2014 VCAA Math Methods CAS Exam 1 Solutions

Q1a $y = x^2 \sin x$, $\frac{dy}{dx} = x^2 \cos x + 2x \sin x$

Q1b $f(x) = \sqrt{x^2+3}$, $f'(x) = \frac{1}{2\sqrt{x^2+3}} \times 2x = \frac{x}{\sqrt{x^2+3}}$

$f'(1) = \frac{1}{\sqrt{1^2+3}} = \frac{1}{2}$

Q2 $\int_4^5 \frac{2}{2x-1}dx = \left[\frac{2\log_e(2x-1)}{2}\right]_4^5 = \log_e 9 - \log_e 7 = \log_e \frac{9}{7}$.: $b = \frac{9}{7}$

Q3 $2\cos(2x) = -\sqrt{3}$ and $0 \le x \le \pi$, $\cos(2x) = -\frac{\sqrt{3}}{2}$

.: $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$, $x = \frac{5\pi}{12}, \frac{7\pi}{12}$

Q4 $2^{3x-3} = 8^{2-x}$, $\left(2^3\right)^{x-1} = 8^{2-x}$, $8^{x-1} = 8^{2-x}$, $x-1 = 2-x$, $x = \frac{3}{2}$

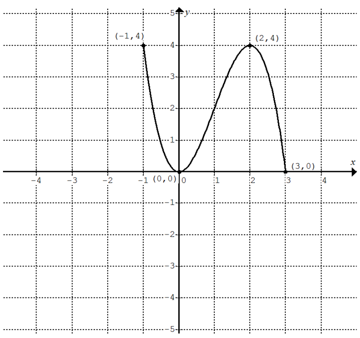
Q5a $f(x) = 3x^2 - x^3$ and $-1 \le x \le 3$

$f'(x) = 6x - 3x^2 = 3x(2-x)$ Let $f'(x) = 0$ for stationary points.

$3x(2-x) = 0$, .: $x = 0$ or $x = 2$, and the corresponding ordinates are $y = 0$ and $y = 4$.

The stationary points are $(0,0)$ and $(2,4)$.

Q5b



Q5c $\int_{-1}^{2}\left[4-\left(3x^2-x^3\right)\right]dx = \left[4x - x^3 + \frac{x^4}{4}\right]_{-1}^{2} = \frac{27}{4}$

Q6 $\log_e x - 3 = \log_e \sqrt{x}$ and $x > 0$

$\log_e x - \log_e \sqrt{x} = 3$, $\log_e \frac{x}{\sqrt{x}} = 3$, $\log_e \sqrt{x} = 3$

.: $\sqrt{x} = e^3$, .: $x = e^6$

Q7 $f'(x) = 2\cos x - \sin(2x)$ and $f\left(\frac{\pi}{2}\right) = \frac{1}{2}$

$f(x) = \int (2\cos x - \sin(2x))dx = 2\sin x + \frac{\cos(2x)}{2} + c$

$f\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} + \frac{\cos(\pi)}{2} + c = \frac{1}{2}$

.: $2 - \frac{1}{2} + c = \frac{1}{2}$, $c = -1$, .: $f(x) = 2\sin x + \frac{\cos(2x)}{2} - 1$

Q8a $\int_0^m \frac{1}{5}e^{-\frac{x}{5}}dx = \frac{1}{2}$, $\left[-e^{-\frac{x}{5}}\right]_0^m = \frac{1}{2}$, $-e^{-\frac{m}{5}} + e^0 = \frac{1}{2}$

.: $e^{-\frac{m}{5}} = \frac{1}{2}$, $e^{\frac{m}{5}} = 2$, $\frac{m}{5} = \log_e 2$, $m = 5\log_e 2$

Q8b For $m > 1$, $\Pr(X < 1 \mid X \le m) = \frac{\Pr(X < 1 \cap X \le m)}{\Pr(X \le m)}$

$= \frac{\Pr(X<1)}{\Pr(X \le m)} = \frac{\left[-e^{-\frac{x}{5}}\right]_0^1}{0.5} = \frac{-e^{-\frac{1}{5}} + e^0}{0.5} = 2\left(1 - e^{-\frac{1}{5}}\right)$

Q9a $\Pr(\text{at least once}) = 1 - \Pr(\text{none}) = 1 - \frac{1}{4} \times \frac{2}{3} = \frac{5}{6}$

Q9bi

$\Pr(\text{walk}) = \Pr(\text{pleasant and walk}) + \Pr(\text{unpleasant and walk})$

$= \frac{5}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{3} = \frac{19}{32}$

Q9bii $\Pr(\text{pleasant} \mid \text{walk}) = \frac{\Pr(\text{pleasant} \cap \text{walk})}{\Pr(\text{walk})} = \frac{\frac{15}{32}}{\frac{19}{32}} = \frac{15}{19}$

Q10a $y = ax^2 + bx$, $\frac{dy}{dx} = 2ax + b$

At $(2,4)$, $4 = a\left(2^2\right) + b(2)$, .: $4a + 2b = 4$, .: $2a + b = 2$ ... (1)

also gradient of the tangent $= 2a(2) + b = 4a + b = \frac{0-4}{6-2}$,

.: $4a + b = -1$ … (2)

Solve (1) and (2) simultaneously, $a = -\frac{3}{2}$ and $b = 5$

Q10bi Gradient of VQ = gradient of QU

.: $\frac{v-4}{0-2} = \frac{4-0}{2-u}$, $v - 4 = \frac{8}{u-2}$, $v = 4 + \frac{8}{u-2}$, $v = \frac{4u}{u-2}$

Q10bii Shaded area $A(u) = \frac{1}{2}uv - 8 = \frac{2u^2}{u-2} - 8$

Let $\frac{dA}{du} = 0$ to find the turning point(s).

$\frac{(u-2)(4u) - \left(2u^2\right)(1)}{(u-2)^2} = 0$.: $2u^2 - 8u = 0$, $2u(u-4) = 0$.

Since $u > 0$, .: $u = 4$ and $A = \frac{2\left(4^2\right)}{4-2} - 8 = 8$

First end point at $u = \frac{5}{2}$, $A = \frac{2\left(\frac{5}{2}\right)^2}{\frac{5}{2} - 2} - 8 = 17$

Second end point at $u = 6$, $A = \frac{2\left(6^2\right)}{6-2} - 8 = 10$

.: $A_{min} = 8$ square units

Q10biii From part ii, $A_{max} = 17$ square units

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors