2015 VCAA Further Mathematics Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
E	B	E	A	A	B	D	C	A	C	A	D	D

SECTION B

Module 1: Number patterns

1	2	3	4	5	6	7	8	9
D	D	E	E	B	D	C	A	D

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
C	A	A	E	C	C	D	E	B

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
B	E	C	A	D	B	C	E	C

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
C	D	C	E	C	D	D	C	D

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
C	C	E	C	D	A	B	A	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
B	E	B	E	D	D	A	A	C

SECTION A Core: Data analysis

Q1 E

Q2 B

Q3 E

Q4 21.4 is two standard deviations lower than the mean.
2.5% of $2400 = 60$ A

Q5 $z = \dfrac{x-\mu}{\sigma}$, $x = \mu + z\sigma = 23.8 + (-1.3)1.2 \approx 22.2$ A

Q6 B

Q7 D

Q8 C

Q9 Gradient of line passing through $(0, 3.5)$ and
$(50, 82) \approx \dfrac{82-3.5}{50-0} \approx 1.6$ A

Q10 $y = a + bx$ where $b = -0.47 \times \dfrac{0.85}{1.2} \approx -0.3329$ and
$a = 7.2 - (-0.3329) \times 1.8 \approx 7.799$ C

Q11 $Weight = -7 + 30\log_{10} 5 \approx 14$ A

Q12 D

Q13

Qu. number	1	2	3	4
Sea. index	1.6	0.6	0.7	1.1
Sales	2800	1032	875	759
Desea. sales	1750	1720	1250	690

By CAS, $deseasonalised\ sales = 2265 - 365 \times quarter\ number$ D

SECTION B

Module 1: Number patterns

Q1 $t_4 = 40$, $r = 2$, $t_5 = 2 \times t_4 = 80$ D

Q2 $t_1 = t_3 - t_2 = 72 - 36 = 36$ D

Q3 $P_1 = 1.5 \times 200 = 300$ E

Q4 $r = \dfrac{80}{64} = 1.25$, $S_5 = \dfrac{a(r^5 - 1)}{r - 1} = \dfrac{64(1.25^5 - 1)}{0.25} = 525.25$ E

Q5 $r = \dfrac{6}{2} = 3$, $t_7 = ar^6 = 2 \times 3^6 = 1458$ B

Q6 $K_{n+1} = 0.99K_n + 250$, $K_{30} = 7550$
$K_{31} = 0.99 \times 7550 + 250 = 7724.5$
$K_{32} = 0.99 \times 7724.5 + 250 = 7897.255$ D

Q7 Pattern of geometric growth: $16, 12, 9, \ldots$
$a = 16$, $r = \dfrac{12}{16} = 0.75$, $S_\infty = \dfrac{a}{1-r} = \dfrac{16}{1-0.75} = 64$
Max. height $= 80 + 64 = 144$ C

Q8 $A_{n+1} = n$, $A_{n+2} = n + 1$, the ratio $\dfrac{A_{n+2}}{A_{n+1}} = \dfrac{n+1}{n}$ is not a constant.
A

Q9 $F_{n+1} = 0.95F_n + a$, $F_0 = 3000$
$F_1 = 0.95F_0 + a = 0.95 \times 3000 + a = 2540$, $\therefore\ a = -310$
$F_2 = 0.95F_1 + a = 0.95 \times 2540 - 310 = 2103$
$F_3 = 0.95F_2 + a = 0.95 \times 2103 - 310 = 1687.85$ D

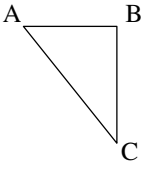
Module 2: Geometry and trigonometry

Q1 Area $= \dfrac{1}{2}(60 + 80)60 = 4200$ C

Q2 Perimeter $= \dfrac{1}{2} \times \pi \times 12 + 4 + 12 + 4 \approx 38.8$ A

Q3 The cosine rule: $\cos x = \dfrac{3^2 + 6^2 - 5^2}{2 \times 3 \times 6}$ A

Q4

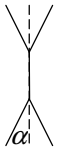


E

Q5 Area $= 3\left(\dfrac{1}{2} \times 2 \times 2 \sin 60°\right) \approx 5.2$ C

Q6 Volume $= \pi \times 6^2 \times 8 - \dfrac{1}{2}\left(\dfrac{4}{3} \times \pi \times 5^3\right) \approx 643$ C

Q7 $\alpha = \dfrac{360°}{12} = 30°$, $\theta = 2\alpha = 60°$ D



Q8 $b+2(180°-a)=180°$, $\therefore\ 2a-b=180°$ (assuming the angles are measured in degrees) E

Q9 $\dfrac{\text{V(smaller)}}{\text{V(larger)}}=\dfrac{1}{2}$, $\dfrac{\text{A}\ell\text{(smaller)}}{\text{A}\ell\text{(larger)}}=\dfrac{\text{A(smaller)}}{\text{A(larger)}}=\left(\dfrac{8-d}{8}\right)^2=\dfrac{1}{2}$
$\therefore\ d\approx 2.3$ B

Module 3: Graphs and relations

Q1 From graph B

Q2 By substitution E

Q3 From graph C

Q4 $\dfrac{x}{10}+\dfrac{y}{4}=1\ \therefore\ 2x+5y=20$ A

Q5 $y\geq\dfrac{x}{15}$ D

Q6 $d=kv^2$, $k=\dfrac{d}{v^2}=\dfrac{d}{60^2}=\dfrac{45}{75^2}\ \therefore\ d=28.8$ B

Q7 When $n=200$, $n+150=0.6n+p\ \therefore\ p=230$ C

Q8 $(35-12)n-190\geq 1000$, $n\geq\dfrac{1190}{23}$, minimum $n=52$ E

Q9 Gradient of objective function $\left(=\dfrac{4}{3}\right)<$ gradient of line segment $QR\ (=5)$.: maximum Z at point R. C

Module 4: Business-related mathematics

Q1 5% of $\$368.40=\$18.42$ C

Q2 $\dfrac{850000-600000}{600000}\times 100\%\approx 41.7\%$ D

Q3 Closing price $=160\times\dfrac{97}{100}\times\dfrac{1045}{100}\approx 162.18$ dollars C

Q4 Total amount after 2 years $=1200\left(1+\dfrac{3.35}{12\times 100}\right)^{24}\approx 1283.03$
Interest $=1283.03-1200=83.03$ dollars E

Q5 Total of 60 repayments $=400\times 60=24000$
Interest $=24000-(20000-5000)=9000$ dollars C

Q6 Min. balance $=4870.50$, interest $=5885.72-5870.50=15.22$
$\therefore\ 4870.50\times r\times\dfrac{1}{12}=15.22$, $r=0.0375=3.75\%$ D

Q7 From graph, initial value $=35000$, constant depreciation rate $=\dfrac{35000-3000}{20000-0}=0.25$ dollars per km D

Q8 By tvm solver, quarterly payment $\approx\$456.79392...$
After 3 years (12 payments), FV(balance of loan) $\approx\$4757.407672$
and .: reduced by $8400-4757.41\approx 3642.59$, i.e. approx. 43% C

Q9 By tvm solver: number of monthly payments ≈ 106.9069347
After 106 payments, FV(balance of loan) $\approx\$1759.881692$
Final payment $\approx 1759.881692\times\left(1+\dfrac{6.18}{12\times 100}\right)\approx\$1768.95$ D

Module 5: Networks and decision mathematics

Q1 C

Q2 $v+f=e+2$, $5+6=e+2$, $e=9$ C

Q3 E

Q4 $5+2+12+7=26$ C

Q5 D

Q6 A

Q7 $80+90+125+60=355$ B

	Abe	Bailey	Chris	Donna
Task 1	80			
Task 2				90
Task 3			125	
Task 4		60		

Q8 A

One-step dominance matrix X Two-step dominance matrix X^2

$$\begin{array}{c} \\ L \\ E \\ R \\ D \\ H \end{array}\begin{array}{c}\begin{array}{ccccc} L & E & R & D & H \end{array}\\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}\end{array} \qquad \begin{array}{c} \\ L \\ E \\ R \\ D \\ H \end{array}\begin{array}{c}\begin{array}{ccccc} L & E & R & D & H \end{array}\\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix}\end{array}$$

Dominance score = sum of one - step and two - step dominances

$$\begin{array}{ccccc} L & E & R & D & H \\ [8 & 4 & 7 & 5 & 0] \end{array}$$.: the ranking from highest to lowest is $LRDEH$

Q9 B

Module 6: Matrices

Q1 B

Q2 E

Q3 B

Q4 E

Q5 Let the transition matrix be

$$X=\begin{array}{c}\begin{array}{ccccc} T & R & C & I & D \end{array}\\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}\end{array}\begin{array}{c} T \\ R \\ C \\ I \\ D \end{array}$$ and the initial state matrix be $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} T \\ R \\ C \\ I \\ D \end{array}$

$X\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}C$, $X\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}D$, $X\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}I$, $X\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}R$, $X\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}T$ D

Q6 $\begin{bmatrix} 4 & 7 & 0 \\ 2 & 0 & 3 \\ 0 & 5 & 1 \end{bmatrix}\begin{bmatrix} t \\ s \\ b \end{bmatrix}=\begin{bmatrix} 33 \\ 12 \\ 10 \end{bmatrix}\ \therefore\ \begin{bmatrix} t \\ s \\ b \end{bmatrix}=\begin{bmatrix} 4 & 7 & 0 \\ 2 & 0 & 3 \\ 0 & 5 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 33 \\ 12 \\ 10 \end{bmatrix}\approx\begin{bmatrix} 4.99 \\ 1.86 \\ 0.68 \end{bmatrix}$ D

Q7 $P.P^{-1}=I$, $(wP).\left(\dfrac{1}{w}P^{-1}\right)=I$, $Q.\left(\dfrac{1}{w}P^{-1}\right)=I\ \therefore\ Q^{-1}=\dfrac{1}{w}P^{-1}$ A

Q8 $x_{ij}=i-j\ \therefore X=\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$ A

Q9 C

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

