a geometric series with $a = 10$ and $r = \frac{1}{3}$.

$S_\infty = \frac{a}{1-r} = \frac{10}{1-\frac{1}{3}} = 15$, .: the total amount will not exceed 15 mL

Q14e



Q15a $2\sin^2 x+\cos x-2=0$, $0\le x\le 2\pi$

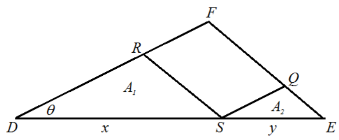
$2(1-\cos^2 x)+\cos x-2=0$

$\cos x-2\cos^2 x=0$, $\cos x(1-2\cos x)=0$

.: $\cos x=0$, $x=\frac{\pi}{2},\frac{3\pi}{2}$ or $1-2\cos x=0$, $x=\frac{\pi}{3},\frac{5\pi}{3}$

The solutions are $x=\frac{\pi}{3},\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{3}$

Q15bi



$RS\,//\,FE$, .: $\angle DRS=\angle DFE$ and $\angle DSR=\angle DEF$ because corresponding angles are equal. θ is a common angle to both triangles. .: ΔDSR and ΔDEF are similar.

Q15bii Ratios of any two pairs of corresponding sides of two similar triangles are equal, .: $\frac{DR}{DF}=\frac{DS}{DE}=\frac{x}{x+y}$

Q15biii $\frac{A_1}{A}=\frac{\frac{1}{2}(DR)(DS)\sin\theta}{\frac{1}{2}(DF)(DE)\sin\theta}=\frac{x}{x+y}.\frac{x}{x+y}=\left(\frac{x}{x+y}\right)^2$

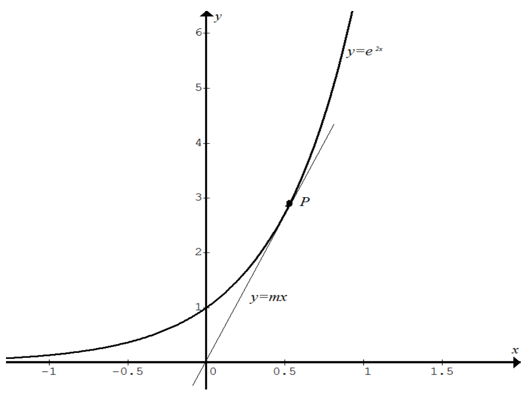
.: $\frac{x}{x+y}=\sqrt{\frac{A_1}{A}}$

Q15biv From biii, $\sqrt{\frac{A_1}{A}}=\frac{x}{x+y}$. Similarly, $\sqrt{\frac{A_2}{A}}=\frac{y}{x+y}$

$\sqrt{\frac{A_1}{A}}+\sqrt{\frac{A_2}{A}}=\frac{x}{x+y}+\frac{y}{x+y}$, .: $\frac{\sqrt{A_1}}{\sqrt{A}}+\frac{\sqrt{A_2}}{\sqrt{A}}=\frac{x+y}{x+y}=1$

.: $\sqrt{A}=\sqrt{A_1}+\sqrt{A_2}$

Q15ci



Q15cii At P, $e^{2x}=mx$ and $\frac{d}{dx}e^{2x}=\frac{d}{dx}mx$, .: $2e^{2x}=m$

.: $2mx=m$, $2x=1$, $x=\frac{1}{2}$, $y=e^{2x}=e$, .: $P\left(\frac{1}{2},e\right)$

Q15ciii At P, $e^{2x}=mx$, .: $e=\frac{m}{2}$, $m=2e$

Q16a

$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\sec x\,dx\approx\frac{\frac{\pi}{3}}{6}\left(\sec\left(-\frac{\pi}{3}\right)+4\sec\left(-\frac{\pi}{6}\right)+2\sec 0+4\sec\frac{\pi}{6}+\sec\frac{\pi}{3}\right)$

$=\frac{\pi}{18}\left(2+\frac{8}{\sqrt{3}}+2+\frac{8}{\sqrt{3}}+2\right)=\frac{\pi}{18}\left(6+\frac{16}{\sqrt{3}}\right)=\frac{\pi}{9}\left(3+\frac{8}{\sqrt{3}}\right)$

Q16bi

	Start of month	End of month
1st	500	$500(1.003)$
2nd	$500(1.003)+500(1.01)$	$500(1.003)^2+500(1.01)(1.003)$

Q16bii

Start of 3rd month $=500(1.003)^2+500(1.01)(1.003)+500(1.01)^2$

End of 3rd month

$=500(1.003)^3+500(1.01)(1.003)^2+500(1.01)^2(1.003)$

$=500(1.003)\left((1.01)^2+(1.01)(1.003)+(1.003)^2\right)$

$=500(1.003)\left(\frac{(1.01)^3-(1.003)^3}{1.01-1.003}\right)$

.: end of the 60th month $=500(1.003)\left(\frac{(1.01)^{60}-(1.003)^{60}}{1.01-1.003}\right)$

$\approx \$44404$

Q16ci Perimeter $=\frac{\pi x}{2}+2x+2y=10$, .: $\frac{\pi x}{4}+x+y=5$

$y=5-x-\frac{\pi x}{4}$, $y=5-x\left(1+\frac{\pi}{4}\right)$

Q16cii Area of the semicircular section $=\frac{1}{2}\pi\left(\frac{x}{2}\right)^2=\frac{\pi x^2}{8}$

Area of the rectangular section $=xy=5x-x^2\left(1+\frac{\pi}{4}\right)$

$L=\frac{\pi x^2}{8}\times 1+\left(5x-x^2\left(1+\frac{\pi}{4}\right)\right)\times 3$.: $L=15x-x^2\left(3+\frac{5\pi}{8}\right)$

Q16ciii $L=15x-x^2\left(3+\frac{5\pi}{8}\right)$ is an inverted parabola, .: the turning point is a maximum.

Let $\frac{dL}{dx}=0$, $15-2x\left(3+\frac{5\pi}{8}\right)=0$, $x=\frac{60}{24+5\pi}\approx 1.511\,\text{m}$

$y=5-\left(\frac{60}{24+5\pi}\right)\left(1+\frac{\pi}{4}\right)=\frac{60+10\pi}{24+5\pi}\approx 2.3022$ m

Please inform mathline@itute.com re conceptual and/or mathematical errors.