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2016 NSW BOS Mathematics Extension 2 Solutions

Section I

1	2	3	4	5	6	7	8	9	10
C	C	A	D	B	C	D	C	A	B

Q1 $f(-x) = f(x)$ **C**

Q2 **C**
$P(x) = x^3(x^2-1) - (x^2-1) = (x^2-1)(x^3-1) = (x-1)^2(x+1)(x^2+x+1)$

Q3 $e = 0$ for a circle and $e < 1$ for an ellipse
.: the sum of the eccentricities < 1. For other pairs, the sum > 1 **A**

Q4 Let $w = -a + bi$, $z = c + di$ where a, b, c and $d \in R^+$, $a > c$ and $b < d$.
$w - z = -(a+c) + (b-d)i$
Both real and imaginary parts of $w - z$ are negative. **D**

Q5 $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{-2i}{2} = -i$, clockwise rotation by $\frac{\pi}{2}$ **B**

Q6 $p(x+1) = (x+1)^{12} + (x+1)^{11} + (x+1)^{10} + (x+1)^9 + (x+1)^8 + \cdots$
Coefficient of $x^8 = {}^{12}C_8 + {}^{11}C_8 + {}^{10}C_8 + {}^{9}C_8 + 1 = 715$ **C**

Q7 $xy = 8$ has $y = x$ as an axis of symmetry. $xy = 8$ intersects $y = x$ at $(2\sqrt{2}, 2\sqrt{2})$, .: distance of intersection from O is 4.
Rotate $xy = 8$ clockwise by $\frac{\pi}{4}$ to obtain $x^2 - y^2 = k$ and the intersection is the x-intercept $(4, 0)$. .: $k = 4^2 = 16$. **D**

Q8 Horizontal component: $F\cos\theta - N\sin\theta = mr\omega^2$
Vertical component: $F\sin\theta + N\cos\theta = mg$ **C**

Q9 Let a and b be the side lengths of the rectangular slice.
$\frac{a-2}{8-2} = \frac{h}{4}$ and $\frac{b-3}{7-3} = \frac{h}{4}$, .: $a = \frac{3h}{2} + 2$ and $b = h + 3$
.: $V = \int_0^4 (h+3)\left(\frac{3h}{2} + 2\right) dh$ **A**

Q10 $x + \frac{1}{x} = -1$, $x^2 + x + 1 = 0$, .: $x = cis\left(\frac{2\pi}{3}\right)$, $\frac{1}{x} = cis\left(-\frac{2\pi}{3}\right)$
or $x = cis\left(-\frac{2\pi}{3}\right)$, $\frac{1}{x} = cis\left(\frac{2\pi}{3}\right)$
.: $x^{2016} = cis(0) = 1$, $\frac{1}{x^{2016}} = cis(0) = 1$, .: $x^{2016} + \frac{1}{x^{2016}} = 2$ **B**

Section II

Q11ai $z = \sqrt{3} - i$, $|z| = 2$, $\theta = \frac{-1}{\sqrt{3}} = -\frac{\pi}{6}$, .: $z = 2\text{cis}\left(-\frac{\pi}{6}\right)$

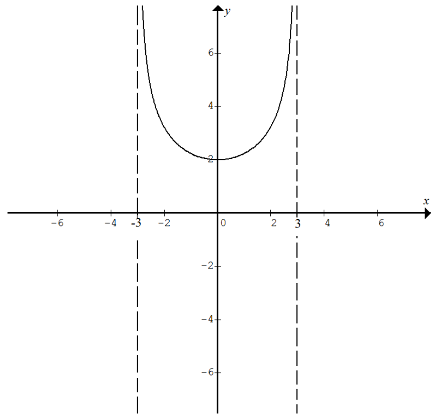
Q11aii $z^6 = 2^6\text{cis}\left(-\frac{6\pi}{6}\right) = -2^6$, a real number

Q11aiii $n = 3$, $z^3 = 2^3\text{cis}\left(-\frac{3\pi}{6}\right) = -8i$, a purely imaginary number

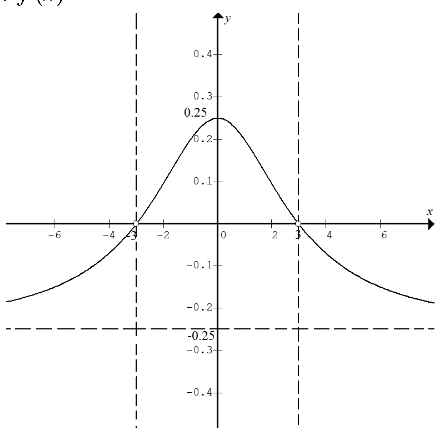
Q11b $\frac{d}{dx}\left(xe^{-2x}\right) = e^{-2x} - 2xe^{-2x}$, .: $xe^{-2x} = \frac{1}{2}e^{-2x} - \frac{1}{2}\frac{d}{dx}\left(xe^{-2x}\right)$
$\int xe^{-2x}\,dx = \int\left(\frac{1}{2}e^{-2x} - \frac{1}{2}\frac{d}{dx}\left(xe^{-2x}\right)\right)dx = -\frac{1}{4}e^{-2x} - \frac{1}{2}xe^{-2x} + c$

Q11c $x^3 + y^3 = 2xy$, $3x^2 + 3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$
$3x^2 - 2y = 2x\frac{dy}{dx} - 3y^2\frac{dy}{dx}$, .: $\frac{dy}{dx} = \frac{3x^2 - 2y}{2x - 3y^2}$

Q11di $y = \sqrt{f(x)}$



Q11dii $y = 1/f(x)$



Q11e Domain: For $\sin^{-1}\left(\frac{x}{2}\right)$ to be defined, $-1 \le \frac{x}{2} \le 1$
.: $-2 \le x \le 2$ is the domain
Range: For $-2 \le x \le 0$, $0 \le y \le \pi$, for $0 < x \le 2$, $0 < y \le \pi$
.: the range is $0 \le y \le \pi$.

Q12ai $\frac{x^2}{9}+\frac{y^2}{4}=1$

Q12aii $4=9\left(1-e^2\right)$, $e^2=\frac{5}{9}$, $e=\frac{\sqrt{5}}{3}$

Q12aiii $\left(-\sqrt{5},0\right)$, $\left(\sqrt{5},0\right)$

Q12aiv $x=-\frac{9\sqrt{5}}{5}$, $x=\frac{9\sqrt{5}}{5}$

Q12bi $\frac{d}{dx}\left(xf(x)-\int xf'(x)dx\right)=xf'(x)+f(x)-xf'(x)=f(x)$

Q12bii Hence $\int f(x)dx=xf(x)-\int xf'(x)dx$

.: $\int \tan^{-1}x\,dx=x\tan^{-1}x-\int\frac{x}{1+x^2}dx=x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)+c$

Q12ci $z=\cos\theta+i\sin\theta$

$z^4=\cos 4\theta+i\sin 4\theta$

$z^4=(\cos\theta+i\sin\theta)^4=\cos^4\theta+4i\cos^3\theta\sin\theta-6\cos^2\theta\sin^2\theta$

$-4i\cos\theta\sin^3\theta+\sin^4\theta$

Equating the real parts: $\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$

Q12cii $\cos 4\theta=\cos^4\theta-6\cos^2\theta\left(1-\cos^2\theta\right)+\left(1-\cos^2\theta\right)^2$

$=8\cos^4\theta-8\cos^2\theta+1$

Q12di $xy=c^2$, $\frac{dy}{dx}=-\frac{c^2}{x^2}$

At $P\left(cp,\frac{c}{p}\right)$, $\frac{dy}{dx}=-\frac{1}{p^2}$, .: gradient of the normal $=p^2$

Equation of the normal: $y-\frac{c}{p}=p^2(x-cp)$, $\frac{y}{p}-\frac{c}{p^2}=p(x-cp)$

$\frac{y}{p}-\frac{c}{p^2}=px-cp^2$, $px-\frac{y}{p}=cp^2-\frac{c}{p^2}$, $px-\frac{y}{p}=c\left(p^2-\frac{1}{p^2}\right)$

Q12dii Multiply the last equation by $x\neq 0$:

$px^2-\frac{xy}{p}=c\left(p^2-\frac{1}{p^2}\right)x$, $px^2-c\left(p^2-\frac{1}{p^2}\right)x-\frac{c^2}{p}=0$

Sum of roots $=cp+cq=-\frac{-c\left(p^2-\frac{1}{p^2}\right)}{p}$

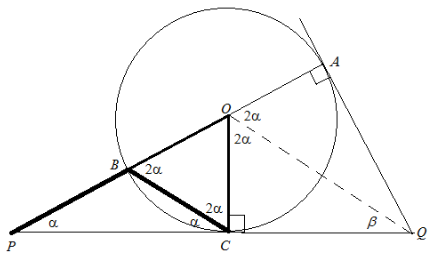
.: $q=\frac{\left(p^2-\frac{1}{p^2}\right)}{p}-p$, $q=-\frac{1}{p^3}$

Q13a $f(x)=x^x$, $\ln\left(f(x)\right)=x\ln x$, $\frac{d}{dx}\ln\left(f(x)\right)=\ln x+1$

To find the minimum of $\ln\left(f(x)\right)=x\ln x$, let $\ln x+1=0$, .: $x=\frac{1}{e}$

.: $f(x)=e^{x\ln x}$ has a minimum at $x=\frac{1}{e}$.

Q13b Let $\angle OPQ=\alpha$. The other relevant angles are shown in terms of α.



$\angle AOQ=\angle ABC$, .: $OQ // BC$, .: $\beta=\alpha$

.: ΔOPQ is isosceles, hence $OP=OQ$

Q13ci Vertical component: $T_1\times\frac{0.4}{0.5}-10M=0$, .: $T_1=12.5M$

Horizontal component: $T_1\times\frac{0.3}{0.5}+T_2=M\times 0.3\omega^2$

$T_2=M\times 0.3\omega^2-7.5M=0.3M\left(\omega^2-25\right)$

Q13cii $\frac{T_2}{T_1}>1$, $0.024\left(\omega^2-25\right)>1$, $\omega^2-25>\frac{125}{3}$

$\omega^2>\frac{200}{3}$, $\omega>\sqrt{\frac{600}{9}}$, $\omega>\frac{10\sqrt{6}}{3}$ radians per second

Q13di $p(x)=ax^3+bx^2+cx+d$, $p'(x)=3ax^2+2bx+c$

For $p(x)$ to have one x-intercept only, $p'(x)=3ax^2+2bx+c\neq 0$ for all x

.: the discriminant of $p'(x)$ must be less than zero

.: $(2b)^2-4(3a)c<0$, .: $b^2-3ac<0$

Q13dii $p'(x)=3ax^2+2bx+c$. If $b^2-3ac=0$, then

$p'(x)=3ax^2+2bx+\frac{b^2}{3a}=3a\left(x^2+\frac{2b}{3a}+\frac{b^2}{9a^2}\right)=3a\left(x+\frac{b}{3a}\right)^2$

$p(x)=a\left(x+\frac{b}{3a}\right)^3+\text{constant}$

If $p\left(-\frac{b}{3a}\right)=0$, then the constant is zero and $p(x)=a\left(x+\frac{b}{3a}\right)^3$

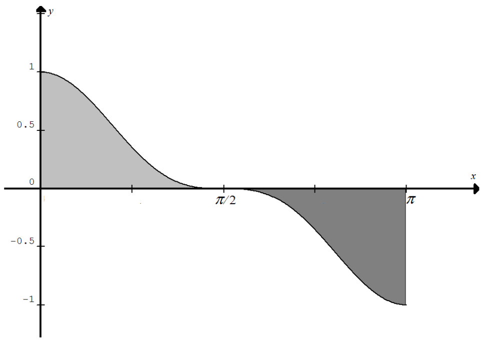
.: the multiplicity of the root $x=-\frac{b}{3a}$ is 3.

Q14ai $\int \sin x \sin^2 x\,dx = \int \sin x\left(1-\cos^2 x\right)dx = \int -\left(1-u^2\right)du$

$= \frac{u^3}{3} - u + c = \frac{1}{3}\cos^3 x - \cos x + c$

Q14aii The graph of $y = \cos^m x$ is shown below for odd $m > 0$ in the interval $[0, \pi]$. The two shaded regions are equal in area.

$\therefore \int_0^{\pi} \cos^m x\,dx = \int_0^{\frac{\pi}{2}} \cos^m x\,dx + \int_{\frac{\pi}{2}}^{\pi} \cos^m x\,dx = 0$



Q14aiii $V = \int_0^{\pi} 2\pi x \sin^3 x\,dx = 2\pi \int_0^{\pi} x \sin^3 x\,dx$

Integration by parts:

$u = x$, $dv = \sin^3 x\,dx$, $du = dx$, $v = \frac{1}{3}\cos^3 x - \cos x$

$V = 2\pi\left(\left[\frac{1}{3}x\cos^3 x - x\cos x\right]_0^{\pi} - \int_0^{\pi}\left(\frac{1}{3}\cos^3 x - \cos x\right)dx\right)$

Since $\int_0^{\pi}\left(\frac{1}{3}\cos^3 x - \cos x\right)dx = 0$ from part aii,

$\therefore V = 2\pi\left(-\frac{\pi}{3} + \pi\right) = \frac{4\pi^2}{3}$ unit^3

Q14bi $I_n = \int_0^1 \frac{x^n}{\left(x^2+1\right)^2}dx$, let $x = \tan\theta$, $\frac{dx}{d\theta} = \sec^2\theta$, $\frac{d\theta}{dx} = \frac{1}{\sec^2\theta}$

When $x = 0$, $\theta = 0$; when $x = 1$, $\theta = \frac{\pi}{4}$

$I_0 = \int_0^1 \frac{1}{\left(\tan^2\theta+1\right)^2}dx = \int_0^1 \frac{1}{\left(\sec^2\theta\right)^2}dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sec^2\theta}d\theta = \int_0^{\frac{\pi}{4}} \cos^2\theta\,d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}}\left(1+\cos 2\theta\right)d\theta = \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}} = \frac{\pi}{8} + \frac{1}{4}$

Q14bii

$I_0 + I_2 = \int_0^1 \frac{1}{\left(x^2+1\right)^2}dx + \int_0^1 \frac{x^2}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{1+x^2}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{1}{x^2+1}dx$

$= \left[\tan^{-1} x\right]_0^1 = \frac{\pi}{4}$

Q14biii $I_4 = \int_0^1 \frac{x^4}{\left(x^2+1\right)^2}dx = \int_0^1 \frac{\left(x^2+1\right)^2 - 2\left(x^2+1\right)+1}{\left(x^2+1\right)^2}dx$

$= \int_0^1 \left(1 - \frac{2}{x^2+1} + \frac{1}{\left(x^2+1\right)^2}\right)dx$

$= 1 - 2\left(I_0 + I_2\right) + I_0 = 1 - 2\times\frac{\pi}{4} + \frac{\pi}{8} + \frac{1}{4} = \frac{5}{4} - \frac{3\pi}{8}$

Q14c It is true that $(x+1)(x-1)^2 \ge 0$ for $x \ge 0$

$\therefore x^3 - x^2 - x + 1 \ge 0$, $x^3 + 1 \ge x^2 + x$

Complete the squares by adding $2x\sqrt{x}$ to both sides:

$x^3 + 2x\sqrt{x} + 1 \ge x^2 + 2x\sqrt{x} + x$, $\therefore \left(x\sqrt{x}+1\right)^2 \ge \left(x + \sqrt{x}\right)^2$

Since $x\sqrt{x} + 1 \ge 1$ for $x \ge 0$, $\therefore x\sqrt{x} + 1 \ge x + \sqrt{x}$ for $x \ge 0$

Q15a Let $x = \alpha^2$, $x = \beta^2$ and $x = \gamma^2$ be the roots of the required cubic equation, $\therefore \alpha = \pm\sqrt{x}$

Since α is a root of $x^3 - 3x + 1 = 0$, $\therefore \left(\pm\sqrt{x}\right)^3 - 3\left(\pm\sqrt{x}\right) + 1 = 0$

$\left(\sqrt{x}\right)^3 - 3\sqrt{x} = \mp 1$, $\sqrt{x}(x-3) = \mp 1$

Squaring both sides: $x\left(x^2 - 6x + 9\right) = 1$, $x^3 - 6x^2 + 9x - 1 = 0$, the required cubic equation.

Q15bi $a = \frac{1}{2}\frac{dv^2}{dx} = -\frac{\mu^2}{x^2}$, $\frac{v^2}{2\mu^2} = \int \frac{-1}{x^2}dx = \frac{1}{x} + c$

At $x = b$, $v = 0$, $\therefore c = -\frac{1}{b}$, $\therefore \frac{v^2}{2\mu^2} = \frac{1}{x} - \frac{1}{b}$, $v^2 = 2\mu^2\left(\frac{1}{x} - \frac{1}{b}\right)$

For $0 < x \le b$, the particle moves towards O,

$\therefore v = \frac{dx}{dt} = -\mu\sqrt{2}\sqrt{\frac{b-x}{bx}}$

Q15bii $x = b\cos^2\theta$, $\frac{dx}{d\theta} = -2b\sin\theta\cos\theta$

At $t = 0$, $x = b$, $\therefore \theta = 0$; at time t, $x = d$, $\therefore \theta = \cos^{-1}\sqrt{\frac{d}{b}}$

$\therefore \frac{dx}{dt} = -\mu\sqrt{2}\sqrt{\frac{b - b\cos^2\theta}{b^2\cos^2\theta}} = -\frac{\mu\sqrt{2}}{\sqrt{b}}\frac{\sin\theta}{\cos\theta}$

$\frac{dt}{dx}\times\frac{dx}{d\theta} = \frac{dt}{d\theta} = \frac{\sqrt{b}\cos\theta}{\mu\sqrt{2}\sin\theta}\times 2b\sin\theta\cos\theta = \frac{b\sqrt{2b}}{\mu}\cos^2\theta$

$\therefore t = \frac{b\sqrt{2b}}{\mu}\int_0^{\cos^{-1}\sqrt{\frac{d}{b}}} \cos^2\theta\,d\theta$

Q15biii Given $t = \frac{1}{\mu}\sqrt{\frac{b}{2}}\left(\sqrt{bd - d^2} + b\cos^{-1}\sqrt{\frac{d}{b}}\right)$

As $d \to 0$, $t \to \frac{1}{\mu}\sqrt{\frac{b}{2}}b\cos^{-1}0 = \frac{1}{\mu}\sqrt{\frac{b}{2}}b\times\frac{\pi}{2} = \frac{\pi}{\mu}\sqrt{\left(\frac{b}{2}\right)^3}$

Q15ci Let $\dfrac{3!}{x(x+1)(x+2)(x+3)}=\dfrac{a_0}{x}+\dfrac{a_1}{x+1}+\dfrac{a_2}{x+2}+\dfrac{a_3}{x+3}$

$$\frac{3!}{x(x+1)(x+2)(x+3)}=\big(a_0(x+1)(x+2)(x+3)+a_1x(x+2)(x+3)$$
$$+a_2x(x+1)(x+3)+a_3x(x+1)(x+2)\big)/\,x(x+1)(x+2)(x+3)$$

Let $x\to 0,-1,-2,-3$ to obtain

$a_0=\dfrac{3!}{(x+1)(x+2)(x+3)}=\dfrac{3!}{(1)(2)(3)}=1$, $a_1=\dfrac{3!}{(-1)(1)(2)}=-3$,

$a_2=\dfrac{3!}{(-2)(-1)(1)}=3$ and $a_3=\dfrac{3!}{(-3)(-2)(-1)}=-1$ respectively.

$\therefore\ \dfrac{3!}{x(x+1)(x+2)(x+3)}=\dfrac{1}{x}-\dfrac{3}{x+1}+\dfrac{3}{x+2}-\dfrac{1}{x+3}$

Q15cii

$$\frac{n!}{x(x+1)(x+2)\cdots(x+n)}=\frac{a_0}{x}+\frac{a_1}{x+1}+\cdots+\frac{a_k}{x+k}+\cdots+\frac{a_n}{x+n}$$

Let $x\to -k$,

$a_k=\dfrac{n!}{x(x+1)(x+2)\cdots(x+n)}$ (without $(x+k)$ in the denominator)

$=\dfrac{n!}{-k(-k+1)(-k+2)\cdots(-k+n)}$ (negative when k is odd)

$=\dfrac{(-1)^k n!}{k(k-1)(k-2)\cdots(1)(1)(2)\cdots(n-k)}=\dfrac{(-1)^k n!}{k!(n-k)!}=(-1)^k\dbinom{n}{k}$

Q15ciii $\dfrac{n!}{x(x+1)(x+2)\cdots(x+n)}=\dfrac{a_0}{x}+\dfrac{a_1}{x+1}+\cdots+\dfrac{a_n}{x+n}$

Let $x=1$,

$$\frac{a_0}{1}+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1}=\frac{n!}{(n+1)!}=\frac{1}{n+1}$$

$$1\binom{n}{0}-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+\frac{(-1)^n}{n+1}\binom{n}{n}=\frac{1}{n+1}$$

$$1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+\frac{(-1)^n}{n+1}=\frac{1}{n+1}$$

$$\therefore\ \lim_{n\to\infty}\left(1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\frac{1}{4}\binom{n}{3}+\cdots+\frac{(-1)^n}{n+1}\right)=\lim_{n\to\infty}\frac{1}{n+1}=0$$

Q16ai $|z|=1$ and $|w|=1$

$1+z+w=0$, $(1+\cos\theta+\cos\alpha)+(\sin\theta+\sin\alpha)i=0$

$\therefore\ \sin\theta+\sin\alpha=0$ and $\cos\theta+\cos\alpha=-1$

$\therefore\ \sin\theta=-\sin\alpha=\sin(-\alpha)$ or $\sin(\pi-\alpha)$

$\therefore\ \theta=-\alpha$ Note: $(\pi-\alpha)$ does not satisfy $\cos\theta+\cos\alpha=-1$.

$\therefore\ \cos(-\alpha)+\cos\alpha=-1$, $\cos\alpha+\cos\alpha=-1$, $\cos\alpha=-\dfrac{1}{2}$

$\therefore\ \alpha=\pm\dfrac{2\pi}{3}$ and $\theta=\mp\dfrac{2\pi}{3}$

$\therefore$ 1, z and w are on the unit circle centred at O, and separated by $\dfrac{2\pi}{3}$.

$\therefore$ 1, z and w form the vertices of an equilateral triangle.

Q16aii Given $1+z+w=0$

Let $z_1=2iz$ and $z_2=2iw$,

$\therefore\ 2i+z_1+z_2=i\,2+i(2z)+i(2w)=2i(1+z+w)=0$

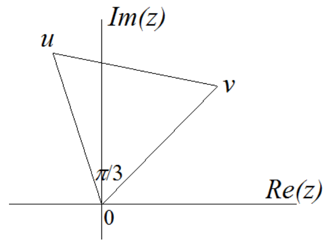
$\therefore\ 2i=2i\times 1$ is $2|1|$ and rotation of 1 by $\dfrac{\pi}{2}$ anticlockwise about O.

$z_1=2iz$ is $2|z|$ and rotation of z by $\dfrac{\pi}{2}$ anticlockwise about O.

$z_2=2iw$ is $2|w|$ and rotation of w by $\dfrac{\pi}{2}$ anticlockwise about O.

$\therefore\ 2i$, z_1 and z_2 form the vertices of an equilateral triangle.

Q16bi



$u=v\,\text{cis}\left(\dfrac{\pi}{3}\right)$, $u^3=v^3\text{cis}\pi=-v^3$

$\therefore\ u^3+v^3=(u+v)(u^2-uv+v^2)=0$

Since $u+v\neq 0$, $\therefore\ u^2-uv+v^2=0$, $u^2+v^2=uv$

Q16bii $v=1$, $u=\text{cis}\left(\dfrac{\pi}{3}\right)=\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

Q16ci If Tom and any one of the remaining $(n-1)$ people have each other's hat, and the other $(n-2)$ people have other people's hats, the number of such derangements $=(n-1)D(n-2)$

Q16cii Tom or any person can pick another person's hat and the remaining $(n-1)$ people have other people's hats, the number of such derangements $=(n)D(n-1)$. However, this count includes the situation (as in part i) that the person swaps hats with one of the remaining $(n-1)$ people in $D(n-1)$ ways.

$\therefore$ number of such derangements

$=nD(n-1)-D(n-1)=(n-1)D(n-1)$

$\therefore\ D(n)=(n-1)D(n-1)+(n-1)D(n-2)$

$=(n-1)[D(n-1)+D(n-2)]$

Q16ciii $D(n)=(n-1)[D(n-1)+D(n-2)]$

$D(n)=nD(n-1)-D(n-1)+(n-1)D(n-2)$

$\therefore\ D(n)-nD(n-1)=-[D(n-1)-(n-1)D(n-2)]$

Q16civ

$D(2)-2D(1)=1$

$D(3)-3D(2)=-[D(2)-2D(1)]=-1$

$D(4)-4D(3)=-[D(3)-3D(2)]=1$

$D(5)-5D(4)=-[D(4)-4D(3)]=-1$ etc

$\therefore\ D(n)-nD(n-1)=(-1)^n$ for $n>1$

Q16cv Given $D(1)=0$

$D(1)=1!\sum_{r=0}^{1}\frac{(-1)^r}{r!}=1-1=0$, true for $n=1$

Assume that $D(k)=k!\sum_{r=0}^{k}\frac{(-1)^r}{r!}$

From part iv, $D(n)=nD(n-1)+(-1)^n$

$\therefore\ D(k+1)=(k+1)D(k)+(-1)^{k+1}=(k+1)k!\sum_{r=0}^{k}\frac{(-1)^r}{r!}+(-1)^{k+1}$

$=(k+1)!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^k}{k!}\right)+(-1)^{k+1}$

$=(k+1)!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^k}{k!}+\frac{(-1)^{k+1}}{(k+1)!}\right)$

$\therefore\ D(k+1)=(k+1)!\sum_{r=0}^{k+1}\frac{(-1)^r}{r!}$

$\therefore\ D(n)=n!\sum_{r=0}^{n}\frac{(-1)^r}{r!}$ for all integers $n\geq 1$

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