2014 VCAA Further Mathematics Exam 2 Solutions

Core – Data analysis

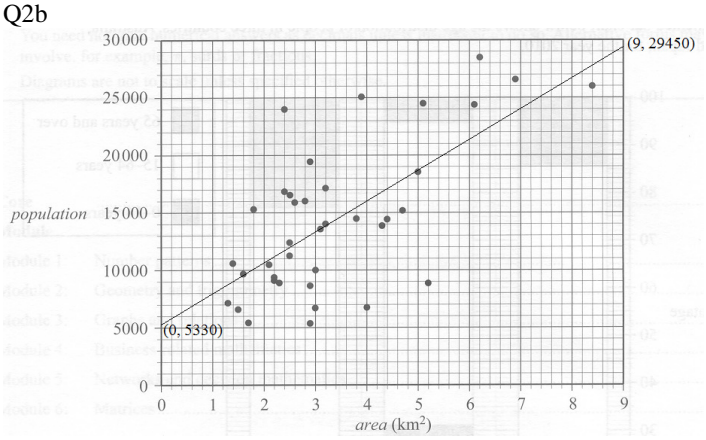
Q1a 19 %

Q1b $128\ 000\ 000 \times \left(\frac{100-77}{100}\right) = 29\ 440\ 000$

Q1c It appears that there is no association between the percentage of people in the 15–64 age group and the country in which they live because the percentage is about the same in all three countries: 67 % in Australia, 64 % in India and 64 % in Japan.

Q2a Population is the dependent variable.

Q2b



Q2c The rate of increase in population is approximately 2680 for every increase of 1 km^2 in area.

Q2di Predicted population $= 5330 + 2680 \times 4 = 16\ 050$

Residual $= 6690 - 16\ 050 = -9360$

Q2dii $r^2 = 0.668^2 \approx 0.446$, i.e. 44.6 %

Q3a $population = 7.7 + 7.7 \times \log_{10}(area)$

Q3b $population = 7.7 + 7.7 \times \log_{10} 90 \approx 23$ thousands

Q4a $r = -\sqrt{0.141} = -0.3755$, .: the association between *population density* and *area* is weak, negative and linear.

Q4bi $z = \frac{3082 - 4370}{1560} \approx -0.8$

Q4bii $38 \times \frac{2.5}{100} \approx 1$

Q4biii $\mu + 2\sigma = 3.4 + 2 \times 1.6 = 6.6$, .: 2 inner suburbs (read from graph)

Module 1: Number patterns

Q1a 20 000

Q1b $1 - 0.99 = 0.01 = 1\%$

Q1c $L_{2015} = 0.99\, L_{2014} = 0.99 \times 20\ 000 = 19\ 800$

Q1d $0.01 L_{2015} = 0.01 \times 19\ 800 = 198$

Q2a $r = \frac{0.68}{0.8} = 0.85$

Q2b $0.578 \times 0.85^3 = 0.35$

Q2c $S_5 = \frac{0.8\left(1 - 0.85^5\right)}{1 - 0.85} \approx 2.97$

Q2d Let $S_n = 4$, $\frac{0.8\left(1 - 0.85^n\right)}{1 - 0.85} = 4$, $n \approx 8.53$, .: in 2022

Q3a $H_{2015} = 0.85\, H_{2014} + 500 = 0.85 \times 14\ 000 + 500 = 12\ 400$

Q3b $E_{2016} = 1.06^2 \times 5000 = 5618$ elephants

$H_{2016} = 0.85 \times 12\ 400 + 500 = 11\ 040$ km^2

Number of km^2 per elephant at the end of 2016 $= \frac{11\ 040}{5618} \approx 1.97 < 2$

.: expected to be overpopulated in 2016

Q3c To remain constant, $P_{2015} = 1.06\, P_{2014} - k = P_{2014}$

.: $k = 0.06\, P_{2014} = 0.06 \times 5\ 000 = 300$

Q3d $1.06 \times 5\ 000 - k = 5\ 000 - 100$, $k = 400$

Q3e Maximum number of elephants $= \frac{11\ 040}{2} = 5\ 520$

$P_{2015} = 1.06 \times 5\ 000 - k = 5\ 300 - k$

$P_{2016} = 1.06\left(5\ 300 - k\right) - k = 5\ 618 - 2.06k = 5\ 520$

.: $k \approx 47.6$, .: minimum number of elephants to be moved is 48.

Module 2: Geometry and trigonometry

Q1a Area of the floor $= \frac{1}{2}(3+5)(2) = 8\ m^2$

Q1b Perimeter $= 3 + 2 + 5 + \sqrt{2^2 + (5-3)^2} \approx 12.8$ m

Q2a $\theta = 180° - (45° + 60°) = 75°$

Q2b $\frac{AX}{\sin 45°} = \frac{3.16}{\sin 75°}$

Q2c $\frac{AX}{\sin 45°} = \frac{3.16}{\sin 75°}$, $AX = \frac{3.16 \times \sin 45°}{\sin 75°} \approx 2.31$

Q2d Area of $\Delta ADX = \frac{1}{2} \times 3.16 \times 2 \approx 3.2 \text{ m}^2$

Q2e Roof area $= \frac{1}{2}(3+5-3.16)(2) = 4.84$

Wall area $= (3+2+5-3.16)(1.8) = 12.312$

Total area $4.84 + 12.312 \approx 17 \text{ m}^2$

Q3a Area $= \frac{1}{2}(4\pi \times 5^2) \approx 157 \text{ cm}^2$

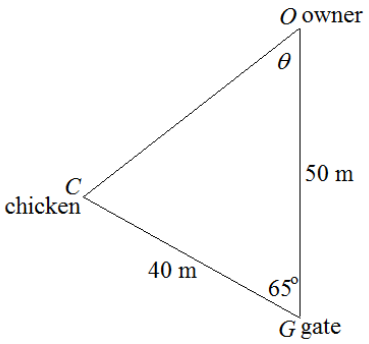
Q3b Max. volume = volume of hemisphere + volume of cylinder

$= \frac{1}{2}\left(\frac{4\pi}{3} \times 5^3\right) + \pi 5^2 \times 15 \approx 1\,440 \text{ cm}^3$

Q3c Volume ratio (feed : water) $= 1 : \frac{3}{4}$, $\therefore$ area ratio $= 1 : \left(\frac{3}{4}\right)^{\frac{2}{3}}$

$\therefore\ x : 628 = 1 : \left(\frac{3}{4}\right)^{\frac{2}{3}}$, $\therefore\ x \approx 761 \text{ cm}^2$

Q4



$OC = \sqrt{40^2 + 50^2 - 2(40)(50)\cos 65°} \approx 49$, $\frac{\sin\theta}{40} = \frac{\sin 65°}{49}$

$\therefore\ \theta \approx 48°$ and the bearing of C from $O = 180° + 48° = 228°$

Module 3: Graphs and relations

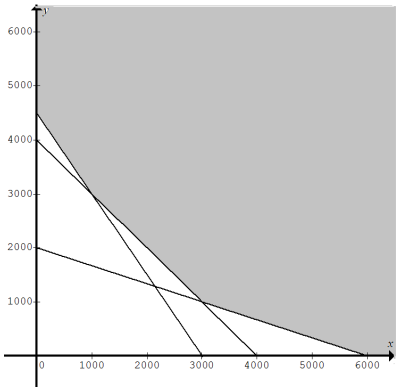
Q1a $0.02 \times 2 = 0.04 \text{ kg}$

Q1b $0.05 \times 100 + 0.05 \times 400 = 25 \text{ kg}$

Q1c $0.06x + 0.04y \geq 180$ kg

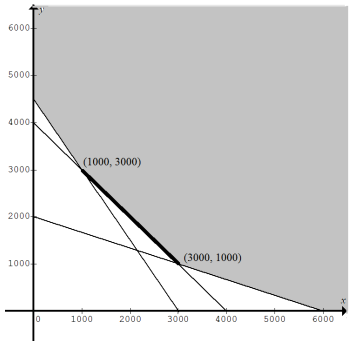
Q1di $y = 2\,000 - \frac{1}{3}x$, it can be expressed as $0.02x + 0.06y = 120$

Q1dii

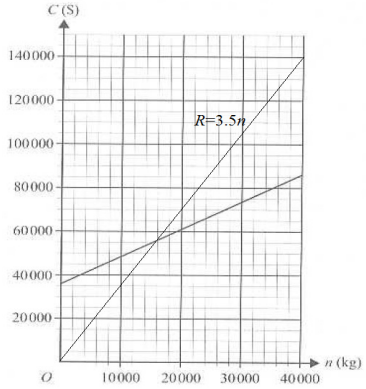


Q1ei $x + y = 4\,000$ is the least amount.

Q1eii The points are shown as a thick solid line in the following diagram. Vertices $(1\,000,\ 3\,000)$ and $(3\,000,\ 1\,000)$ are included.



Q2a



Q2b Profit in dollars

$= R - C = 3.5 \times 20\,000 - (1.25 \times 20\,000 + 36\,000) = 9\,000$

Q3a Revenue $= 10.8 + 4(8-2) = 34.8$ dollars

Q3b Assuming the revenue is a continuous function of n,

$10.8 + 4(n-2) = a + 2(n-10)$ when $n = 10$

$\therefore\ a = 10.8 + 4(10-2) = 42.8$

Q3c When $n \leq 10$, $R - C > 0$. $R - C$ decreases to zero as n increases above 10.

Let $R - C = 0$, i.e. $42.8 + 2(n-10) - 3.5n = 0$, $n = 15.2$.

$\therefore$ the maximum number of kg is 15.2 to break even.

Module 4: Business-related mathematics

Q1a $\frac{30}{150} = 0.20 = 20\%$

Q1b $150 + 15 \times 12 = 330$ dollars

Q1c $150 \times 5\% \times 2 = 15$ dollars

Q1di $59\,700 - 42\,700 = 17\,000$ dollars

Q1dii $42\,700 \times r\% \times \frac{1}{12} = 125.12$, $\therefore\ r \approx 3.5$

Q2a $20\,000 \times r\% = 750$, $r = 3.75$

Q2b \$20 000

Q2ci $\$750\times(1+0.03)=\$772.50$

Q2cii Let $\$x$ be the 2014 value. $x\times1.03^{10}=750$, $\therefore\ x\approx558$

Q3a Interest $=60\,000-45\,550=14\,450$ dollars

Q3b By TVM Solver:
$N=16, PV=-45\,550, PMT=0, FV=60\,000, P/Y=4, C/Y=4$
$I\%=6.948\,002\,081$
The annual rate of interest was 6.9%.

Q3ci account balance $=60\,000\times\left(1+\dfrac{7.2}{100\times12}\right)+885$

Q3cii Account balance $\approx\$75\,443$

Q4 Consider the balance of $\$143\,585.33$.
After the next monthly repayment,
the balance $=143\,585.33\times\left(1+\dfrac{4.5}{100\times12}\right)-2500\approx141\,623.775$
The loan is reduced by $143\,585.33-141\,623.775\approx1\,961.555$
Percentage of the monthly repayment $=\dfrac{1\,961.555}{2\,500}\approx0.78=78\%$

Module 5: Networks and decision mathematics

Q1a 2

Q1b miniature trains

Q2a

Task	Andrew	Brianna	Charlie	Devi
publicity	3	2	0	0
finances	0	1	2	2
equipment	0	4	3	2
catering	1	2	3	0

Q2b The second column does not contain zeros.

Q2ci equipment

Q2cii $8+10+10+8=36$ hours

Q3ai Bower, Eden ***or*** Eden, Bower

Q3aii Distance
$=160+130+80+70+60+40+100+150+120=910$ km

Q3b Shortest Hamiltonian path: B-C-D-E-A
Distance $=70+60+40+100=270$ km

Q3c Remove the train line directly connecting ***Bower*** and ***Derrin*** to make all vertices even.

Q4a $5+2=7$ hours after the restoration is started.

Q4b $21-3=18$ hours after the restoration is started.

Q4c Float time of $J=15-11-2=2$ hours

Q4d $5+6-7=4$ hours after the restoration is started.

Q4e Crashing activity A by 3 hours, cost $=\$90\times3=\$270$

Module 6: Matrices

Q1a 4×2

Q1b 1850

Q1c The total number of adult females living in the small city.

Q1d The number of columns in V = the number of rows in P

Q1e $w=1360\times0.45+1460\times0.55=1415$

Q1f $1\,415+1\,812+988+1\,806=6\,021$

Q2ai 20%

Q2aii $5\%+20\%=25\%$

Q2b $6\,000\times5\%+2\,160\times40\%=1164$

Q2ci $S_3=T^2S_1=\begin{bmatrix}0.75&0.10&0.20\\0.05&0.80&0.40\\0.20&0.10&0.40\end{bmatrix}^2\begin{bmatrix}6\,000\\3\,840\\2\,160\end{bmatrix}=\begin{bmatrix}4\,900\\4\,634\\2\,466\end{bmatrix}\begin{matrix}A\\B\\C\end{matrix}$

Q2cii The number of voters who are expected to have a preference for each candidates A, B and C in that order in March.

Q2d $S_6=T^5S_1=\begin{bmatrix}0.75&0.10&0.20\\0.05&0.80&0.40\\0.20&0.10&0.40\end{bmatrix}^5\begin{bmatrix}6\,000\\3\,840\\2\,160\end{bmatrix}\approx\begin{bmatrix}4\,334\\5\,303\\2\,363\end{bmatrix}\begin{matrix}A\\B\\C\end{matrix}$

B is the winner and expected to receive 5303 votes.

Q3a 10% of B changes to C before C would withdraw.
After C would withdraw, the 10% would split equally, 5% to A (from 10% to 15%) and 5% to B (from 80% to 85%).
The required percentage $=\dfrac{5\%}{10\%}=0.5=50\%$

Q3b $S_5=T^4S_1=\begin{bmatrix}0.75&0.10&0.20\\0.05&0.80&0.40\\0.20&0.10&0.40\end{bmatrix}^4\begin{bmatrix}6\,000\\3\,840\\2\,160\end{bmatrix}\approx\begin{bmatrix}4\,454\\5\,154\\2\,392\end{bmatrix}\begin{matrix}A\\B\\C\end{matrix}$

$S_6=\begin{bmatrix}0.75&0.15&0.6\\0.25&0.85&0.4\\0&0&0\end{bmatrix}\begin{bmatrix}4\,454\\5\,154\\2\,392\end{bmatrix}\approx\begin{bmatrix}5\,549\\6\,451\\0\end{bmatrix}\begin{matrix}A\\B\\C\end{matrix}$

B is expected to receive 6 451 votes.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors