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2016
Mathematical
Methods

Year 12
Problem Solving Task

Time allowed: 2.5 hours

You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator

Working must be shown for questions worth 2 or more marks. Total: 70 marks

Theme: Moving furnishings inside an art gallery

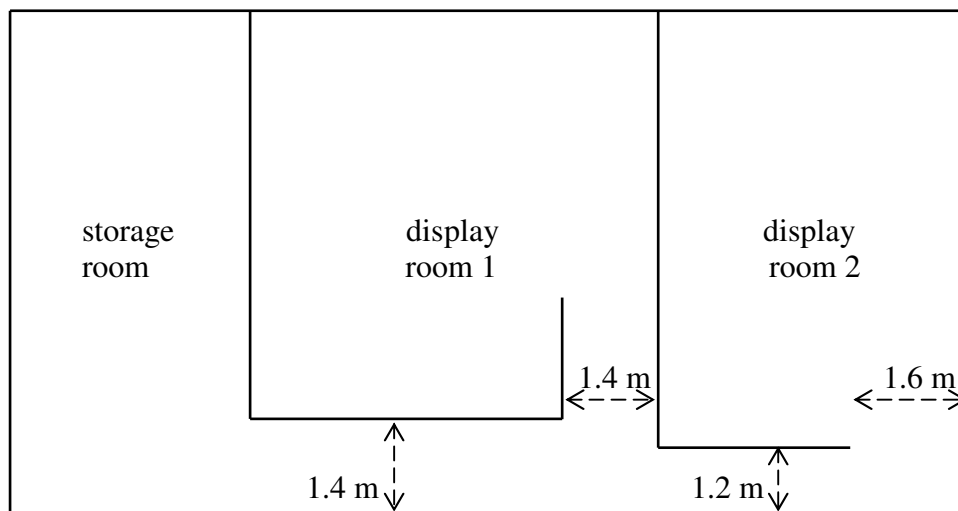
An art gallery has a storage room and two display rooms.

The floor plan is shown below and it is not drawn to scale.

Display room 1 has a 1.4 m wide doorway. The doorway in display room 2 is 1.6 m.

The ceiling of the gallery is 3.0 m above the floor.

All length measurements are in metres.



Problem 1

Some rigid curtain rods are to be moved from the storage room into the display rooms. Assume that the rods are 1-dimensional, i.e. the rods are straight line segments.

- a. What is the length of the longest curtain rod which can be moved **horizontally** from the storage room into display room 1?

Hint: The rod needs to touch the doorway edge to be the longest. \therefore you need to find the length of the shortest horizontal line segment touching the doorway edge, the wall inside display room 1 and the corridor wall. The line segment making a 45° angle with each of the two walls will be the shortest.

2 marks

- b. What is the length of the longest curtain rod which can be moved **at an angle to the floor** from the storage room into display room 1?

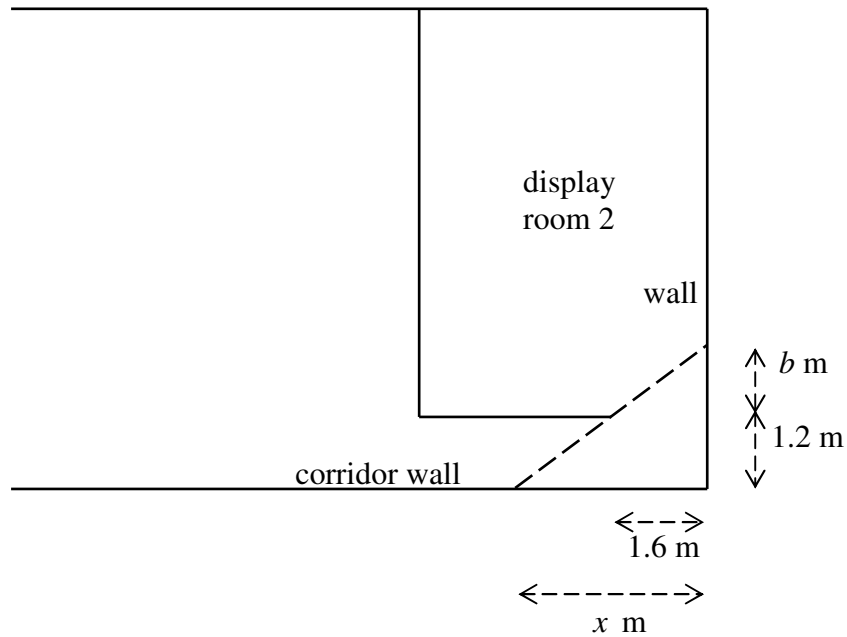
2 marks

Now move the longest rod into display room 2. The task is to find the length of the longest rod.

Hint: Find the length of the shortest horizontal line segment (the dotted line in the following diagram).

Choose a variable which is related to the length of the dotted line.

Let x metres be the variable as shown below.



- c. Calculate the length of the dotted line when (i) $x = 3.8$ (ii) $x = 2.9$ and (iii) $x = 2.2$.

2 + 1 + 1 = 4 marks

- d. Comment on the variation of the length of the dotted line with respect to the variation in x .

2 marks

- e. State the minimum value of x .

1 mark

f. Show that $b = \frac{1.92}{x-1.6}$.

2 marks

g. Hence show that the length of the dotted line is $\frac{x\sqrt{x^2 - 3.2x + 4}}{x-1.6}$.

3 marks

h. What is the length of the shortest dotted line possible?

1 mark

i. What is the length of the longest curtain rod which can be moved **horizontally** from the storage room into display room 2?

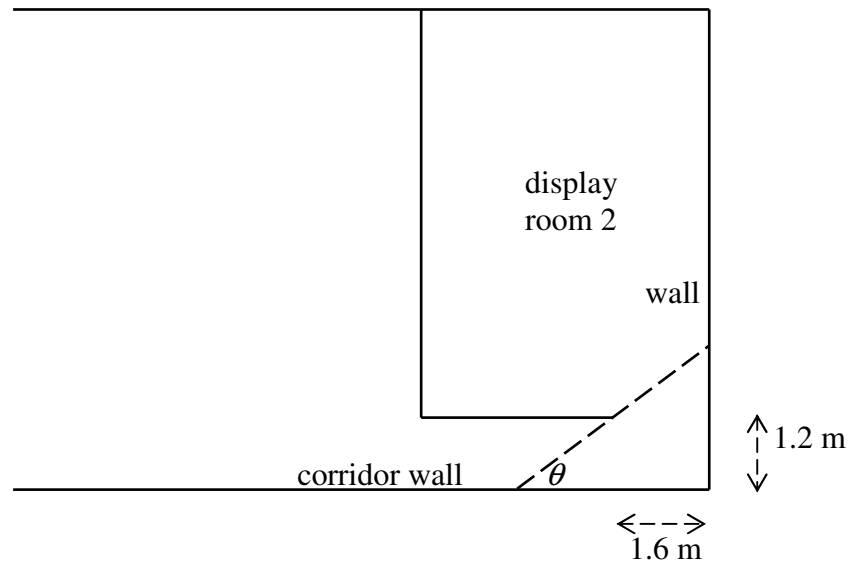
1 mark

j. What is the length of the longest curtain rod which can be moved **at an angle to the floor** from the storage room into display room 2?

1 mark

Problem 2

A simpler approach in finding the longest rod which can be moved horizontally into display room 2 is to choose θ as the variable. Refer to the following diagram.



- a. Show that the length of the dotted line is $\frac{2(4\sin\theta + 3\cos\theta)}{5\sin\theta\cos\theta}$. 3 marks

- b. What is the value of angle θ when the dotted line is shortest?
Write your answer in decimal degree, correct to 1 decimal place. 2 marks

Now some rigid rectangular paintings are to be moved from the storage room into the display rooms. Assume that the paintings are 2-dimensional, i.e. the paintings have no thickness.

Problem 3

a. If the paintings are to be moved **vertically**, what is the area of the largest painting (in area) which can be moved into

(i) display room 1?

2 marks

(ii) display room 2?

1 mark

b. If the paintings are to be moved **horizontally**, what is the area of the largest painting (in area) which can be moved into display room 1?

2 marks

c. Some paintings with a height of 0.8 m are to be moved **horizontally** into display room 1. What is the exact length of the longest one?

Drawing a neat diagram may help to solve this problem.

4 marks

d. Some paintings with a height of 0.8 m are to be moved **horizontally** into display room 2.

What is the length (in metres, correct to 2 decimal places) of the longest one?

7 marks

Drawing a neat diagram may help to solve this problem.

Make use of the results in **Problem 1** part **i** and **Problem 2** part **b**

Calculus is not needed to solve this problem.

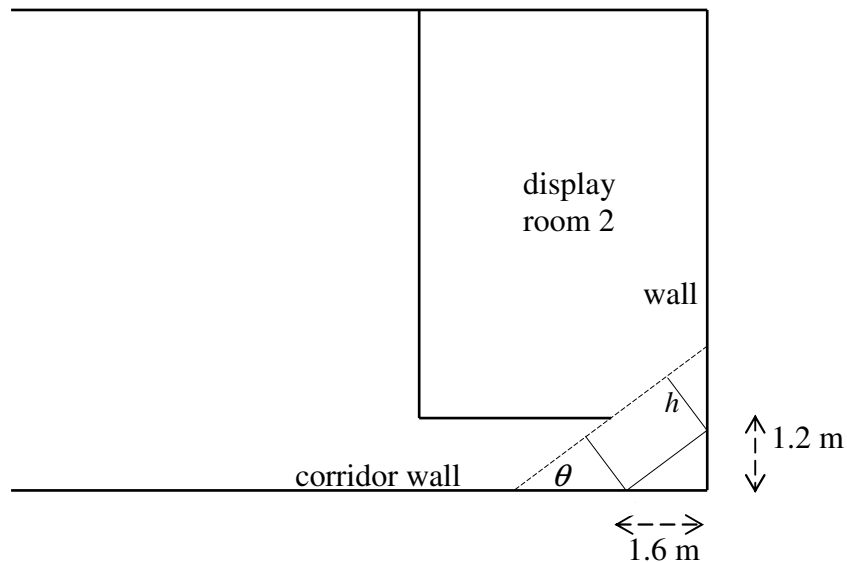
Problem 4 Some paintings in the storage room have different heights.

If these paintings are to be moved **horizontally**, what is the area of the largest painting (in area) which can be moved into display room 2?

Hint: This question is similar to **Problem 3** part **d**.

Make use of the results in **Problem 1** part **i** and **Problem 2** part **b**

Use variable h instead of using 0.8 m as the height.



- What value for θ should be used to solve the problem? 1 mark
- What length for the dotted line should be used to solve the problem? 1 mark
- Show that the area A (m^2) of the painting is $3.946h - 2.009h^2$ approximately. 5 marks

d. State the domain of A .

2 marks

e. State the length and the height of the largest (in area) painting which can be moved **horizontally** into display room 2.

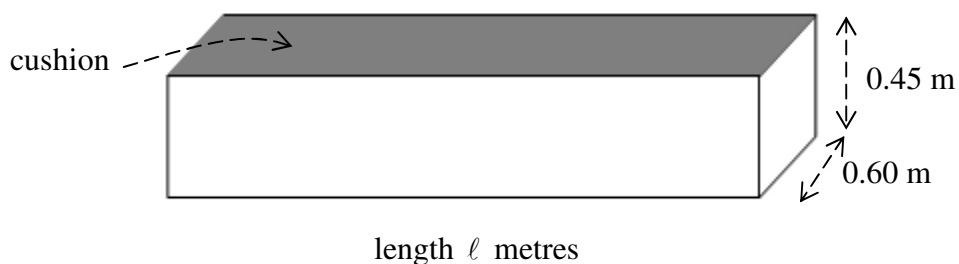
2 marks

f. State the length and the height of the largest (in area) painting which can be moved into display room 2.

3 marks

Problem 5

Now some rigid rectangular solid shape seats are to be moved from the storage room into the display rooms.



a. How would you move them to allow the longest ones into the display rooms?
You may use previous results to explain your answer.

2 marks

b. Find the length of the longest seat which can be moved **horizontally** into display room 2, with the cushion facing the ceiling.

2 marks

c. Find the length of the longest seat which can be moved **horizontally** into display room 2, with the cushion facing the wall, i.e. the seat on its side.

2 marks

d. Find the length of the longest seat which can be moved into display room 2, with the cushion facing the ceiling, one end on the floor and the other end 1.0 m above the floor.

5 marks

e. Find the length of the longest seat which can be moved into display room 2, with the cushion facing the ceiling, one end on the floor and the other end touching the ceiling.

5 marks

END OF TASK