2015 Mathematical Methods

Year 12 Modelling Task

Time allowed: 2.5 hours

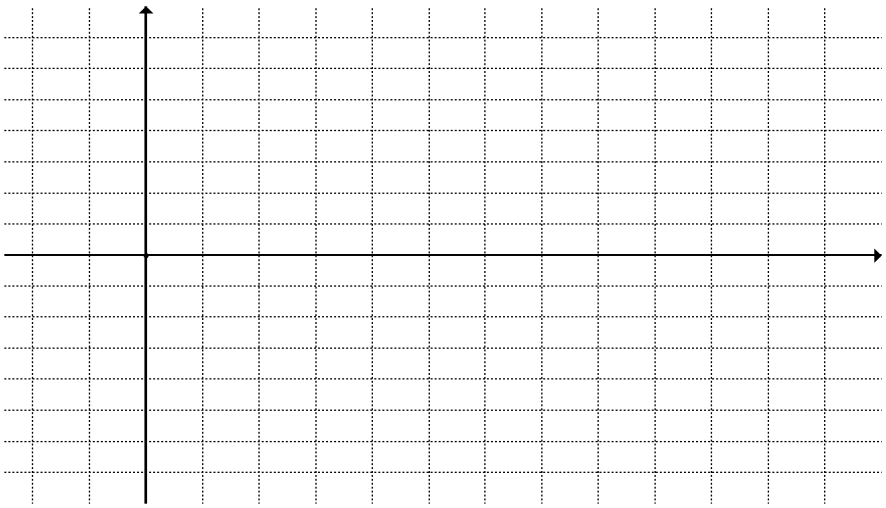
You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator
Working must be shown for questions worth 2 or more marks. Total: 70 marks

Theme: Medication dosages

Question 1

a) Consider a function given as $f(t) = -(t-10) + 0.01(t-10)^3$.
Sketch the graph of $y = f(t)$, labelling coordinates of axis-intercepts (give exact values) and turning points (correct to 2 decimal places).

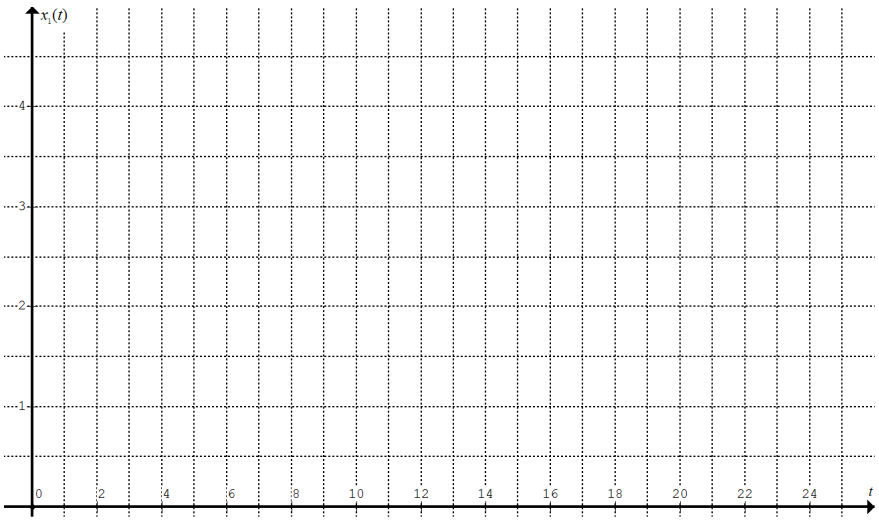
4 marks



Medication B is given to a patient and the concentration in the patient's body can be modelled by function $x_{B1}(t) = -(t-10) + 0.01(t-10)^3$ units, where $t = 0$ is the time (in hours) the medication is first taken.

b) What is a suitable domain for function $x_{B1}(t)$? 1 mark

c) Sketch the graph of function $x_{B1}(t)$ for appropriate values of t. 2 marks



d) **Calculate** the maximum concentration of Medication B and the time when it happens, correct to 2 decimal places.

4 marks

e) Find the rate of change of the concentration of Medication B in the body at $t = 1$, and $t = 6$. Interpret the answers in terms of absorption/clearance of the medication in the patient's body.

4 marks

f) Medication B is considered to be effective if the concentration is at least 1.8 units. Specify the time interval when it happens. Express your answer in hours and nearest minutes.

2 marks

Question 2

A second dose $x_{B2}(t)$ of Medication B is given to the patient at $t = 8$.

a) The second dose $x_{B2}(t)$ of the medication is a **translation** of $x_{B1}(t)$, i.e. $x_{B2}(t) = x_{B1}(t - \alpha)$. State the value of α.

1 mark

b) State the domain of $x_{B2}(t)$. 1 mark

c) State the rule of $x_{B2}(t)$. 2 marks

d) During certain time interval both doses are in the patient's body. The total concentration of the two doses is modelled by $x_{B1}(t) + x_{B2}(t)$.

i) Find $x_{B1}(t) + x_{B2}(t)$ in terms of t. 1 mark

ii) Express $x_{B1}(t) + x_{B2}(t)$ as a polynomial function of t, i.e. in the form $at^3 + bt^2 + ct + d$ where a, b, c and d are real constants.

2 marks

iii) State the maximal domain of $x_{B1}(t) + x_{B2}(t)$ for it to be defined. 1 mark

e) For the time between $t = 9$ and $t = 10$, determine the maximum medication concentration and when it happens, correct to 2 decimal places.

2 marks

Question 3

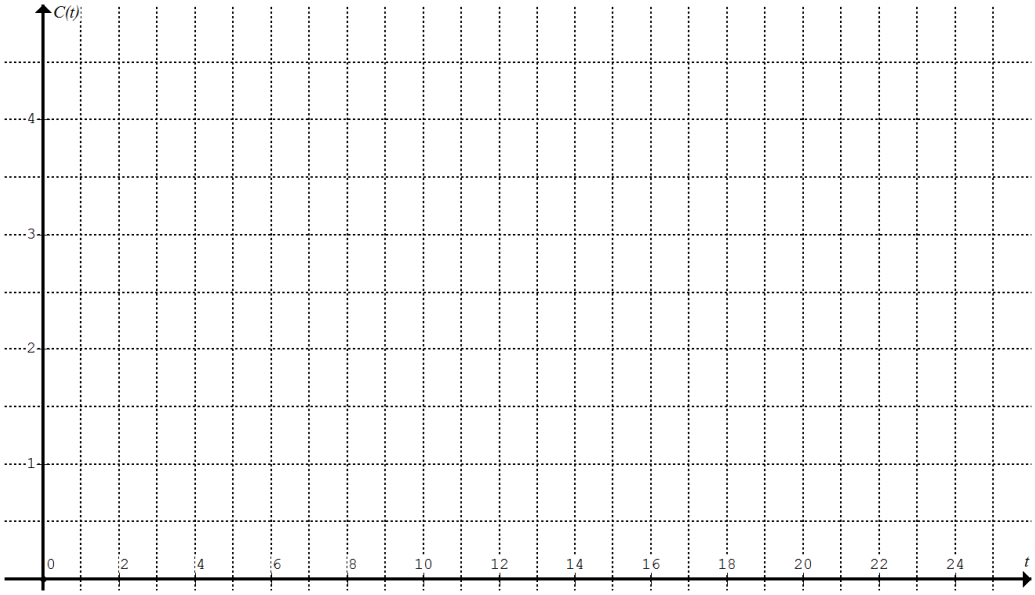
A third dose is given at $t = 16$, and a fourth dose at $t = 24$.

a) Using your responses from Questions 1 and 2, define a hybrid function $C_B(t)$ which models the Medication B concentration in the patient's body for the first 24 hours.
You **do not** need to express each part as a polynomial function of t.

10 marks

b) Sketch the graph which models the concentration of Medication B in the patient's body in the first 24 hours (just before the fourth dose), labelling the points with coordinates where parts are joined (cusps).

5 marks



Medication A was taken by the patient before Medication B.
The concentration of Medication A in the patient's blood can be modelled by $x_A = t - 0.01t^3$, where $t = 0$ is the time (hours) the medication is first taken.

Medication A is considered to be effective if the concentration is at least 1.8 units.

Let the first dose, the second dose, the third dose, … be $x_{A1}, x_{A2}, x_{A3}, \ldots$

Typically, absorption of a drug by a body is required to be swift, whilst clearance is more gradual.

Question 4

a. Compare Medication A and Medication B in relation to the rates of absorption and clearance. Justify your answer.

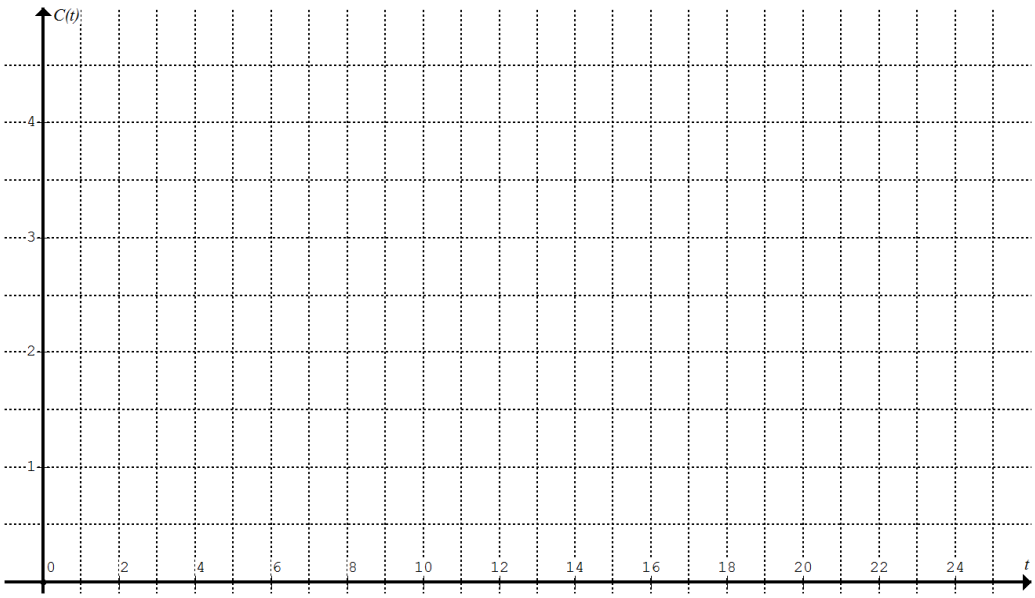
8 marks

b. Find the latest time in hours after the first dose (correct to 4 decimal places) that the second dose must be taken if the concentration of Medication A is at least 1.8 units.
Find $x_{A1}(t) + x_{A2}(t)$ and state its domain. Verify that $x_{A1}(t) + x_{A2}(t) \geq 1.8$.

8 marks

c. Sketch the graph which models the concentration of Medication A in the patient's blood for the first three doses if the concentration of Medication A is at least 1.8 units.
Show the graph and equation(s) from $t = 0$ to just before the fourth dose.
Label the points with coordinates where parts are joined.

12 marks



End of task