# 2015 VCAA Further Mathematics Exam 2 Solutions



**Core – Data analysis**

Q1a Between 70 and 75

Q1b $5+9=14$

Q1c $\frac{30}{183}\times 100\% \approx 16.4\%$

Q2a Negatively skewed

Q2b

| Year | 1953 | 1973 | 1993 |
|---|---|---|---|
| Median | 51 | 63 | 69 |

There is a positive correlation between the median of life expectancy and the year.

Q3a The male life expectancy increases by 0.88 years with each year of increase in the female life expectancy.

Q3b $male = 3.6 + 0.88\times 35 = 34.4$ years

Q3c About 95% of the variability in the male life expectancy is explained by the variability of the female life expectancy using the linear regression model.

Q4a Strong, positive and linear

Q4b $male = 9.69 + 0.81\times female$

Q5a 82 (in 2010) − 60 (in 1920) = 22 years

Q5bi Australia: $life\ expectancy = -451.7 + 0.2657\times 2030 \approx 87.67$
UK: $life\ expectancy = -350.4 + 0.2143\times 2030 \approx 84.63$
$Difference = 87.67 - 84.63 \approx 3$ years

Q5bii Year 2030 is far away from the range 1975 to 2010. It is a long range prediction using two linear models (either one or both may change) and therefore it may be of limited reliability.

**Module 1: Number patterns**

Q1a $2150 - 2000 = 150$

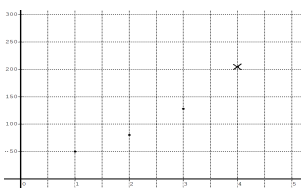
Q1b $2300 + 2\times 150 = 2600$

Q1c $S_8 = \frac{8}{2}(2\times 2000 + (8-1)\times 150) = 20200$

Q1d $S_n = \frac{n}{2}(2\times 2000 + (n-1)\times 150) > 14000$, $n > 5.911$
∴ in the sixth week

Q1e $C_{n+1} = C_n + 150$, $C_1 = 2000$

Q2a number of whiteflies per square metre at the beginning of week $4 = 50\times 1.6^3 \approx 205$



Q2b $50\times 1.6^{n-1} > 500$, $n > 5.9$
∴ at the beginning of the sixth week

Q2c The number is 1.6 times the previous number, an increase of 0.6 times, i.e. 60%

Q2di To remain constant, $W_{n+1} = W_n$, ∴ $1.6\times W_n - k = W_n$
∴ $k = 0.6\times W_n = 0.6\times 128 = 76.8$

Q2dii $W_4 = 1.6\times W_3 - 50 = 1.6\times 128 - 50 = 154.8$

Q2diii $W_4 = 1.6\times W_3 - k = 1.6\times 128 - k$
$W_5 = 1.6\times W_4 - k = 1.6\times(1.6\times 128 - k) - k$
$W_3 - W_5 = 30$ ∴ $128 - (1.6\times(1.6\times 128 - k) - k) = 30$ ∴ $k = 88.3$

Q3a $m$ represents the number of ladybirds in the greenhouse released into the field at the end of each week.

Q3b To maintain constant population in both the greenhouse and the field, let $1.25\times 1200 - m = 1200$ and $0.65\times f + m = f$.
Solve the two simultaneous equations, $f \approx 857$

**Module 2: Geometry and trigonometry**

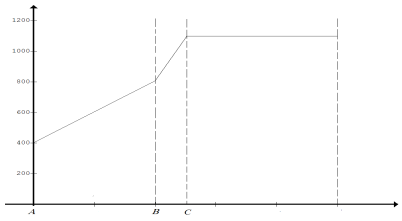
Q1a $800 - 400 = 400$ metres

Q1b Average slope $= \frac{1100-800}{500} = 0.6$

Q1c Let $x$ metres be the distance on the map.
$x : 400 = 1 : 50000$, $\frac{x}{400} = \frac{1}{50000}$ ∴ $x = 0.008$

Q1d $\cos\theta = \frac{2500}{2596}$ ∴ $\theta \approx 16°$

Q1e




Q2a $950\cos 60° = 475$ metres

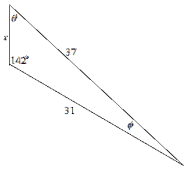
Q2bi The cosine rule:
$length = \sqrt{950^2 + 1400^2 - 2(950)(1400)\cos 40°} \approx 908.2 \approx 908$ m

Q2bii Let $\angle CTE = \phi$. $\cos\phi = \frac{950^2 + 908.2^2 - 1400^2}{2(950)(908.2)}$ ∴ $\phi \approx 98°$
∴ the true bearing of *C* from *T* is $360 - (180 - 60) - 98 = 142°$

Q3a $h = \sqrt{2.4^2 - 2^2} + 2.4 \approx 3.73$

Q3b Cross-section: Area $= \frac{1}{2}(2.4 + 3.73)4 = 12.26$ m$^2$
Total area to be painted $= 2\times(12.26) + 4\times(2.4\times 5.4) \approx 76$ m$^2$

Q4 The sine rule $\frac{\sin\theta}{31} = \frac{\sin 142°}{37}$, $\theta \approx 31.05°$




∴ $\phi \approx 180 - 142 - 31.05 = 6.95°$
$x \approx \sqrt{37^2 + 31^2 - 2(37)(31)\cos 6.95°} \approx 7.3$ m

**Module 3: Graphs and relations**

Q1a $1200

Q1b 8

Q2a 18000 yen

Q2b $18000 \div 200 = 90$

Q3a $8075 = 95 \times 100 - k$, $k = 1425$

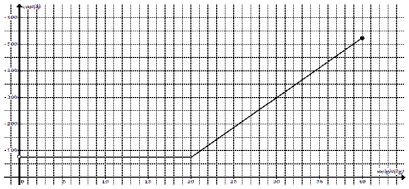
Q3bi $0 = 95 \times dollars - 1425$, $dollars = 15$

Q3bii Commission/transaction fee

Q4a $250

Q4b $online\ cost = 22.5 \times 30 - 375 = 300$ dollars

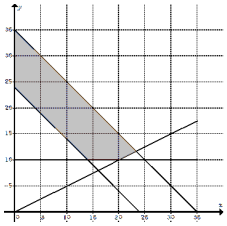
Q4c



Q4d $22.5 \times weight - 375 = 250$, $weight = 27.8$ kg

Q5a Ben must attend at least 10 lessons.

Q5b



Q5c The point $(23, 12)$ satisfies all the inequalities .: max. $x = 23$

**Module 4: Business-related mathematics**

Q1a $GST = 88 \div 1.10 = 8$ dollars

Q1b $Total\ amount = 88 \times 4 = 352$ dollars

Q1c $New\ hourly\ rate\ including\ GST = 88 \times (1 + 0.125) = 99$ dollars

Q2a $(3800 - 3150) \div 2 = 325$ dollars

Q2b Let $n$ be the number of years. $3800 - 325 \times n = 550$, $n = 10$

Q2c Let $r\%$ be the annual depreciation rate.

$$2100\left(1 - \frac{r}{100}\right)^5 = 1040 \ .:\ r = 13.11$$

Q3a Let $\$A$ be the minimum amount.
$3.68\% \times A = 460$ .: $A = 12500$

Q3b Infinitely many years

Q4a $$Amount = 4000 \times \left(1 + \frac{3.6}{12 \times 100}\right)^6 \approx 4072.54 \text{ dollars}$$

Q4b $0.008 \times 100\% \times 4 = 3.2\%$

Q4c By tvm solver, $amount \approx \$6664.63$

Q5a By tvm solver, annual interest rate $\approx 5.91\%$

Q5b Bal. after a year $\approx 47049$, paid off $\approx 50000 - 47049 = \$2951$

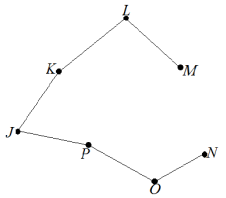
Q5c Bal. after 6 years and then one-off payment of $3500 \approx 25876$
Time required to repay = 62 months, saving = $72 - 62 = 10$ months

**Module 5: Networks and decision mathematics**

Q1a $300 Q1b $440 + 480 = 920$ dollars

Q1c Start at *P* and finish at *N*, or start at *N* and finish at *P*.

Q1di




Q1dii Remove *P* and join *N* to *M*. Save $200 + 400 - 480 = \$120$

Q2ai factory $- T - S - Q - R - S - U -$ factory

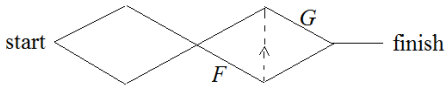
Q2aii The van passes through town *S* more than once.

Q2b Hamiltonian circuit: factory $- T - S - R - Q - U -$ factory
$Length\ (km) = 44 + 38 + 12 + 8 + 38 + 22 = 162$

Q3a 3 Q3b 4 Q3c 3 Q3d *A C F H I*

Q3e Adding 7 hours to the usual minimum preparation time

Q3fi




Q3fii Adding 2 hours to the usual minimum preparation time

**Module 6: Matrices**

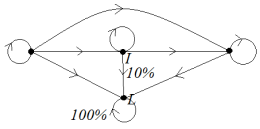
Q1a $20 + 60 + 40 = 120$ Q1bi $e_{12} = 10$, $e_{23} = 9$ Q1bii $e_{22} = 30$

Q1ci *CQ* Q1cii $15 \times 2 + 25 \times 6 + 40 \times 4 = 340$ dollars

Q2a 100% of the advanced-level students stay at the same level after assessment.

Q2bi $S_1 = T_1 S_0 \approx \begin{bmatrix} 10 \\ 58 \\ 52 \end{bmatrix}$ Q2bii $0.20 \times 60 = 12$

Q3a




Q3b $T_2 R_0 = \begin{bmatrix} 6 \\ 50 \\ 43 \\ 21 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$ .: $percentage = \frac{21}{120} \times 100\% = 17.5\%$

Q3c $T_2^2 R_0 = \begin{bmatrix} 1.8 \\ 37.4 \\ 42.55 \\ 38.25 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$ .: 43 advanced-level students

Q3d After 5 assessments: $T_2^5 R_0 \approx \begin{bmatrix} 0.05 \\ 13.40 \\ 30.08 \\ 76.47 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$

Q3e $R_1 = \begin{bmatrix} 10 \\ 52 \\ 46 \\ 21 \end{bmatrix}$, $R_2 \approx \begin{bmatrix} 7 \\ 42.4 \\ 48.4 \\ 40.2 \end{bmatrix}$, $R_3 \approx \begin{bmatrix} 6.1 \\ 34.5 \\ 48.1 \\ 58.3 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$

After the next assessment, number of *I* students expected to become *A* students $= 0.20 \times 34.5 \approx 7$

*Please inform mathline@itute.com re conceptual, mathematical and/or typing errors*