2016
Math Methods/ Specialist Maths

Year 11
Investigation Task

Time allowed: 2 hours

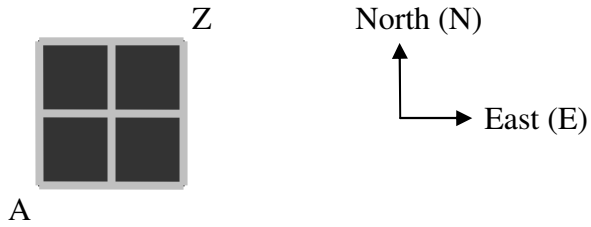
You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator
Working must be shown for questions worth 2 or more marks. Total: 60 marks

Theme: Moving forwards

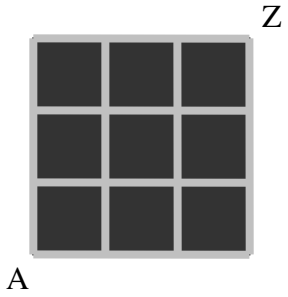
Question 1 In this question the dark squares represent building blocks, and the light grid lines represent streets.

a. Count the number of ways to walk from corner A to corner Z without walking backwards, i.e. always walking towards corner Z.

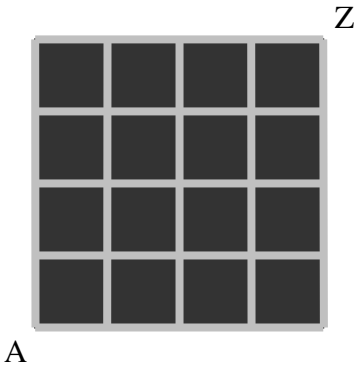
1 mark



b. Count the number of ways to walk from corner A to corner Z without walking backwards. 2 marks

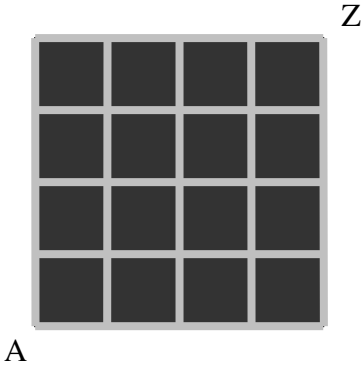


c. Count the number of ways to walk from corner A to corner Z without walking backwards. 2 marks



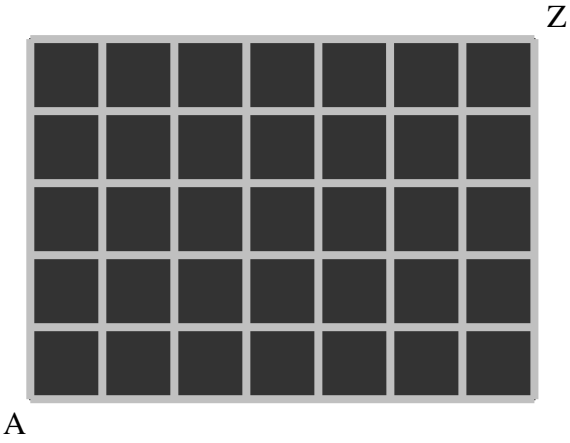
d. Use the following method to check your answer to part c.
Write next to each street-intersection the number of ways to arrive there from corner A without walking backwards.

3 marks



e. Use the method in part d to find the number of ways to walk from corner A to corner Z without walking backwards.

3 marks

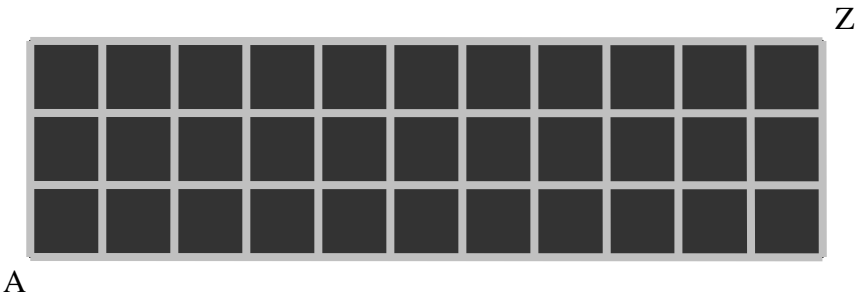


f. In part e, the number of ways to walk from corner A to corner Z without walking backwards can be denoted by ${}^nC_r$. Try different values of n and r in ${}^nC_r$ to obtain the correct values of n and r.

2 marks

g. Use ${}^nC_r$ to find the number of ways to walk from corner A to corner Z without walking backwards.

1 mark

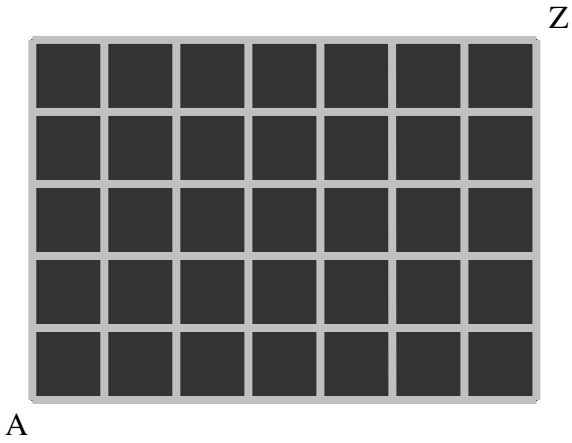


h. There are x squares in the easterly direction and y squares in the northerly direction in the rectangular array of the building blocks between corner A and corner Z.
${}^nC_r$ is the number of ways to walk from corner A to corner Z without walking backwards.
Find the values of n and r in terms of x and y. 3 marks

i. Explain why ${}^nC_r$ gives the number of ways to walk from corner A to corner Z without walking backwards.

1 mark

Question 2 In this question, write N to stand for 'walking 1 block to the north', and E to stand for 'walking 1 block to the east'. Consider the following rectangular array of building blocks (dark squares) between corner A and corner Z.



a. Write a string of N's and E's to represent a path to walk all the way to the east and then all the way to the north from corner A to corner Z.

1 mark

b. Write a string of N's and E's to represent another possible path from corner A to corner Z.

1 mark

c. Write an expression using factorials for finding the total number of ways to walk from corner A to corner Z. Explain your answer.

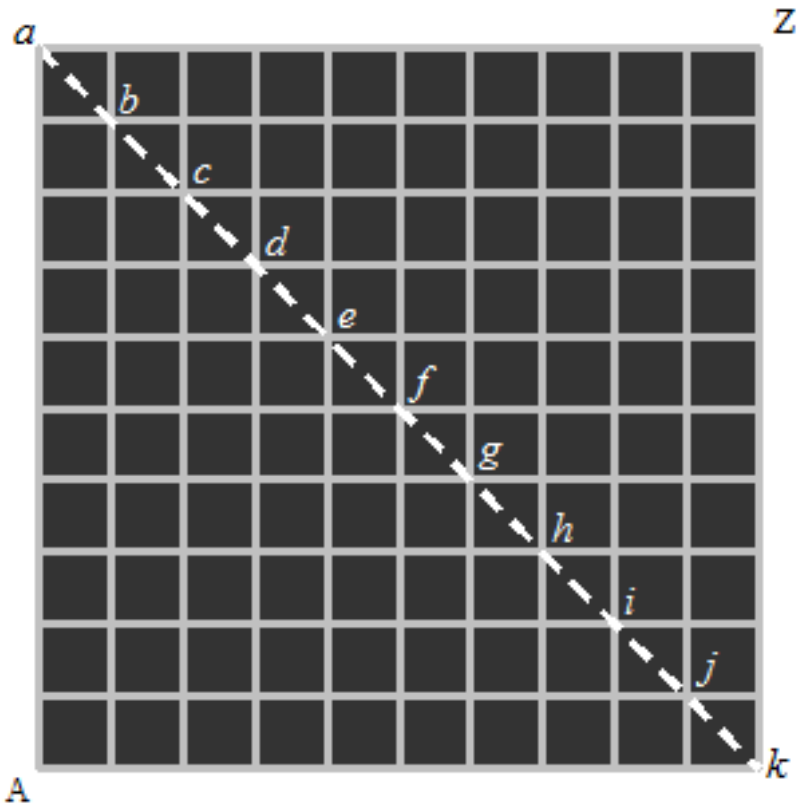
3 marks

d. Evaluate the expression in part c. 1 mark

e. Given $\frac{w!}{u!v!} = {}^{n}C_{r}$, write down a set of possible values for u, v and w in terms of n and r.

3 marks

Question 3 Consider the following square array of building blocks (dark squares) between corner A and corner Z. The dotted line is a diagonal of the square array of building blocks. The street-intersections on the diagonal are marked as $a, b, c, d, e, f, g, h, i, j$ and k.



a. ***Use the result*** in **Question 1** h to find the number of ways to walk from corner A (or corner Z) to each of the street-intersections on the diagonal marked as $a, b, c, d, e, f, g, h, i, j$ and k without walking backwards.

10 marks

b. Find the total number of ways to walk from corner A (or corner Z) to each of the street-intersections on the diagonal marked as $a, b, c, d, e, f, g, h, i, j$ and k without walking backwards.

1 mark

c. Express your answer in part b in the form 2^p where p is a natural number. 1 mark

d. Explain why the total number of ways to walk from corner A (or corner Z) to each of the street-intersections on the diagonal without walking backwards can be expressed as 2^p.

2 marks

Question 4 Consider the expansion of $(x+y)^7$ shown in part a below.

a. The coefficients of three terms in the expansion are missing.
Insert the missing coefficients, ${}^nC_r$, in front of the terms. 2 marks

$$(x+y)^7 = {}^7C_0\,x^7y^0 + {}^7C_1\,x^6y^1 + \square\,x^5y^2 + \square\,x^4y^3 + {}^7C_4\,x^3y^4 + {}^7C_5\,x^2y^5 + \square\,x^1y^6 + {}^7C_7\,x^0y^7$$

b. Hence show that the sum of the coefficients in the expansion above equals 2^7. 2 marks

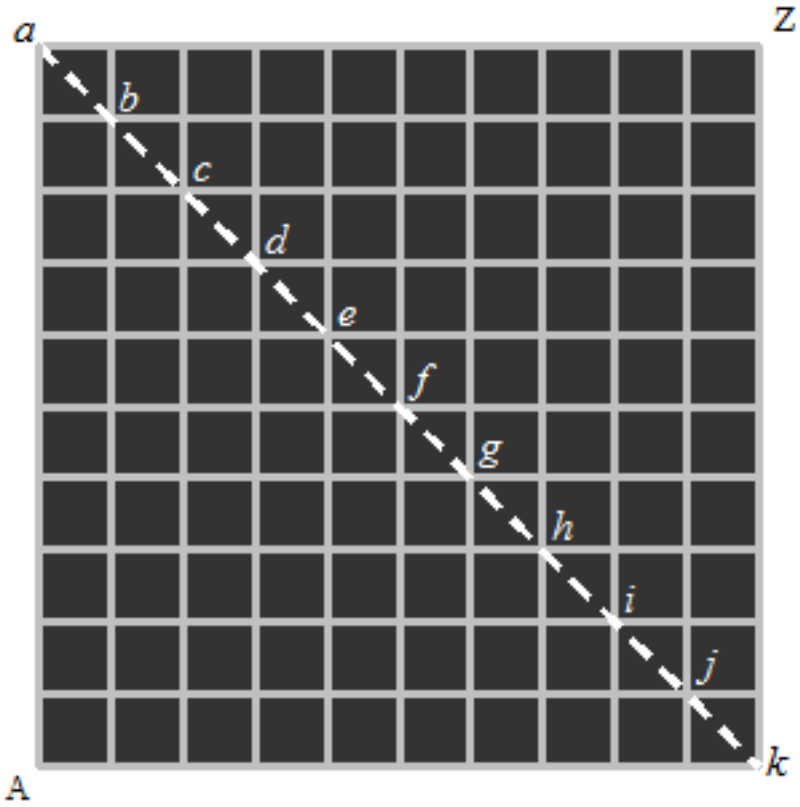
c. Write down the first three terms and the last three terms of the expansion of $(x+y)^n$ where n is a natural number greater than 2.

3 marks

d. Discuss the symmetry in the coefficients of the expansion of $(x+y)^n$ and the number of ways to walk from corner A (or corner Z) to each of the street-intersections on the diagonal of a $n \times n$ square array of building blocks without walking backwards.

2 marks

Question 5 Aaron walks towards the diagonal from corner A. Zach also walks towards the diagonal but from corner Z. Both start at the same time and walk at the same speed.



a. Calculate the number of ways that they meet at the intersection marked as *e*. 2 marks

b. Calculate the number of ways that they meet at the diagonal. 3 marks

c. Calculate the number of ways that they **do not** meet at the diagonal. 3 marks

d. Assuming that each chooses the walking route towards the diagonal at random, calculate the probability that they meet at the diagonal.

2 marks

End of task