2015 NSW BOS Mathematics Exam Solutions

Section I

1	2	3	4	5	6	7	8	9	10
D	C	A	A	B	C	B	C	D	A

Q1 $0.00523359 = 5.234\times 10^{-3}$ (4 significant figures) D

Q2 $2x-4y+3=0$, $4y=2x+3$, $y=\frac{1}{2}x+\frac{3}{4}$ C

Q3 $a=3, d=4$, $t_{15}=3+(15-1)4=59$ A

Q4 $\left(1-\frac{5}{7}\right)\left(1-\frac{2}{3}\right)=\frac{2}{21}$ A

Q5 $\frac{1}{2}\left(e^1+\frac{3}{2}e^{1.5}\right)\frac{1}{2}+\frac{1}{2}\left(\frac{3}{2}e^{1.5}+2e^2\right)\frac{1}{2}+\frac{1}{2}\left(2e^2+\frac{5}{2}e^{2.5}\right)\frac{1}{2}$

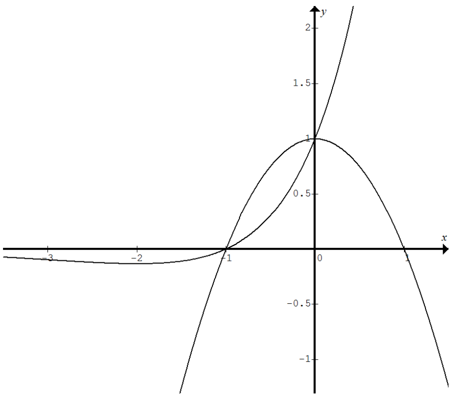
$+\frac{1}{2}\left(\frac{5}{2}e^{2.5}+3e^3\right)\frac{1}{2}=\frac{1}{4}\left(e^1+3e^{1.5}+4e^2+5e^{2.5}+3e^3\right)$ B

Q6 $\frac{dy}{dx}=6\cos 3x-3\sec^2 x$

At $x=0$, $\frac{dy}{dx}=6\times 1-3\times 1=3$ C

Q7 $\int_0^2\left(4x-x^2-2x\right)dx=\int_0^2\left(2x-x^2\right)dx$ B

Q8 The graphs of $y=e^x(1+x)$ and $y=1-x^2$ intersect at two points. C



Q9 Initial position $x=2$, positive displacement of $\frac{1}{2}\times 4\times 8=16$ from $t=0$ to $t=4$, .: $x_{max}=2+16=18$ D

Q10 $\int_1^d \frac{2}{x}dx=2$, $\left[2\log_e x\right]_1^d=2$, $2\log_e d=2$, $d=e$ A

Section II

Q11a $4x-(8-6x)=4x-8+6x=10x-8$

Q11b $3x^2-27=3\left(x^2-3^2\right)=3(x-3)(x+3)$

Q11c $\frac{8}{2+\sqrt{7}}=\frac{8}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}=\frac{8\left(\sqrt{7}-2\right)}{3}$

Q11d $a=1$, $r=-\frac{1}{4}$, $S_\infty=\frac{1}{1-\left(-\frac{1}{4}\right)}=\frac{1}{\frac{5}{4}}=\frac{4}{5}$

Q11e $\frac{d}{dx}\left(e^x+x\right)^5=5\left(e^x+x\right)^4\left(e^x+1\right)$

Q11f $\frac{dy}{dx}=(x+4)\times\frac{1}{x}+1\times\ln x=1+\frac{4}{x}+\ln x$

Q11g $\int_0^{\frac{\pi}{4}}\cos 2x\,dx=\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}=\frac{1}{2}\sin\frac{\pi}{2}-0=\frac{1}{2}$

Q11h $\int\frac{x}{x^2-3}dx=\int\frac{1}{2u}du=\frac{1}{2}\ln|u|+c=\frac{1}{2}\ln\left|x^2-3\right|+c$

Q12a $\sin\theta=\frac{1}{2}$, $\theta=\frac{\pi}{6}, \frac{5\pi}{6}$

Q12bi Gradient of ℓ_2 is $-\frac{1}{3}$, .: gradient of ℓ_1 is 3.

Equation of ℓ_1: $y-11=3(x-7)$, i.e. $y=3x-10$

Q12bii Solve simultaneous equations $y=3x-10$ and $y=-\frac{x}{3}$.

$3x-10=-\frac{x}{3}$, $9x-30=-x$, $10x=30$, $x=3$, $y=-1$.: $D(3,-1)$

Q12c $f(x)=\frac{x^2+3}{x-1}$

$f'(x)=\frac{(x-1)(2x)-\left(x^2+3\right)(1)}{(x-1)^2}=\frac{x^2-2x-3}{(x-1)^2}=\frac{(x-3)(x+1)}{(x-1)^2}$

Q12d For real roots, $\Delta\ge 0$, $(-8)^2-4(1)k\ge 0$, $4k\le 64$, $k\le 16$

Q12ei $\frac{dy}{dx}=x$, at $x=1$, $m_t=1$

Equation of the tangent: $y-\frac{1}{2}=1(x-1)$, .: $y=x-\frac{1}{2}$

Q12eii $y=-\frac{1}{2}$

Q12eiii Solve simultaneously $y=x-\frac{1}{2}$ and $y=-\frac{1}{2}$, .: $x=0$

.: $Q\left(0,-\frac{1}{2}\right)$, .: Q lies on the y - axis

Q12eiv $\overline{PS}=1-0=1$, $\overline{PQ}=\frac{1}{2}-\left(-\frac{1}{2}\right)=1$.: ΔPQS is isosceles.

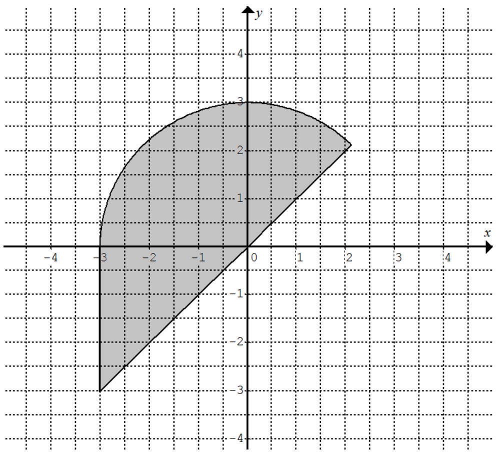
Q13ai $\cos A = \dfrac{8^2 + 6^2 - 4^2}{2(8)(6)} = \dfrac{7}{8}$

Q13aii $\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\dfrac{7}{8}\right)^2} = \dfrac{\sqrt{15}}{8}$

Area (cm^2) of $\Delta ABC = \dfrac{1}{2} \times 8 \times 6 \times \sin A = 3\sqrt{15}$

Q13bi $y = \sqrt{9 - x^2}$ is a semi-circle of radius 3 units centred at the origin O in the first and second quadrants.
.: the domain is $[-3, 3]$ and the range is $[0, 3]$

Q13bii



Q13ci $y = x^3 - x^2 - x + 3$, $\dfrac{dy}{dx} = 3x^2 - 2x - 1 = (3x+1)(x-1)$

At the stationary points, $\dfrac{dy}{dx} = 0$, .: $x = -\dfrac{1}{3}$, $x = 1$ and $y = \dfrac{86}{27}$, $y = 2$ respectively.

x	-1	$-\frac{1}{3}$	0	1	2
$\frac{dy}{dx}$	+	0	−	0	+
Nature		local max		local min	

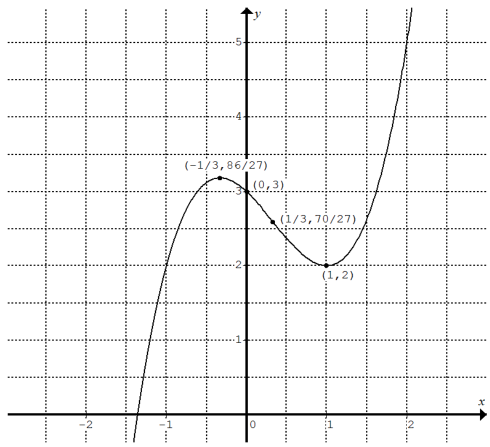
The stationary points are $\left(-\dfrac{1}{3}, \dfrac{86}{27}\right)$, a local maximum, and $(1, 2)$, a local minimum.

Q13cii $\dfrac{d^2y}{dx^2} = 6x - 2$. Let $\dfrac{d^2y}{dx^2} = 0$, .: $x = \dfrac{1}{3}$

x	0	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{dy}{dx}$	−	−	−
$\frac{d^2y}{dx^2}$	−	0	+
Nature		point of inflection	

P is a point of inflection.

Q13ciii



Q14ai Given $\ddot{x} = -10$, $\dot{x} = 0$ and $x = 110$ when $t = 0$,
.: $\dot{x} = \int -10\,dt = -10t$ and $x = \int -10t\,dt = -5t^2 + c = -5t^2 + 110$

Q14aii Given $a = \ddot{x} = -10$, $v = -37$, $u = 0$, find displacement s by $v^2 = u^2 + 2as$, $s = \dfrac{v^2 - u^2}{2a} = \dfrac{(-37)^2 - 0}{2(-10)} = -68.45$
Distance (m) fallen $= 68.45$

Q14bi $\Pr(\text{Sat. being dry}) = \dfrac{1}{2} \times \dfrac{5}{6} + \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{8}{12} = \dfrac{2}{3}$

Q14bii
$\Pr(\text{both Sat. and Sun. being wet}) = \dfrac{1}{2} \times \dfrac{1}{6} \times \dfrac{1}{6} + \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{18}$

Q14biii $\Pr(\text{at least one of Sat. and Sun. being dry})$
$= 1 - \Pr(\text{both Sat. and Sun. being wet}) = 1 - \dfrac{1}{18} = \dfrac{17}{18}$

Q14ci and Q14cii

n	$P + I$	$\$A_n$
0	0	100000
1	$100000(1.006)$	$100000(1.006) - M$
2	$(100000(1.006) - M)1.006$ $= 100000(1.006)^2 - M1.006$	$100000(1.006)^2 - M1.006 - M$ $= 100000(1.006)^2 - M(1 + 1.006)$
3	$\left(100000(1.006)^2 - M(1 + 1.006)\right)1.006$ $= 100000(1.006)^3 - M\left(1.006 + 1.006^2\right)$	$100000(1.006)^3 - M\left(1.006 + 1.006^2\right) - M$ $= 100000(1.006)^3 - M\left(1 + 1.006 + 1.006^2\right)$ $= 100000(1.006)^3 - M\left(\dfrac{1.006^3 - 1}{1.006 - 1}\right)$
⋮ ⋮	⋮ ⋮	⋮ ⋮
n		$100000(1.006)^n - M\left(\dfrac{1.006^n - 1}{0.006}\right)$

Q14ciii $A_n = 100000\,(1.006)^n - M\left(\frac{1.006^n - 1}{0.006}\right)$

$A_{120} = 100000\,(1.006)^{120} - 780\left(\frac{1.006^{120} - 1}{0.006}\right) = 68499.46 \approx 68500$

Q14civ Let $A_n = 48500\,(1.006)^n - 780\left(\frac{1.006^n - 1}{0.006}\right) = 0$

$48500\,(1.006)^n - 130000\,(1.006^n - 1) = 0$, $81500\,(1.006)^n = 130000$

.: $1.006^n \approx 1.595092$, $n = \frac{\ln(1.595)}{\ln(1.006)} \approx 78.055$

After 78 months, amount owing $\approx \$43$.
.: after 79 months in total will the amount owing be completely repaid if the interest rate and monthly repayment remain unchanged.

Q15ai $C = Ae^{-0.14t}$, $\frac{dC}{dt} = -0.14Ae^{-0.14t}$, .: $\frac{dC}{dt} = -0.14C$

Q15aii $C = Ae^{-0.14t}$, $130 = Ae^{-0.14\times 0} = A$, $A = 130$

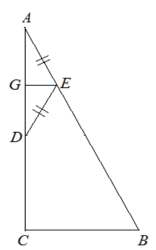
Q15aiii When $t = 7$, $C = 130e^{-0.14\times 7} \approx 48.79$

Q15aiv $0.5 = e^{-0.14t}$, $-0.14t = \ln 0.5$, $t \approx 4.951$
Time taken ≈ 4.951 hours

Q15bi $\angle EDA = \angle EAD$ because ΔEAD is isosceles
$\angle FDC = \angle EDA$ because they are vertically opposite angles
.: $\angle FDC = \angle EAD$
Also $\angle FCD = \angle BCA = 90°$, hence the third angles ($\angle ABC$ and $\angle DFC$) of ΔABC and ΔDFC respectively are equal.
.: ΔABC and ΔDFC are similar since all corresponding angles are equal.

Q15bii Since $\angle ABC = \angle DFC$, .: ΔEFB is isosceles.

Q15biii



G is the midpoint of AD, and D is the midpoint of AC, .: $AC = 4AG$
ΔAGE and ΔACB are similar, $AB = 4AE$
.: $EB = 3AE$

Q15ci $\frac{dV}{dt} < 0$ first when $\pi < 0.5t < 2\pi$, .: $2\pi < t < 4\pi$
The volume starts to decrease after 2π hours.

Q15cii $V = \text{initial} + \text{change} = 1200 + \int_0^3 80\sin(0.5t)\,dt$

$= 1200 + \left[-\frac{80\cos(0.5t)}{0.5}\right]_0^3$

$= 1200 - 160\cos 1.5 + 160\cos 0 \approx 1349$ litres

Q15ciii $V_{greatest} = 1200 + 2\times 160 = 1520$ litres

Q16ai $y = x^2 - 7x + 10 = (x-2)(x-5)$
x-coordinates of points A and B are 2 and 5 respectively.

Q16aii Point $C(a, 10)$, $y = x^2 - 7x + 10 = 10$
.: $a^2 - 7a = 0$, .: $a = 7$ and $C(7, 10)$.

Q16aiii $\int_0^2 (x^2 - 7x + 10)\,dx = \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x\right]_0^2 = \frac{26}{3}$

Q16aiv Area of shaded region in (unit)^2
= area of triangular region bounded by line AC and the x - axis $-\frac{26}{3}$

$= \frac{1}{2}(7-2)\times 10 - \frac{26}{3} = \frac{49}{3}$

Q16b $y = 8\log_e(x-1)$, $x = e^{\frac{y}{8}} + 1$

Volume (in (unit)^3) $= \int_0^6 \pi\left(e^{\frac{y}{8}} + 1\right)^2 dy = \pi\int_0^6 \left(e^{\frac{y}{4}} + 2e^{\frac{y}{8}} + 1\right) dy$

$= \pi\left[4e^{\frac{y}{4}} + 16e^{\frac{y}{8}} + y\right]_0^6 \approx 118.7$

Q16ci Similar triangles: $\frac{H-y}{x} = \frac{H}{R}$, .: $y = \frac{H}{R}(R-x)$

.: $V = \pi x^2 y = \frac{H}{R}\pi x^2 (R-x)$

Q16cii Let $\frac{dV}{dx} = 0$ for maximum V.

$\frac{H}{R}\pi(2Rx - 3x^2) = 0$, $x(2R - 3x) = 0$

V is maximum when $x = \frac{2R}{3}$

.: $V_{max} = \frac{H}{R}\pi\left(\frac{2R}{3}\right)^2\left(R - \frac{2R}{3}\right) = \frac{H}{R}\pi\frac{4R^2}{9}\times\frac{R}{3} = \frac{4}{9}\left(\frac{1}{3}\pi R^2 H\right)$,

i.e. $\frac{4}{9}$ of the volume of the cone.

.: the volume of any inscribed cylinder does not exceed $\frac{4}{9}$ of the volume of the cone.

Please inform mathline@itute.com re conceptual and/or mathematical errors.

